NCERT Solutions for Class-XI Maths

Chapter-8 Exercise-Miscellaneous

NCERT Math Class 11

1. Find a, b and n in the expansion of $(a+b)^n$ if the first three terms of the expansion are 729,7290 and 30375, respectively.

1. It is known that $(r+1)^{th}$ term, $\left(T_{r+1}\right)$, in the binomial expansion of $(a+b)^n$ is given by $T_{r+1} = {}^nC_r a^{n-r} b^r$.

The first three terms of the expansion are given as 729,7290 , and 30375 respectively.

Therefore, we obtain

$T_1 = {}^nC_0 a^{n-0}\ b^0 = a^n = 729$

$T_2 = {}^nC_1 a^{n-1} b^1 = na^{n-1}\ b = 7290$

$T_3 = {}^nC_2 a^{n-2}\ b^2 = \frac{n(n-1)}{2} a^{n-2}\ b^2 = 30375$

Dividing (2) by (1), we obtain

$$\frac{na^{n-1}b}{a^n} = \frac{7290}{729}$$

$$\Rightarrow \frac{nb}{a} = 10$$

Dividing (3) by (2), we obtain

$$\frac{n(n-1)a^{n-2}b^2}{2na^{n-1}b} = \frac{30375}{7290}$$

$$\Rightarrow \frac{(n-1)b}{2a} = \frac{30375}{7290}$$

$$\Rightarrow \frac{(n-1)b}{a} = \frac{30375\times 2}{7290} = \frac{25}{3}$$

$$\Rightarrow \frac{nb}{a} - \frac{b}{a} = \frac{25}{3}$$

$$\Rightarrow 10 - \frac{b}{a} = \frac{25}{3}$$

$$\Rightarrow \frac{b}{a} = 10 - \frac{25}{3} = \frac{5}{3}$$

From (4) and (5), we obtain

$$n \cdot \frac{5}{3} = 10$$

$$\Rightarrow n = 6$$

Substituting $n = 6$ in equation (1), we obtain a^6

$= 729$

$$\Rightarrow a = \sqrt[6]{729} = 3$$

From (5), we obtain

$$\frac{b}{3} = \frac{5}{3} \Rightarrow b = 5$$

Thus, $a = 3, b = 5$, and $n = 6$.

2. Find a if the coefficients of x^2 and x^3 in the expansion of $(3 + ax)^9$ are equal.

2. We know that-

General term of expansion $(a+b)^n$ is

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r$$

For $(3+ax)^9$

Putting a = 3, b = ax & n = 9

General term of $(3+ax)^9$ is

$$T_{r+1} = \binom{9}{r} 3^{n-r} (ax)^r$$

$$T_{r+1} = \binom{9}{r} 3^{n-r} a^r x^r$$

Since we need to find the coefficients of x^2 and x^3, therefore

For r = 2

$$T_{2+1} = \binom{9}{2} 3^{n-2} a^2 x^2$$

Thus, the coefficient of $x^2 = \binom{9}{2} 3^{n-2} a^2$

For r = 3

$$T_{3+1} = \binom{9}{3} 3^{n-3} a^3 x^3$$

Thus, the coefficient of $x^3 = \binom{9}{3} 3^{n-3} a^3$

Given that-

Coefficient of x^2 = Coefficient of x^3

$$\Rightarrow \binom{9}{2} 3^{n-2} a^2 = \binom{9}{3} 3^{n-3} a^3$$

$$\Rightarrow \frac{9!}{2!(9-2)!} \times 3^{n-2} a^2 = \frac{9!}{3!(9-3)!} \times 3^{n-3} a^3$$

$$\Rightarrow \frac{3^{n-2} a^2}{3^{n-3} a^3} = \frac{2!(9-2)!}{3!(9-3)!}$$

$$\Rightarrow \frac{3^{(n-2)-(n-3)}}{a} = \frac{2!7!}{3!6!}$$

$$\Rightarrow \frac{3}{a} = \frac{7}{3}$$

$\therefore$ a = 9/7

Hence, a = 9/7

3. Find the coefficient of x^5 in the product $(1+2x)^6(1-x)^7$ using binomial theorem.

3. Using Binomial Theorem, the expressions, $(1+2x)^6$ and $(1-x)^7$, can be expanded as

$$(1+2x)^6 = {}^6C_0 + {}^6C_1(2x) + {}^6C_2(2x)^2 + {}^6C_3(2x)^3 + {}^6C_+(2x)^4 + {}^6C_5(2x)^5 + {}^6C_6(2x)^6$$

$$= 1 + 6(2x) + 15(2x)^2 + 20(2x)^3 + 15(2x)^4 + 6(2x)^5 + (2x)^6$$

$$= 1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + 64x^6$$

$$(1-x)^7 = {}^7C_0 - {}^7C_1(x) + {}^7C_2(x)^2 - {}^7C_3(x)^3 + {}^7C_4(x)^4 - {}^7C_5(x)^5 + {}^7C_6(x)^6 - {}^7C_7(x)^7$$

$$= 1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7$$

$$\therefore (1+2x)^6(1-x)^7$$

$$= \left(1 + 12x + 60x^2 + 160x^3 + 240x^4 + 192x^5 + 64x^6\right)$$

$$\left(1 - 7x + 21x^2 - 35x^3 + 35x^4 - 21x^5 + 7x^6 - x^7\right)$$

The complete multiplication of the two brackets is not required to be carried out. Only those terms, which involve x^5, are required.

The terms containing x^5 are

$$1\left(-21x^5\right) + (12x)\left(35x^4\right) + \left(60x^2\right)\left(-35x^3\right) + \left(160x^3\right)\left(21x^2\right) + \left(240x^4\right)(-7x) + \left(192x^5\right)(1)$$

$$= 171x^5$$

Thus, the coefficient of x^5 in the given product is 171 .

4. If a and b are distinct integers, prove that a – b is a factor of $a^n - b^n$, whenever n is a positive integer. [Hint write $a^n = (a - b + b)^n$ and expand]

4. We can write a^n as

$a^n = (a-b+b)^n$

We know that-

$$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n-1}a^1 b^{n-1} + \binom{n}{n}a^0 b^n$$

putting a = b & b = a-b, we get-

$$(b+(a-b))^n = \binom{n}{0}b^n(a-b)^0 + \binom{n}{1}b^{n-1}(a-b)^1 + \binom{n}{2}b^{n-2}(a-b)^2$$

$$+\ldots+\binom{n}{n-1}b^1(a-b)^{n-1} + \binom{n}{n}b^0(a-b)^n$$

$$a^n = \binom{n}{0}b^n \times 1 + \binom{n}{1}b^{n-1}(a-b) + \binom{n}{2}b^{n-2}(a-b)^2$$

$$+\ldots+\binom{n}{n-1}b(a-b)^{n-1} + \binom{n}{n}b^0(a-b)^n$$

$$a^n = b^n + \binom{n}{1}b^{n-1}(a-b) + \binom{n}{2}b^{n-2}(a-b)^2$$

$$+\ldots+\binom{n}{n-1}b(a-b)^{n-1} + (a-b)^n$$

$$a^n - b^n = \binom{n}{1}b^{n-1}(a-b) + \binom{n}{2}b^{n-2}(a-b)^2$$

$$+\ldots+\binom{n}{n-1}b(a-b)^{n-1} + (a-b)^n$$

$$a^n - b^n = (a-b)\left[\binom{n}{1}b^{n-1} + \binom{n}{2}b^{n-2}(a-b)^1 + \ldots + (a-b)^{n-1}\right]$$

$$a^n - b^n = (a-b)k$$

where $k = \left[\binom{n}{1}b^{n-1} + \binom{n}{2}b^{n-2}(a-b)^1 + \ldots + (a-b)^{n-1}\right]$

Hence (a-b) is a factor of (a^n-b^n).

5. Evaluate. $(\sqrt{3}+\sqrt{2})^6 - (\sqrt{3}-\sqrt{2})^6$

5. Firstly, the expression $(a+b)^6 - (a-b)^6$ is simplified by using Binomial Theorem. This can be done as

$$(a+b)^6 = {}^6C_0 a^6 + {}^6C_1 a^5\ b + {}^6C_2 a^4\ b^2 + {}^6C_3 a^3\ b^3 + {}^6C_4 a^2\ b^4 + {}^6C_5 a^1\ b^5 + {}^6C_6\ b^6$$

$$= a^6 + 6a^5\ b + 15a^4\ b^2 + 20a^3\ b^3 + 15a^2\ b^4 + 6ab^5 + b^6$$

$(a-b)^6 = {}^6C_0 a^6 - {}^6C_1 a^5\ b + {}^6C_2 a^4\ b^2 - {}^6C_3 a^3\ b^3 + {}^6C_4 a^2\ b^4 - {}^6C_5 a^1\ b^5 + {}_6C_6\ b^6$

$= a^6 - 6a^5\ b + 15a^4\ b^2 - 20a^3\ b^3 + 15a^2\ b^4 - 6ab^5 + b^6$

$\therefore (a+b)^6 - (a-b)^6 = 2\left[6ab\ b + 20a^3\ b^3 + 6ab^5\right]$

Putting $a = \sqrt{3}$ and $b = \sqrt{2}$, we obtain

$(\sqrt{3}+\sqrt{2})^6 - (\sqrt{3}-\sqrt{2})^6 = 2\left[6(\sqrt{3})^5\left(\sqrt{2}\right) + 20(\sqrt{3})^3(\sqrt{2})^3 + 6\left(\sqrt{3}\right)(\sqrt{2})^5\right]$

$= 2\left[54\sqrt{6} + 120\sqrt{6} + 24\sqrt{6}\right]$

$= 2 \times 198\sqrt{6}$

$= 396\sqrt{6}$

6. Find the value of $\left(a^2+\sqrt{a^2-1}\right)^4 + \left(a^2-\sqrt{a^2-1}\right)^4$

6. We know that-

$(a+b)^n = \binom{n}{0}a^n b^0 + \binom{n}{1}a^{n-1}b^1 + \binom{n}{2}a^{n-2}b^2 + \ldots + \binom{n}{n-1}a^1 b^{n-1} + \binom{n}{n}a^0 b^n$

Hence

$(a+b)^4 = \binom{4}{0}a^4 b^0 + \binom{4}{1}a^3 b^1 + \binom{4}{2}a^2 b^2 + \binom{4}{3}a^1 b^3 + \binom{4}{4}a^0 b^4$

$= \frac{4!}{0!(4-0)!}a^4 \times 1 + \frac{4!}{1!(4-1)!}a^3 b^1 + \frac{4!}{2!(4-2)!}a^2 b^2 + \frac{4!}{3!(4-3)!}a^1 b^3$

$+ \frac{4!}{4!(4-4)!}b^4 \times 1$

$= \frac{4!}{0! \times 4!}a^4 + \frac{4!}{1! \times 3!}a^3 b + \frac{4!}{2! \times 2!}a^2 b^2 + \frac{4!}{3! \times 1!}ab^3 + \frac{4!}{4! \times 0!}b^4$

$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

Thus, $(a + b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

Replacing b with -b

$(a + (-b))^4 = a^4 + 4a^3(-b) + 6a^2(-b)^2 + 4a(-b)^3 + (-b)^4$

$(a-b)^4 = a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4$

Now,

$(a + b)^4 + (a - b)^4$

$= (a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4)$

$+ (a^4 - 4a^3b + 6a^2b^2 - 4ab^3 + b^4)$

$= 2(a^4 + 6a^2b^2 + b^4)$

Putting $a = a^2$ & $b = \sqrt{(a^2-1)}$

$(a^2 + \sqrt{(a^2-1)})^4 + (a^2 - \sqrt{(a^2-1)})^4$

$= 2[(a^2)^4 + 6(a^2)^2(\sqrt{(a^2-1)})^2 + (\sqrt{(a^2-1)})^4]$
$= 2[\ a^8 + 6(a^4)(a^2-1) + (a^2-1)^2]$
$= 2[\ a^8 + 6\ a^6 - 6a^4 + a^4 - 2\ a^2 + 1]$
$= 2[\ a^8 + 6\ a^6 - 5a^4 - 2\ a^2 + 1]$

7. Find an approximation of $(0.99)^5$ using the first three terms of its expansion.

7. $0.99 = 1 - 0.01$

$\therefore (0.99)^5 = (1-0.01)^5$

$= {}^5C_0(1)^5 - {}^5C_1(1)^4(0.01) + {}^5C_2(1)^3(0.01)^2$

$= 1 - 5(0.01) + 10(0.01)^2$

$= 1 - 0.05 + 0.001$

$= 1.001 - 0.05$

$= 0.951$

Thus, the value of $(0.99)^5$ is approximately 0.951.

8. Find n, if the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$

8. We know that
General term of expansion $(a + b)^n$

$T_{r+1} = \binom{n}{r} a^{n-r} b^r$

We need to calculate fifth term from beginning of expansion

$\left(\sqrt[4]{2} + \frac{1}{\sqrt[4]{3}}\right)^n$

$\therefore$ putting r = 4, a = $\sqrt[4]{2}$, and b = $\frac{1}{\sqrt[4]{3}}$, we get-

$T_{4+1} = \binom{n}{4}\left(\sqrt[4]{2}\right)^{n-4}\left(\frac{1}{\sqrt[4]{3}}\right)^4$

$T_{4+1} = \binom{n}{4}(2)^{\frac{n-4}{4}}\left(\frac{1}{3}\right)^{\frac{4}{4}}$

$T_5 = \binom{n}{4}(2)^{\frac{n-4}{4}}\left(\frac{1}{3}\right)$

Now,

In the expression of $(a + b)^n$

r^{th} term from the end = $(n-r+2)^{th}$ term from the begining

Hence, 5th term from the end

= $(n-5+2)^{th}$ term from the beginning

= $(n-3)^{th}$ term from the beginning

Now, We need to calculate $(n-3)^{th}$ term from beginning of expansion $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$

$$T_{r+1}=\binom{n}{r}a^{n-r}b^r$$

putting r = (n-3)-1 = n-4, a = $\sqrt[4]{2}$, and b = $\frac{1}{\sqrt[4]{3}}$, we get-

$$T_{(n-4+1)}=\binom{n}{n-4}\left(\sqrt[4]{2}\right)^{n-(n-4)}\left(\frac{1}{\sqrt[4]{3}}\right)^{n-4}$$

$$T_{n-3}=\binom{n}{n-4}\left(\sqrt[4]{2}\right)^{4}\left(\frac{1}{3}\right)^{\frac{n-4}{4}}$$

$$T_{n-3}=\binom{n}{n-4}(2)^{\frac{4}{4}}\left(\frac{1}{3}\right)^{\frac{n-4}{4}}$$

$$T_{n-3}=\binom{n}{n-4}(2)\left(\frac{1}{3}\right)^{\frac{n-4}{4}}$$

Given that-

$$\frac{T_5}{T_{n-3}}=\frac{\sqrt{6}}{1}$$

$$\Rightarrow \frac{\binom{n}{4}(2)^{\frac{n-4}{4}}\left(\frac{1}{3}\right)}{\binom{n}{n-4}(2)\left(\frac{1}{3}\right)^{\frac{n-4}{4}}}=\frac{\sqrt{6}}{1}$$

$$\Rightarrow \frac{\binom{n}{4}}{\binom{n}{n-4}}\times(2)^{\frac{n-4}{4}-1}\times\left(\frac{1}{3}\right)^{1-\frac{n-4}{4}}=\frac{\sqrt{6}}{1}$$

$$\Rightarrow \frac{\binom{n}{4}}{\binom{n}{n-4}} \times (2)^{\frac{n-8}{4}} \times \left(\frac{1}{3}\right)^{\frac{8-n}{4}} = 6^{\frac{1}{2}}$$

$$\Rightarrow \frac{\binom{n}{4}}{\binom{n}{n-4}} \times (2)^{\frac{n-8}{4}} \times (3)^{\frac{n-8}{4}} = 6^{\frac{1}{2}}$$

$$\Rightarrow \frac{\binom{n}{4}}{\binom{n}{n-4}} \times (2 \times 3)^{\frac{n-8}{4}} = 6^{\frac{1}{2}}$$

$$\Rightarrow \frac{\binom{n}{4}}{\binom{n}{n-4}} \times (6)^{\frac{n-8}{4}} = 6^{\frac{1}{2}}$$

Comparing powers of 6

$\frac{n-8}{4} = \frac{1}{2}$

$\Rightarrow$ 2(n-8) = 4

$\Rightarrow$ 2n-16 = 4

$\Rightarrow$ 2n = 20

$\therefore$ n = 20/2 = 10

Thus, the value of n is 10.

9. Expand using Binomial Theorem $\left(1+\frac{x}{2}-\frac{2}{x}\right)^4, x \neq 0$

9. Using Binomial Theorem, the given expression $\left(1+\frac{x}{2}-\frac{2}{x}\right)^4$ can be expanded as

$$\left[\left(1+\frac{x}{2}\right)-\frac{2}{x}\right]^4$$

$$= {}^4C_0\left(1+\frac{x}{2}\right)^4 - {}^4C_1\left(1+\frac{x}{2}\right)^3\left(\frac{2}{x}\right) + {}^4C_2\left(1+\frac{x}{2}\right)^2\left(\frac{2}{x}\right)^2 - {}^4C_3\left(1+\frac{x}{2}\right)\left(\frac{2}{x}\right)^3 + {}^4C_4\left(\frac{2}{x}\right)^4$$

$$=\left(1+\frac{x}{2}\right)^4-4\left(1+\frac{x}{2}\right)^3\left(\frac{2}{x}\right)+6\left(1+x+\frac{x^2}{4}\right)\left(\frac{4}{x^2}\right)-4\left(1+\frac{x}{2}\right)\left(\frac{8}{x^3}\right)+\frac{16}{x^4}$$

$$=\left(1+\frac{x}{2}\right)^4-\frac{8}{x}\left(1+\frac{x}{2}\right)^3+\frac{24}{x^2}+\frac{24}{x}+6-\frac{32}{x^3}-\frac{16}{x^2}+\frac{16}{x^4}$$

$$=\left(1+\frac{x}{2}\right)^4-\frac{8}{x}\left(1+\frac{x}{2}\right)^3+\frac{8}{x^2}+\frac{24}{x}+6-\frac{32}{x^3}+\frac{16}{x^4}$$

Again by using Binomial Theorem, we obtain

$$\left(1+\frac{x}{2}\right)^4={}^4C_0(1)^4+{}^4C_1(1)^3\left(\frac{x}{2}\right)+{}^4C_2(1)^2\left(\frac{x}{2}\right)^2+{}^4C_3(1)^1\left(\frac{x}{2}\right)^3+{}^4C_4\left(\frac{x}{2}\right)^4$$

$$=1+4\times\frac{x}{2}+6\times\frac{x^2}{4}+4\times\frac{x^3}{8}+\frac{x^4}{16}$$

$$=1+2x+\frac{3x^2}{2}+\frac{x^3}{2}+\frac{x^4}{16}$$

$$\left(1+\frac{x}{2}\right)^3={}^3C_0(1)^3+{}^3C_1(1)^2\left(\frac{x}{2}\right)+{}^3C_2(1)\left(\frac{x}{2}\right)^2+{}^3C_3\left(\frac{x}{2}\right)^3$$

$$=1+\frac{3x}{2}+\frac{3x^2}{4}+\frac{x^3}{8}$$

From (1), (2), and (3), we obtain

$$\left[\left(1+\frac{x}{2}\right)-\frac{2}{x}\right]^4$$

$$=1+2x+\frac{3x^2}{2}+\frac{x^3}{2}+\frac{x^4}{16}-\frac{8}{x}\left(1+\frac{3x}{2}+\frac{3x^2}{4}+\frac{x^3}{8}\right)+\frac{8}{x^2}+\frac{24}{x}+6-\frac{32}{x^3}+\frac{16}{x^4}$$

$$=1+2x+\frac{3}{2}x^2+\frac{x^3}{2}+\frac{x^4}{16}-\frac{8}{x}-12-6x-x^2+\frac{8}{x^2}+\frac{24}{x}+6-\frac{32}{x^3}+\frac{16}{x^4}$$

$$=\frac{16}{x}+\frac{8}{x^2}-\frac{32}{x^3}+\frac{16}{x^4}-4x+\frac{x^2}{2}+\frac{x^3}{2}+\frac{x^4}{16}-5$$

10. Find the expansion of $(3x^2 - 2ax + 3a^2)^3$ using binomial theorem.

10. We know that-

$(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

putting $a = 3x^2$ & $b = -a(2x-3a)$, we get-

$[3x^2 + (-a(2x-3a))]^3$

$= (3x^2)^3+3(3x^2)^2(-a(2x-3a)) + 3(3x^2)(-a(2x-3a))^2 + (-a(2x-3a))^3$

$= 27x^6 - 27ax^4(2x-3a) + 9a^2x^2(2x-3a)^2 - a^3(2x-3a)^3$

$= 27x^6 - 54ax^5 + 81a^2x^4 + 9a^2x^2(4x^2-12ax+9a^2)$

$- a^3[(2x)^3 - (3a)^3 - 3(2x)^2(3a) + 3(2x)(3a)^2]$

$= 27x^6 - 54ax^5 + 81a^2x^4 + 36a^2x^4 - 108a^3x^3 + 81a^4x^2$

$-8a^3x^3 + 27a^6 + 36a^4x^2 - 54a^5x$

$= 27x^6 - 54ax^5 + 117a^2x^4 - 116a^3x^3 + 117a^4x^2 - 54a^5x + 27a^6$

Thus, $(3x^2 – 2ax + 3a^2)^3$

$= 27x^6 - 54ax^5 + 117a^2x^4 - 116a^3x^3 + 117a^4x^2 - 54a^5x + 27a^6$