NCERT Solutions for Class-XI Maths

Chapter-10 Exercise-10.2

NCERT Math Class 11

1. Write the equations for the x and y-axes.

1. The y-coordinate of every point on the x-axis is 0.

Therefore, the equation of the x-axis is $y = 0$.

The x-coordinate of every point on the y-axis is 0.

Therefore, the equation of the y-axis is $y = 0$.

2. Passing through the point (– 4, 3) with slope $\frac{1}{2}$

2. Given point = (-4, 3) and slope, m = ½

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), if and only if, its coordinates satisfy the equation $y - y_0 = m(x - x_0)$

$\therefore y - 3 = \frac{1}{2}(x - (-4))$

$\Rightarrow y - 3 = \frac{1}{2}(x + 4)$

$\Rightarrow 2(y - 3) = x + 4$

$\Rightarrow 2y - 6 = x + 4$

$\Rightarrow x + 4 - (2y - 6) = 0$

$\Rightarrow x + 4 - 2y + 6 = 0$

$\Rightarrow x - 2y + 10 = 0$

The equation of the line is $x - 2y + 10 = 0$.

3. Find the equation of the line which passes though $(0,0)$ with slope m.

3. We know that the equation of the line passing through point (x_0, y_0), whose slope is m, is

$(y - y_0) = m(x - x_0)$.

Thus, the equation of the line passing through point $(0,0)$, whose slope is m, is

$(y - 0) = m(x - 0)$

i.e., $y = mx$

4. Find the equation of the line which passes though $\left(2, 2\sqrt{3}\right)$ and is inclined with the x-axis at an angle of 75^{o}.

4. The slope of the line that inclines with the x-axis at an angle of 75^{o} is m

$= m = \tan 75^{o}$

We know that the equation of the line passing through point $\left(x_0, y_0\right)$

whose slope is m, is

$\left(y - y_0\right) = m\left(x - x_0\right)$.

Thus, if a line passes though $\left(2, 2\sqrt{3}\right)$ and inclines with the x-axis at an angle of 75^{o},

then the equation of the line is given as

$$\left(y - 2\sqrt{3}\right) = \frac{\sqrt{3}+1}{\sqrt{3}-1}(x-2)$$

$$\left(y - 2\sqrt{3}\right)\left(\sqrt{3}-1\right) = \left(\sqrt{3}+1\right)(x-2)$$

$$y\left(\sqrt{3}-1\right) - 2\sqrt{3}\left(\sqrt{3}-1\right) = x\left(\sqrt{3}+1\right) - 2\left(\sqrt{3}+1\right)$$

$$\left(\sqrt{3}+1\right)x - \left(\sqrt{3}-1\right)y = 2\sqrt{3} + 2 - 6 + 2\sqrt{3}$$

$$\left(\sqrt{3}+1\right)x - \left(\sqrt{3}-1\right)y = 4\sqrt{3} - 4$$

i.e., $\left(\sqrt{3}+1\right)x - \left(\sqrt{3}-1\right)y = 4\left(\sqrt{3}-1\right)$

5. Find the equation of the line which intersects the x-axis at a distance of 3 units to the left of origin with slope -2.

5. It is known that if a line with slope m makes x-intercept d, then the equation of the line is given as $y = m(x-d)$

For the line intersecting the x-axis at a distance of 3 units to the left of the origin, $d = -3$.

The slope of the line is given as $m = -2$

Thus, the required equation of the given line is $y = -2\left[x - (-3)\right] y = -2x - 6$

i.e., $2x + y + 6 = 0$

6. Intersecting the y-axis at a distance of 2 units above the origin and making an angle of 30^{o} with positive direction of the x-axis.

6. We know that the point (x, y) on the line with slope m and y-intercept c lies on the line if and only if y = mx + c.

If distance is 2 units above the origin, c = +2

Given $\theta = 30°$

We know that slope, $m = \tan\theta$

$\therefore m = \tan 30° = (1/\sqrt{3})$

$\therefore y = (1/\sqrt{3})x + 2$

$\Rightarrow y = (x + 2\sqrt{3}) / \sqrt{3}$

$\Rightarrow \sqrt{3}\, y = x + 2\sqrt{3}$

$\Rightarrow x - \sqrt{3}\, y + 2\sqrt{3} = 0$

The equation of the line is $x - \sqrt{3}y + 2\sqrt{3} = 0$.

7. Find the equation of the line which passes through the points $(-1,1)$ and $(2,-4)$.

7. It is known that the equation of the line passing through points (x_1, y_1) and (x_2, y_2) is

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1).$$

Therefore, the equation of the line passing through the points $(-1,1)$ and $(2,-4)$ is

$$(y-1) = \frac{-4-1}{2+1}(x+1)$$

$$(y-1) = \frac{-5}{3}(x+1)$$

$$3(y-1) = -5(x+1)$$

$$3y - 3 = -5x - 5$$

$$\text{i.e.,} 5x + 3y + 2 = 0$$

8. Perpendicular distance from the origin is 5 units and the angle made by the perpendicular with the positive x-axis is 30^{o}.

8. We know that the equation of the line having normal distance p from the origin and angle ω which the normal makes with the positive direction of x-axis is given by $x \cos\omega + y \sin\omega = p$.

Given $p = 5$ and $\omega = 30°$

Substituting the values in the equation, we get

$\Rightarrow x \cos 30° + y \sin 30° = 5$

$\Rightarrow x(\sqrt{3} / 2) + y(½) = 5$

$\Rightarrow \sqrt{3}\, x + y = 5(2) = 10$

$\Rightarrow \sqrt{3}\, x + y – 10 = 0$

The equation of the line is $\sqrt{3}\, x + y – 10 = 0$.

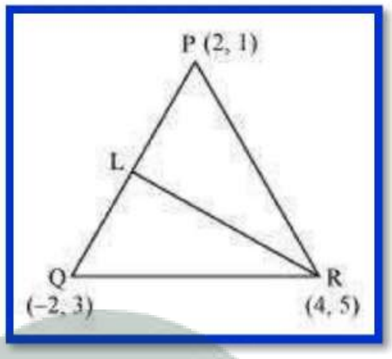
9. The vertices of $\triangle PQR$ are $P(2,1), Q(-2,3)$ and $R(4,5)$. Find equation of the median through the vertex R.

9. It is given that the vertices of $\triangle PQR$ are $P(2,1), Q(-2,3)$, and $R(4,5)$.

Let RL be the median through vertex R.

Accordingly, L is the mid-point of PQ.

By mid-point formula, the coordinates of point L are given by $\left(\frac{2-2}{2}, \frac{1+3}{2}\right) = (0,2)$



It is known that the equation of the line passing through points (x_1, y_1) and (x_2, y_2) is $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$.

Therefore, the equation of RL can be determined by substituting $(x_1, y_1) = (4,5)$ and $(x_2, y_2) = (0,2)$

Hence, $y - 5 = \frac{2-5}{0-4}(x-4)$

$\Rightarrow y - 5 = \frac{-3}{-4}(x-4)$

$\Rightarrow 4(y-5) = 3(x-4)$

$\Rightarrow 4y - 20 = 3x - 12$

$\Rightarrow 3x - 4y + 8 = 0$

Thus, the required equation of the median through vertex R is $3x - 4y + 8 = 0$.

10. Find the equation of the line passing through (–3, 5) and perpendicular to the line through the points (2, 5) and (–3, 6).

10. Given points are (2, 5) and (-3, 6).

We know that slope, m = $\frac{y_2 - y_1}{x_2 - x_1}$

$\therefore m = \frac{6-5}{(-3)-2} = \frac{1}{-5} = \frac{-1}{5}$

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.

$\therefore\ m = (-1/m) = \frac{-1}{\frac{-1}{5}} = 5$

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), if and only if, its coordinates satisfy the equation $y - y_0 = m(x - x_0)$

$\therefore y - 5 = 5(x - (-3))$

$\Rightarrow y - 5 = 5x + 15$

$\Rightarrow 5x + 15 - y + 5 = 0$

$\Rightarrow 5x - y + 20 = 0$

Ans. The equation of the line is 5x – y + 20 = 0.

11. A line perpendicular to the line segment joining the points $(1,0)$ and $(2,3)$ divides it in the ratio $1:n$. Find the equation of the line.

11. According to the section formula, the coordinates of the point that divides the line segment joining the points $(1,0)$ and $(2,3)$ in the ratio $1:n$ is given by

$$\left(\frac{n(1)+1(2)}{1+n}, \frac{n(0)+1(3)}{1+n}\right) = \left(\frac{n+2}{n+1}, \frac{3}{n+1}\right)$$

The slope of the line joining the points $(1,0)$ and $(2,3)$ is

$$m = \frac{3-0}{2-1} = 3$$

We know that two non-vertical lines are perpendicular to each other if and only if their slopes are negative reciprocals of each other.

Therefore, slope of the line that is perpendicular to the line joining the points

$(1,0)$ and $(2,3) = -\frac{1}{m} = -\frac{1}{3}$

Now, the equation of the line passing through $\left(\frac{n+2}{n+1}, \frac{3}{n+1}\right)$ and whose slope is $-\frac{1}{3}$ is given by

$$\left(y - \frac{3}{n+1}\right) = \frac{-1}{3}\left(x - \frac{n+2}{n+1}\right)$$

$$\Rightarrow 3\left[(n+1)y - 3\right] = -\left[x(n+1) - (n+2)\right]$$

$$\Rightarrow 3(n+1)y - 9 = -(n+1)x + n + 2$$

$\Rightarrow (1+n)x+3(1+n)y=n+11$

12. Find the equation of a line that cuts off equal intercepts on the coordinate axes and passes through the point (2, 3).

12. We know that equation of the line making intercepts a and b on x-and y-axis, respectively, is $\frac{x}{a}+\frac{y}{b}=1$

Given that the line cuts off equal intercepts on the coordinate axes i.e. a = b.

So, $\frac{x}{a}+\frac{y}{a}=1$

$\Rightarrow x + y = a \ldots (1)$

Given point = (2, 3)

$\therefore 2 + 3 = a$

$\Rightarrow a = 5$

Substituting a value in (1),

$\Rightarrow x + y = 5$

$\Rightarrow x + y - 5 = 0$

Ans. The equation of the line is $x + y - 5 = 0$.

13. Find equation of the line passing through the point $(2,2)$ and cutting off intercepts on the axes whose sum is 9.

13. The equation of a line in the intercept form is

$$\frac{x}{a}+\frac{y}{b}=1$$

Here, a and b are the intercepts on x and y axes respectively.

It is given that $a+b=9 \Rightarrow b=9-a\ldots$ (ii) From equations (i) and (ii), we obtain

$$\frac{x}{a}+\frac{y}{9-a}=1$$

It is given that the line passes through point $(2,2)$. Therefore, equation (iii) reduces to

$$\frac{2}{a}+\frac{2}{9-a}=1$$

$$\Rightarrow 2\left(\frac{1}{a}+\frac{1}{9-a}\right)=1$$

$$\Rightarrow 2\left(\frac{9-a+a}{a(9-a)}\right)=1$$

$$\Rightarrow \frac{18}{9a-a^2}=1$$

$$\Rightarrow 18=9a-a^2$$

$\Rightarrow a^2 - 9a + 18 = 0$

$\Rightarrow a^2 - 6a - 3a + 18 = 0$

$\Rightarrow a(a-6) - 3(a-6) = 0$

$\Rightarrow (a-6)(a-3) = 0$

$\Rightarrow a = 6$ or $a = 3$

If $a = 6$ and $b = 9 - 6 = 3$, then the equation of the line is

$\frac{x}{6} + \frac{y}{3} = 1 \Rightarrow x + 2y - 6 = 0$

If $a = 3$ and $b = 9 - 3 = 6$, then the equation of the line is $\frac{x}{3} + \frac{y}{6} = 1 \Rightarrow 2x + y - 6 = 0$

14. Find equation of the line through the point (0, 2) making an angle $\frac{2\pi}{3}$ with the positive x-axis. Also, find the equation of line parallel to it and crossing the y-axis at a distance of 2 units below the origin.

14. Given point = (0, 2) and $\theta = 2\pi/3$

We know that $m = \tan \theta$

$\therefore m = \tan (2\pi/3) = -\sqrt{3}$

We know that the point (x, y) lies on the line with slope m through the fixed point (x_0, y_0), if and only if, its coordinates satisfy the equation $y - y_0 = m (x - x_0)$

$\therefore y - 2 = -\sqrt{3}\,(x - 0)$

$\Rightarrow y - 2 = -\sqrt{3}\,x$

$\Rightarrow \sqrt{3}\,x + y - 2 = 0$

Given, equation of line parallel to above obtained equation crosses the y-axis at a distance of 2 units below the origin.

So, the point = (0, -2) and $m = -\sqrt{3}$

From point slope form equation,

$\Rightarrow y - (-2) = -\sqrt{3}\,(x - 0)$

$\Rightarrow y + 2 = -\sqrt{3}\,x$

$\Rightarrow \sqrt{3}\,x + y + 2 = 0$

Ans. The equation of line is $\sqrt{3}\,x + y - 2 = 0$ and the line parallel to it is $\sqrt{3}\,x + y + 2 = 0$.

15. The perpendicular from the origin to a line meets it at the point $(-2, 9)$, find the equation of the line.

15. The slope of the line joining the origin $(0,0)$ and point $(-2,9)$ is $m_1 = \frac{9-0}{-2-0} = -\frac{9}{2}$

Accordingly, the slope of the line perpendicular to the line joining the origin and point $(-2,9)$ is $m_2 = -\frac{1}{m_1} = -\frac{1}{\left(-\frac{9}{2}\right)} = \frac{2}{9}$

Now, the equation of the line passing through point $(-2,9)$ and having a slope m_2 is

$$(y-9) = \frac{2}{9}(x+2)$$

$$9y - 81 = 2x + 4$$

$$\text{i.e.,} 2x - 9y + 85 = 0$$

16. The length L (in centimetre) of a copper rod is a linear function of its Celsius temperature C. In an experiment, if L = 124.942 when C = 20 and L= 125.134 when C = 110, express L in terms of C.

16. Assuming L along X-axis and C along Y-axis, we have two points (124.942, 20) and (125.134, 110) in XY-plane.

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$$\therefore\ C - 20 = \frac{110-20}{125.134 - 124.942}(L - 124.942)$$

$$\Rightarrow\ C - 20 = \frac{90}{0.192}(L - 124.942)$$

$$\Rightarrow 0.192\ (C - 20) = 90\ (L - 124.942)$$

$$\Rightarrow L = \frac{0.192}{90}(C-20) + 124.942$$

The required relation is $L = \frac{0.192}{90}(C-20) + 124.942$.

17. The owner of a milk store finds that, he can sell 980 litres of milk each week at Rs 14 /litre and 1220 litres of milk each week at Rs 16/ litre. Assuming a linear relationship between selling price and demand, how many litres could he sell weekly at Rs 17/ /litre?

17. The relationship between selling price and demand is linear.

Assuming selling price per litre along the x-axis and demand along the y-axis, we have two points i.e., $(14,980)$ and $(16,1220)$ in the XY plane that satisfy the linear relationship between selling price and demand.

Therefore, the linear relationship between selling price per litre and demand is the equation of the line passing through points $(14,980)$ and $(16,1220)$.

$y - 980 = \frac{1220 - 980}{16 - 14}(x - 14)$

$y - 980 = \frac{240}{2}(x - 14)$

$y - 980 = 120(x - 14)$

i.e., $y = 120(x - 14) + 980$

When $x =$ Rs $17/$ litre,

$y = 120(17 - 14) + 980$

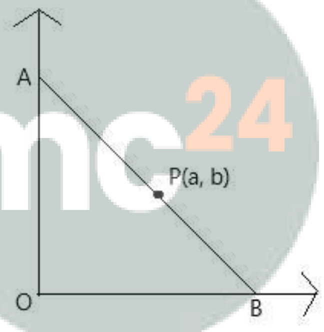
$\Rightarrow y = 120 \times 3 + 980 = 360 + 980 = 1340$

Thus, the owner of the milk store could sell 1340 litres of milk weekly at Rs $17/$ litre.

18. P (a, b) is the mid-point of a line segment between axes. Show that equation of the line is $\frac{x}{a} + \frac{y}{b} = 2$

18. Let AB be a line segment whose midpoint is P (a, b).

Let the coordinates of A and B be (0, y) and (x, 0) respectively.



We know that the midpoint formula is given by $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Since P is the midpoint of (a, b),

$\Rightarrow \left(\frac{0 + x}{2}, \frac{y + 0}{2}\right) = (a,\ b)$

$\Rightarrow \left(\frac{x}{2}, \frac{y}{2}\right) = (a,\ b)$

$\Rightarrow$ a = x/2 and b = y/2

$\Rightarrow$ x = 2a and y = 2b

$\therefore$ A = (0, 2b) and B = (2a, 0)

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\therefore\ y-2b=\frac{0-2b}{2a-0}(x-0)$

$\Rightarrow\ y-2b=\frac{-2b}{2a}(x)$

$\Rightarrow\ y-2b=\frac{-b}{a}(x)$

$\Rightarrow a(y-2b)=-bx$

$\Rightarrow ay-2ab=-bx$

$\Rightarrow bx+ay=2ab$

Dividing by ab on both sides,

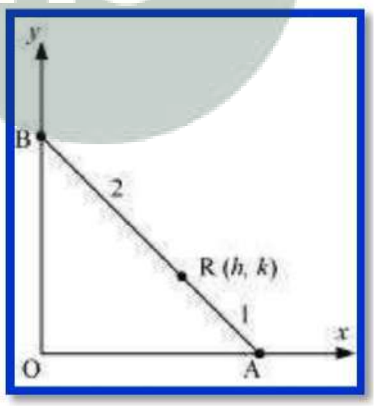
$\Rightarrow\ \frac{bx}{ab}+\frac{ay}{ab}=\frac{2ab}{ab}$

$\Rightarrow\ \frac{x}{a}+\frac{y}{b}=2$

The equation of line is $\frac{x}{a}+\frac{y}{b}=2$.

19. Point $R(h,k)$ divides a line segment between the axes in the ratio $1:2$. Find equation of the line.

19. Let AB be the line segment between the axes such that point $R(h,k)$ divides AB in the ratio 1: 2.



Let the respective coordinates of A and B be $(x,0)$ and $(0,y)$.

Since point $R(h,k)$ divides AB in the ratio $1:2$, according to the section formula,

$(h,k)=\left(\frac{1\times 0+2\times x}{1+2},\frac{1\times y+2\times 0}{1+2}\right)$

$\Rightarrow(h,k)=\left(\frac{2x}{3},\frac{y}{3}\right)$

$\Rightarrow h = \frac{2x}{3}$ and $k = \frac{y}{3}$

$\Rightarrow x = \frac{3h}{2}$ and $y = 3k$

Therefore, the respective coordinates of A and B are $\left(\frac{3h}{2}, 0\right)$ and $(0, 3k)$.

Now, the equation of line AB passing through points $\left(\frac{3h}{2}, 0\right)$ and $(0, 3k)$ is

$$(y - 0) = \frac{3k - 0}{0 - \frac{3h}{2}}\left(x - \frac{3h}{2}\right)$$

$$y = -\frac{2k}{h}\left(x - \frac{3h}{2}\right)$$

$$hy = -2kx + 3hk$$

i.e., $2kx + hy = 3hk$

Thus, the required equation of the line is $2kx + hy = 3hk$

20. By using the concept of equation of a line, prove that the three points (3, 0), (– 2, – 2) and (8, 2) are collinear.

20. If we have to show that the given three points (3, 0), (– 2, – 2) and (8, 2) are collinear, we have to show that the line passing through the points (3, 0) and (– 2, – 2) also passes through the point (8, 2).

We know that the equation of the line passing through the points (x_1, y_1) and (x_2, y_2) is given by $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

$\therefore\ y - 0 = \frac{-2 - 0}{-2 - 3}(x - 3)$

$\Rightarrow\ y = \frac{-2}{-5}(x - 3)$

⇒ -5y = -2 (x – 3)

⇒ -5y = -2x + 6

⇒ 2x – 5y = 6

If 2x – 5y = 6 passes through (8, 2),

LHS = 2x – 5y = 2(8) – 5(2) = 16 – 10 = 6 = RHS

The line passing through the points (3, 0) and (– 2, – 2) also passes through the point (8, 2).

Hence proved.

The given three points are collinear.