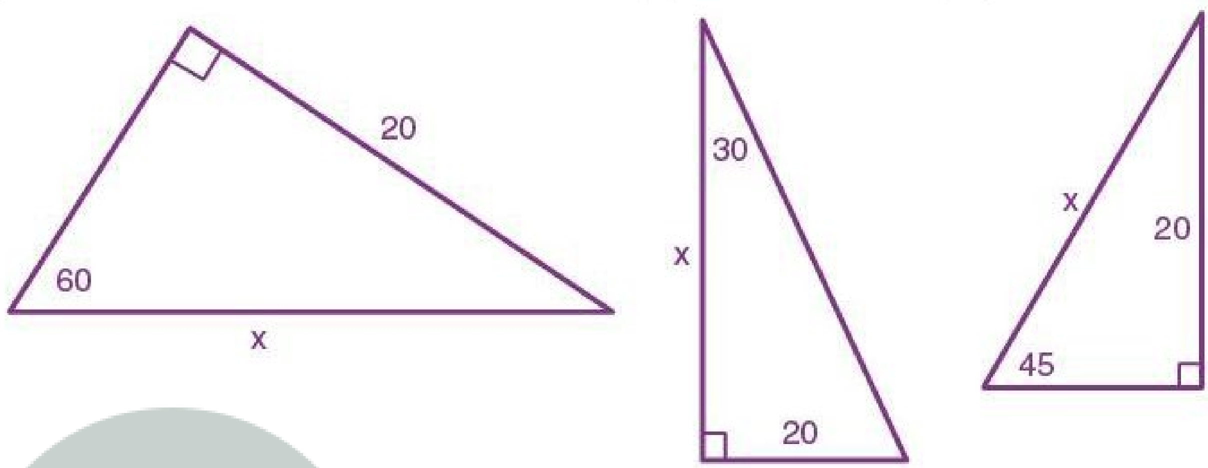
Exercise 24

1. Find 'x' if:



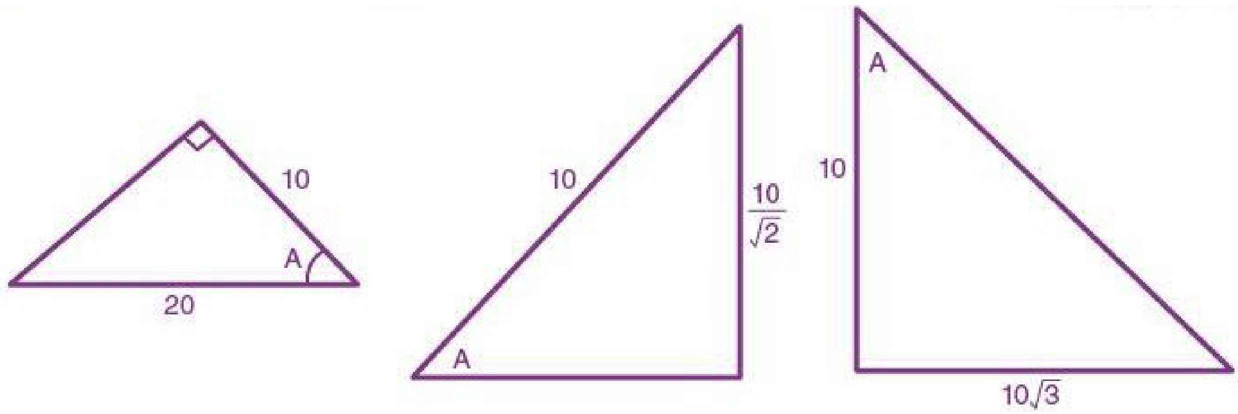
Solution:

(i) From the figure, we have
sin 60° = 20/x
√3/2 = 20/x
∴ x = 40/√3

(ii) From the figure, we have
tan 30° = 20/x
1/√3 = 20/x
∴ x = 20√3

(iii) From the figure, we have
sin 45° = 20/x
1/√2 = 20/x
∴ x = 20√2

2. Find angle 'A' if:



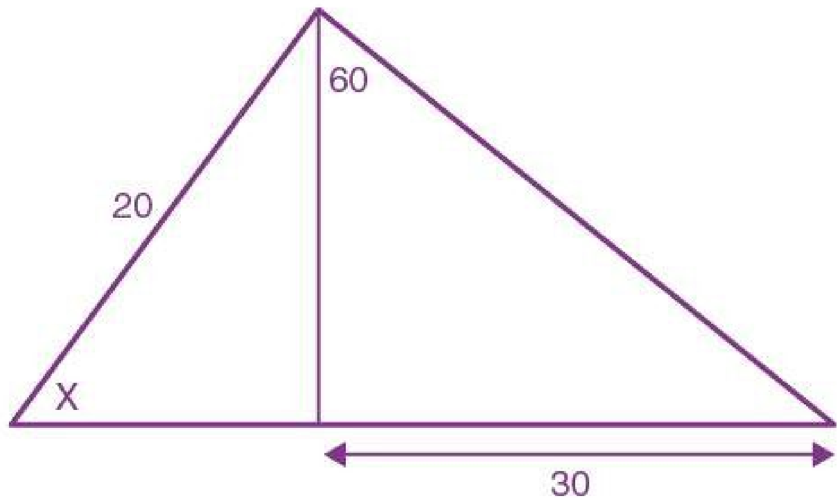
Solution:

(i) From the figure, we have
cos A = 10/20
= ½
cos A = cos 60^o
Hence,
A = 60^o

(ii) From the figure, we have
sin A = (10/√2)/10
= 1/√2
sin A = sin 45^o
Hence,
A = 45^o

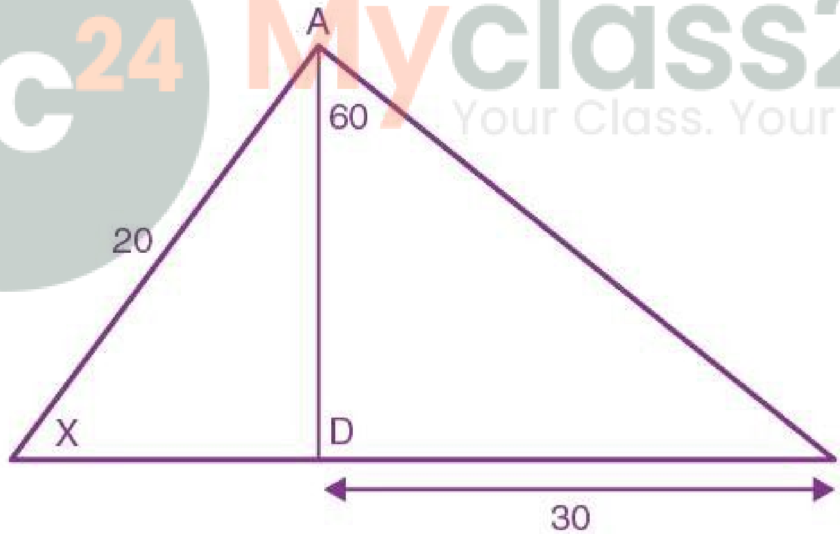
(iii) From the figure, we have
tan A = (10√3)/10
= √3
tan A = tan 60^o
Hence,
A = 60^o

3. Find angle 'x' if:



Solution:

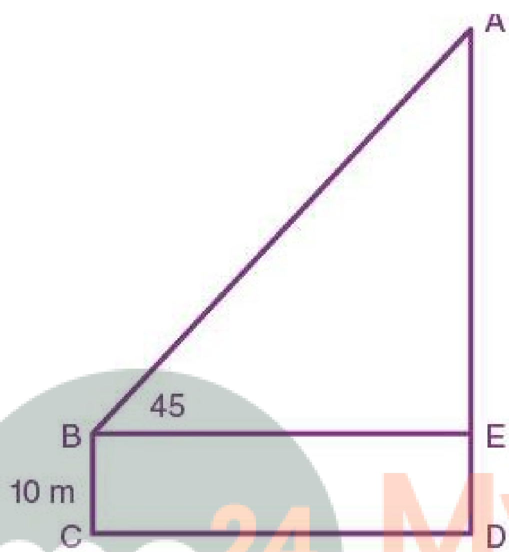
The given figure is drawn as follows:

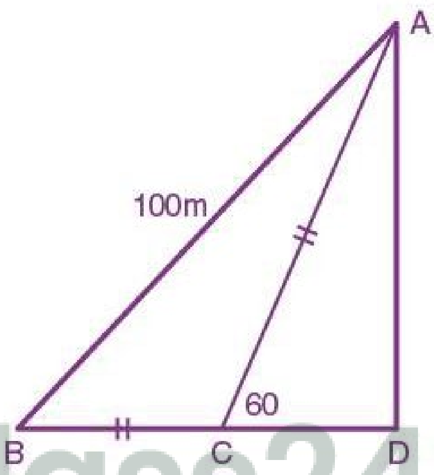


We have,
tan 60^o = 30/AD
√3 = 30/AD
AD = 30/√3
Again,
sin x = AD/20
AD = 20 sin x
Now,
20 sin x = 30/√3
sin x = 30/20√3
sin x = 3/2√3

sin x = √3/2
sin x = sin 60°
⇒ x = 60°

4. Find AD, if:



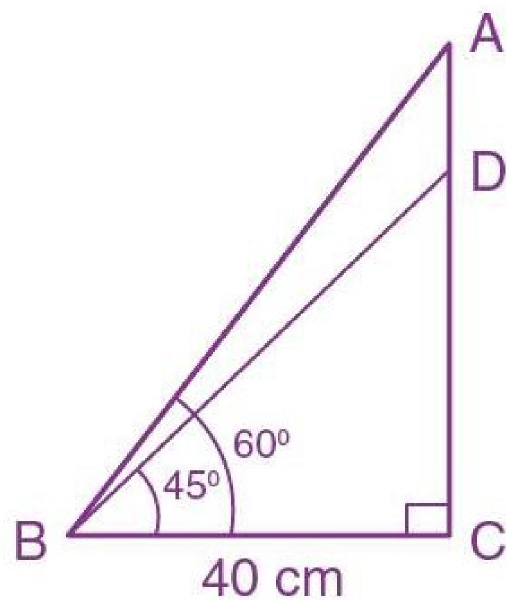


Solution:

(i) In right ΔABE, we have
tan 45° = AE/BE
1 = AE/BE
AE = BE
Thus, AE = BE = 50 m
Now,
In rectangle BCDE, we have
DE = BC = 10 m
Thus, the length of AD is given by
AD = AE + DE
= 50 + 10
= 60 m

(ii) In right ΔABD, we have
sin B = AD/AB
sin 30° = AD/100 [As ∠ACD is the exterior angle of ΔABC]
½ = AD/100
⇒ AD = 50 m

5. Find the length of AD.
Given: ∠ABC = 60°, ∠DBC = 45° and BC = 40 cm

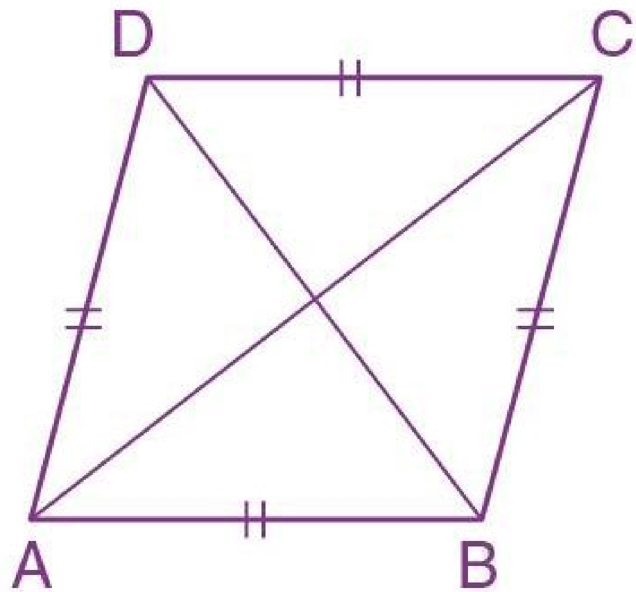


Solution:

In right ΔABC, we have
tan 60° = AC/BC
√3 = AC/40
AC = 40√3 cm
Next,
In right ΔBDC, we have
tan 45° = DC/BC
1 = DC/40
DC = 40 cm

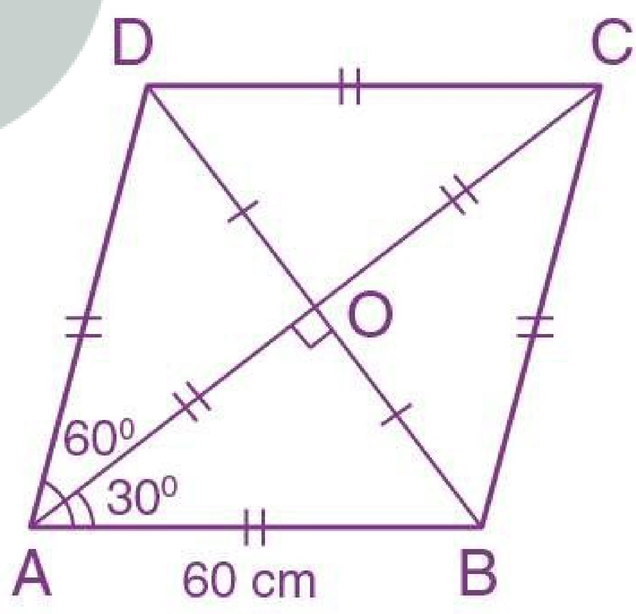
Now, from the figure it's clearly seen that
AD = AC – DC
= 40√3 - 40
= 40(√3 - 1)
Hence, the length of AD is 29.28 cm

6. Find the lengths of diagonals AC and BD. Given AB = 60 cm and ∠BAD = 60°.



Solution:

We know that, diagonals of a rhombus bisect each other at right angles and also bisect the angle of vertex.
Considering the figure as shown below:



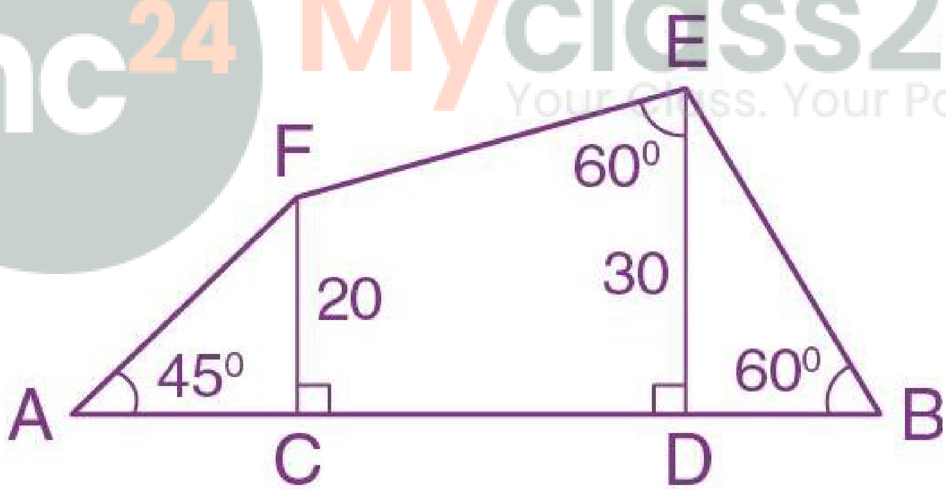
Now, we have
OA = OC = ½ AC,
OB = OD = ½ BD
∠AOB = 90° and
∠OAB = 60°/2 = 30°

Also given that AB = 60 cm

Now,
In right ΔAOB, we have
sin 30° = OB/AB
½ = OB/60
OB = 30
Also,
cos 30° = OA/AB
√3/2 = OA/60
OA = 51.96 cm

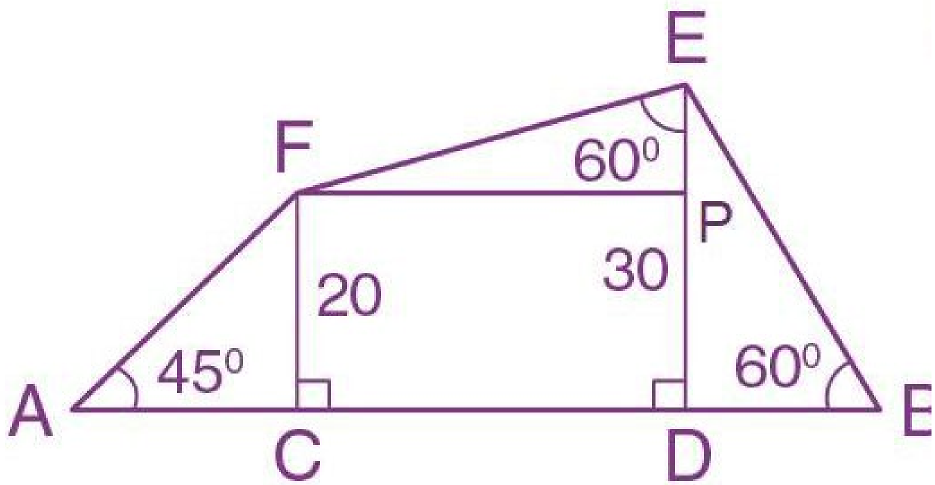
Therefore,
Length of diagonal AC = 2 x OA
= 2 x 51.96
= 103.92 cm
Length of diagonal BD = 2 x OB
= 2 x 30
= 60 cm

7. Find AB.



Solution:

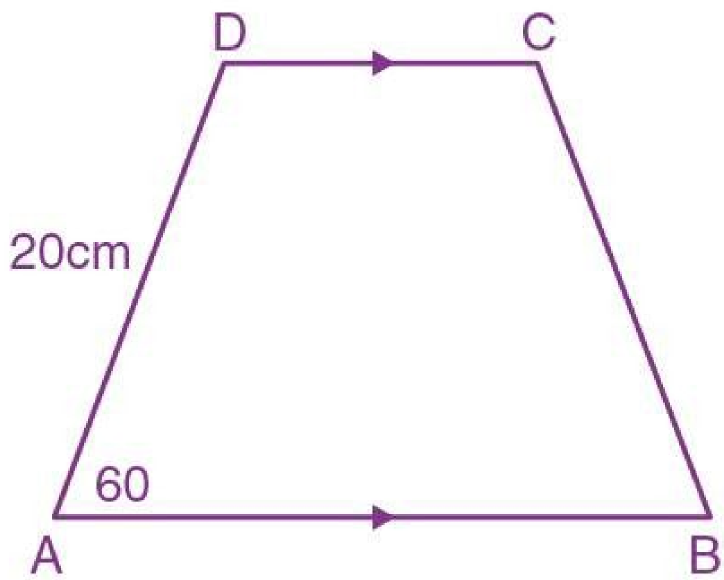
Considering the given figure, let's construct FP ⊥ ED



Now,
In right ΔACF, we have
tan 45^o = 20/AC
1 = 20/AC
AC = 20
Next,
In right ΔDEB, we have
tan 60^o = 30/BD
√3 = 30/BD
BD = 30/√3
= 17.32 cm
Also, given FC = 20 and ED = 30
So, EP = 10 cm
Thus,
tan 60^o = FP/EP
√3 = FP/10
FP = 10√3
= 17.32 cm
And, FP = CD

Therefore, AB = AC + CD + BD
= 20 + 17.32 + 17.32
= 54.64 cm

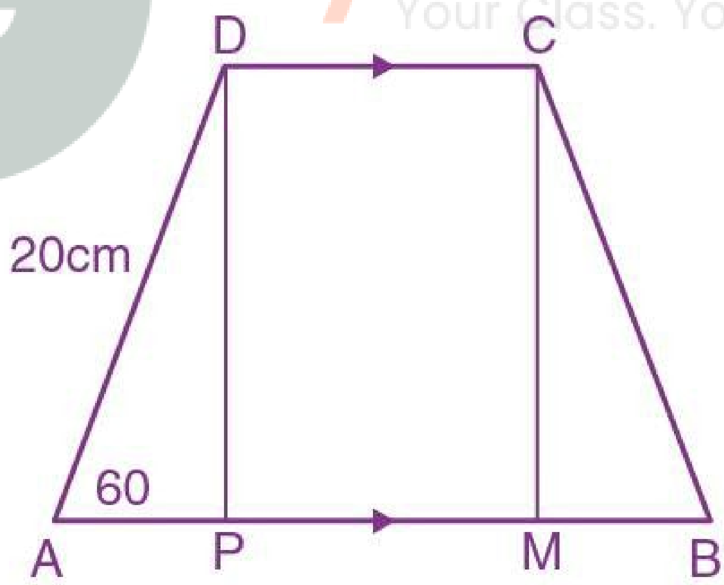
8. In trapezium ABCD, as shown, AB || DC, AD = DC = BC = 20 cm and ∠A = 60^o. Find:
(i) length of AB
(ii) distance between AB and DC



Solution:

Constructing two perpendiculars to AB from the point D and C respectively.
Now, since AB||CD we have PMCD as a rectangle

Considering the figure,



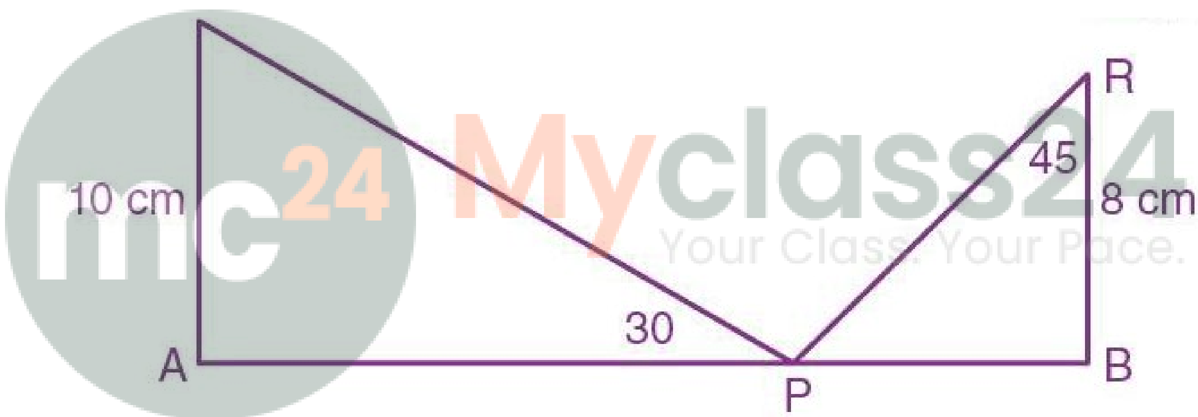
(i) From right ΔADP, we have
$\cos 60^{o} = AP/AD$
$\frac{1}{2} = AP/20$
$AP = 10$
Similarly,
In right ΔBMC, we have

BM = 10 cm

Now, from the rectangle PMCD we have
CD = PM = 20 cm
Therefore,
AB = AP + PM + MB
= 10 + 20 + 10
= 40 cm

(ii) Again, from the right ΔAPD, we have
sin 60° = PD/20
√3/2 = PD/20
PD = 10√3
Hence, the distance between AB and CD is 10√3 cm

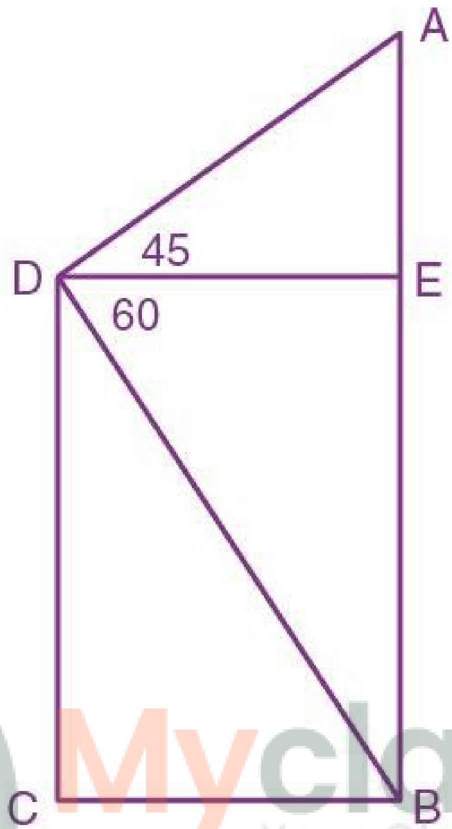
9. Use the information given to find the length of AB.



Solution:

In right ΔAQP, we have
tan 30° = AQ/AP
1/√3 = 10/AP
AP = 10√3
Also,
In right ΔPBR, we have
tan 45° = PB/BR
1 = PB/8
PB = 8
Therefore, AB = AP + PB
= 10√3 + 8

10. Find the length of AB.



Solution:

In right ΔADE, we have

tan 45° = AE/DE

1 = AE/30

AE = 30 cm

Also, in right ΔDBE, we have

tan 60° = BE/DE

√3 = BE/30

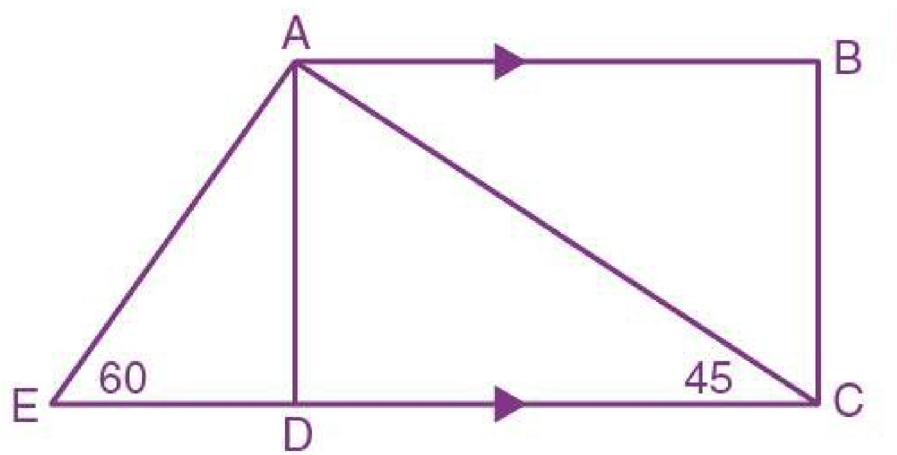
BE = 30√3 cm

Therefore, AB = AE + BE

= 30 + 30√3

= 30(1 + √3) cm

11. In the given figure, AB and EC are parallel to each other. Sides AD and BC are 2 cm each and are perpendicular to AB.



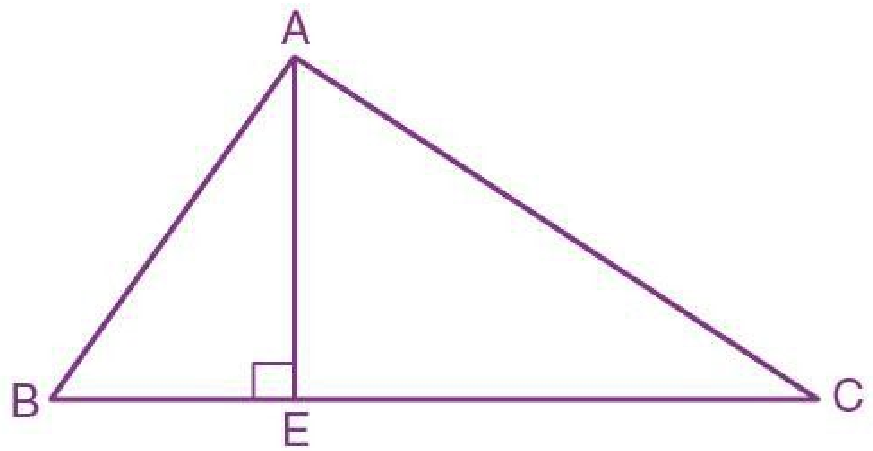
Given that ∠AED = 60° and ∠ACD = 45°. Calculate:
(i) AB (ii) AC (iii) AE
Solution:

(i) In right ΔADC, we have
tan 45° = AD/DC
1 = 2/DC
DC = 2 cm
And, as AD || DC and AD ⊥ EC, ABCD is a parallelogram
So, opposite sides are equal
Hence, AB = DC = 2 cm

(ii) Again, in right ΔADC
sin 45° = AD/AC
1/√2 = AD/AC
AC = 2√2 cm

(iii) In right ΔADE, we have
sin 60° = AD/AE
√3/2 = 2/AE
AE = 4/√3

12. In the given figure, ∠B = 60°, AB = 16 cm and BC = 23 cm,
Calculate:
(i) BE (ii) AC



Solution:

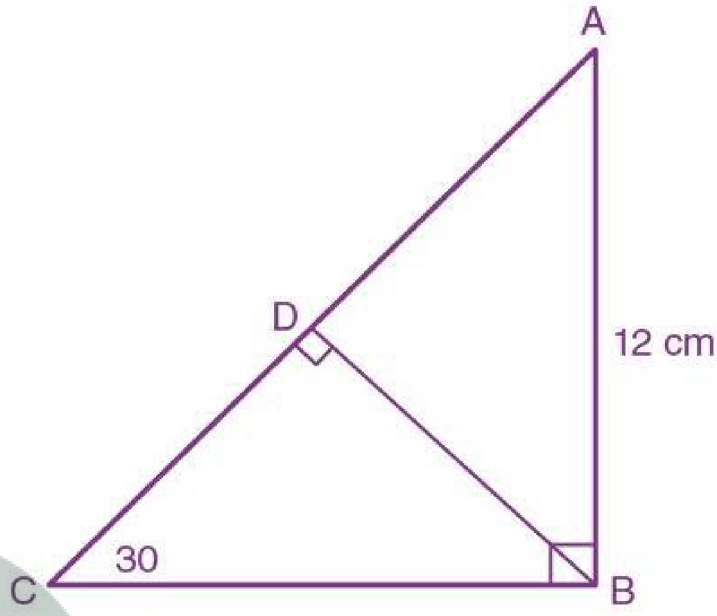
In ∆ABE, we have
sin 60° = AE/AB
√3/2 = AE/16
AE = √3/2 x 16
= 8√3 cm

(i) In ∆ABE, we have ∠AEB = 90°
So, by Pythagoras Theorem, we get
$BE^2 = AB^2 - AE^2$
$BE^2 = (16)^2 - (8\sqrt{3})^2$
$BE^2 = 256 - 192$
$BE^2 = 64$
Taking square root on both sides, we get
BE = 8 cm

(ii) EC = BC - BE
= 23 - 8
= 15
In ∆AEC, we have
∠AEC = 90°
So, by Pythagoras Theorem, we get
$AC^2 = AE^2 + EC^2$
$AC^2 = (8\sqrt{3})^2 + (15)^2$
$AC^2 = 192 + 225$
$AC^2 = 147$
Taking square root on both sides, we get
AC = 20.42 cm

13. Find
(i) BC

(ii) AD
(iii)
(iv) AC



Solution:

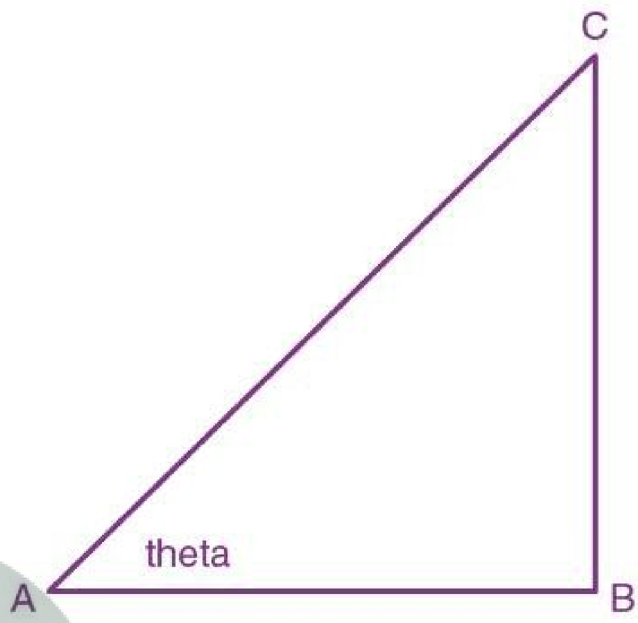
(i) In right ΔAEC, we have
tan 30° = AB/BC
1/√3 = 12/BC
BC = 12√3 cm

(ii) In right ΔABD, we have
cos A = AD/AB
cos 60° = AD/AB
½ = AD/12
AD = 12/2
AD = 6 cm

(iii) In right ΔABC, we have
sin B = AB/AC
sin 30° = AB/AC
½ = 12/AC
AC = 12 x 2
AC = 24 cm

14. In right-angled triangle ABC; ∠B = 90°. Find the magnitude of angle A, if:
(i) AB is √3 times of BC
(ii) BC is √3 times of AB
Solution:

Considering the figure below:

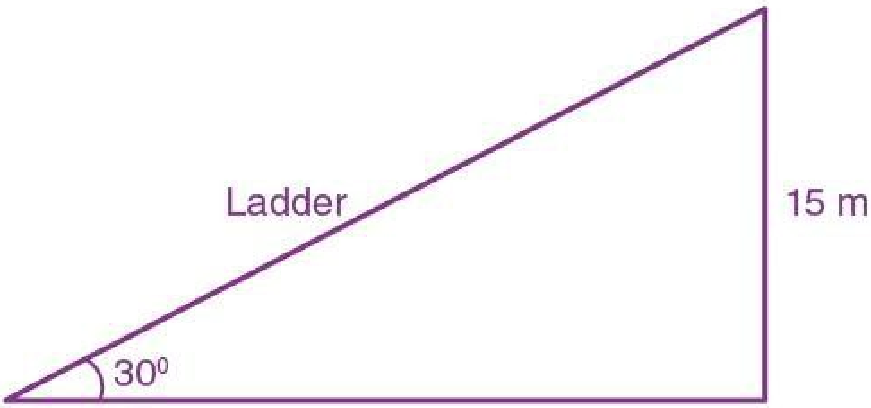


(i) We have, AB is √3 times of BC
AB/BC = √3
cot A = cot 30°
A = 30°
Hence, the magnitude of angle A is 30°

(ii) Again, from the figure
BC/AB = √3
tan A = √3
tan A = tan 60°
A = 60°
Hence, the magnitude of angle A is 60°

15. A ladder is placed against a vertical tower. If the ladder makes an angle of 30° with the ground and reaches upto a height of 15 m of the tower; find length of the ladder.
Solution:

Given that the ladder makes an angle of 30° with the ground and reaches upto a height of 15 m of the tower
Let's consider the figure shown below:

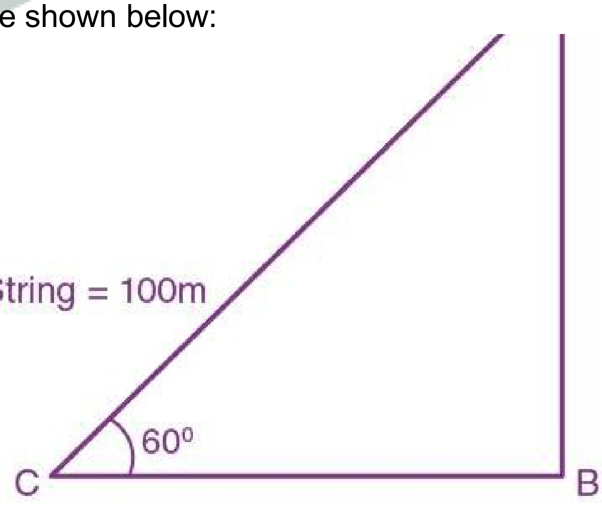


Let's assume the length of the ladder is x metre
Now, from the figure we have
15/x = sin 30^o [As perpendicular/hypotenuse = sine]
15/x = ½
x = 30 m

Hence, the length of the ladder is 30 m.

16. A kite is attached to a 100 m long string. Find the greatest height reached by the kite when its string makes an angles of 60^o with the level ground.
Solution:

Given that the kite is attached to a 100 m long string and it makes an angle of 60^o with the ground level
Let's consider the figure shown below:

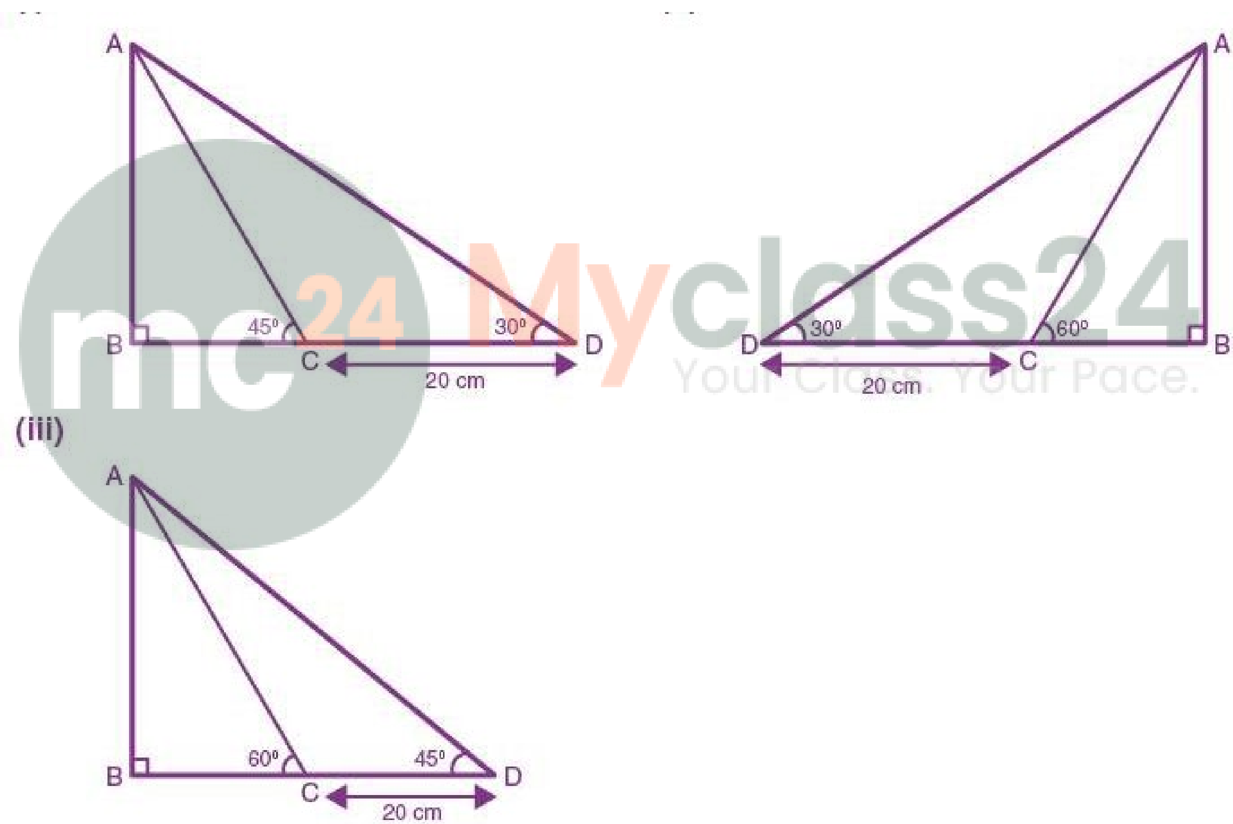


Let's

assume the greatest height to be *x* metre
Now, from the figure we have
x/100 = sin 60° [As perpendicular/hypotenuse = sine]
x/100 = √3/2
x = 100 x (√3/2)
= 86.6 m

Hence, the greatest height reached by the kite is 86.6 m.

17. Find AB and BC, if:



(i) Let assume BC to be x cm
BD = BC + CD = (x + 20) cm
Now,
In ΔABD, we have
tan 30° = AB/BD
1/√3 = AB/(x + 20)
x + 20 = √3AB ... (1)
And,

In ΔABC, we have
$\tan 45^\circ = AB/BC$
$1 = AB/x$
$AB = x$ ... (2)

Now, using (1) in (2) we get
$AB + 20 = \sqrt{3}AB$
$AB(\sqrt{3} - 1) = 20$
$AB = 20/(\sqrt{3} - 1)$
$= 20/(\sqrt{3} - 1) \times [(\sqrt{3} + 1)/(\sqrt{3} + 1)]$
$= 20(\sqrt{3} + 1)/(3 - 1)$
$= 27.32$ cm

Hence from (2), we have
$AB = BC = x = 27.32$ cm
Therefore, AB = 27.32cm, BC = 27.32cm

(ii) Let's assume BC to be x m
$BD = BC + CD$
$= (x + 20)$ cm
In ΔABD, we have
$\tan 30^\circ = AB/BD$
$1/\sqrt{3} = AB/(x + 20)$
$x + 20 = \sqrt{3}\, AB$ … (1)

In ΔABC, we have
$\tan 60^\circ = AB/BC$
$\sqrt{3} = AB/x$
$x = AB/\sqrt{3}$ … (2)

Now, from (1) we have
$AB/\sqrt{3} + 20 = \sqrt{3}AB$
$AB + 20\sqrt{3} = 3AB$
$2AB = 20\sqrt{3}$
$AB = 20\sqrt{3}/2$
$= 10\sqrt{3}$
$= 17.32$ cm
And, from (2) we get
$x = AB/\sqrt{3}$
$= 17.32/\sqrt{3}$
$= 10$ cm

Hence, BC = x = 10cm
Therefore,
AB = 17.32 cm and BC = 10cm

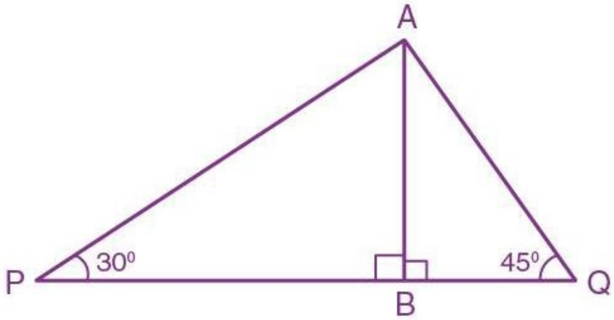
(iii) Let's assume BC to be x cm
BD = BC + CD
= (x + 20) cm
In ∆ABD, we have
tan 45° = AB/BD
1 = AB/(x + 20)
x + 20 = AB … (1)
Also,
In ∆ABD, we have
tan 60° = AB/BC
√3 = AB/x
x = AB/√3 … (2)

Now, from (1) we have
AB/√3 + 20 = AB
AB + 20√3 = √3AB
AB (√3 - 1) = 20√3
AB = 20√3/(√3 - 1)
On rationalizing, we get
AB = 20√3(√3 + 1)/(3 -1)
= 10√3(√3 + 1)
= 47.32 cm
And, from (2) we get
x = AB/√3
= 47.32/√3
= 27.32 cm
Hence, BC = x = 27.32 cm

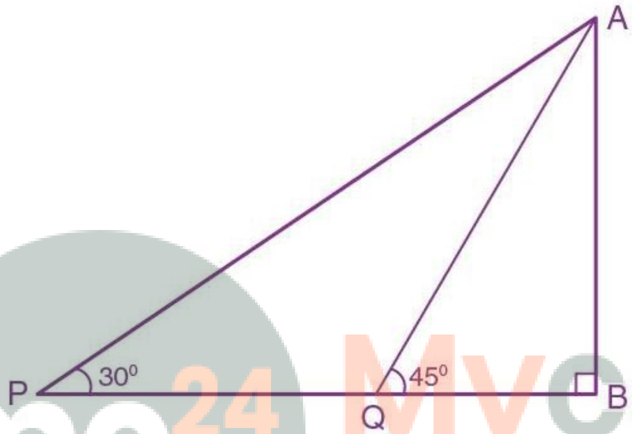
Therefore,
AB = 47.32 cm and BC = 27.32 cm

18. Find PQ, if AB = 150 m, ∠P = 30° and ∠Q = 45°.

(i)



(ii)



Solution:

(i) In ΔAPB, we have

tan 30° = AB/PB

1/√3 = 150/PB

PB = 150√3

= 259.80 m

Also, in ΔABQ

tan 45° = AB/BQ

1 = 150/BQ

BQ = 150 m

Therefore,

PQ = PB + BQ

= 259.80 + 150

= 409.80 m

(ii) In ΔAPB, we have

tan 30° = AB/PB

1/√3 = 150/PB

PB = 150√3

= 259.80 m

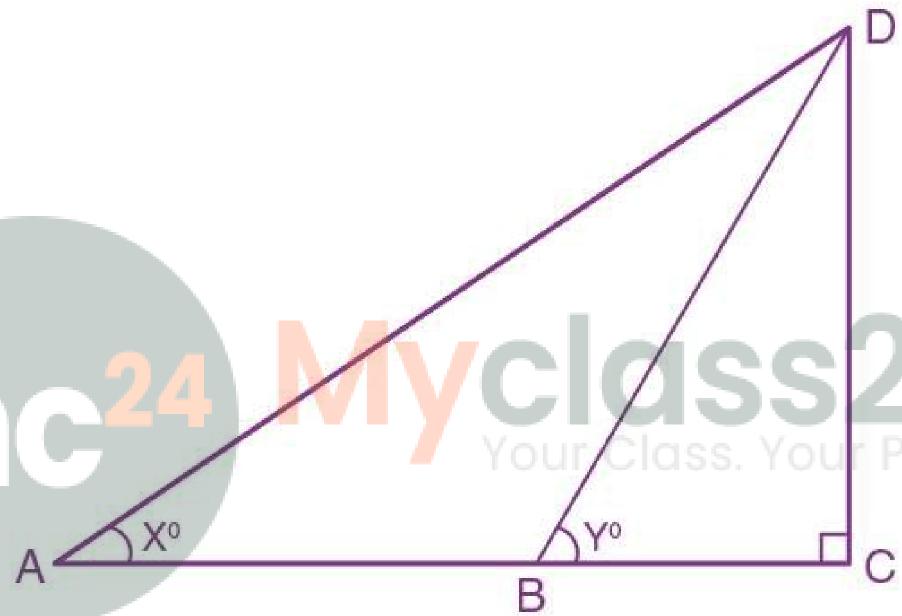
Also, in in ΔAPB, we have

tan 45° = AB/BQ
1 = 150/BQ
BQ = 150 cm

Therefore, PQ = PB – BQ
= 259.80 – 150
= 109.80 m

19.

20. If tan x° = 5/12, tan y° = ¾ and AB = 48 m; find the length of CD.



Solution:

Given,
tan x° = 5/12, tan y° = ¾ and AB = 48 m;
Let's consider the length of BC to x metre
In ∆ADC, we have
tan x° = DC/AC
5/12 = DC/(48 + x)
5(48 + x) = 12DC
240 + 5x = 12DC … (1)

Also, in ∆BDC we have
tan y° = CD/BC
¾ = CD/x
x = 4CD/3 … (2)
Also, from (1) we get
240 + 5(4CD/3) = 12CD

240 + 20CD/3 = 12CD

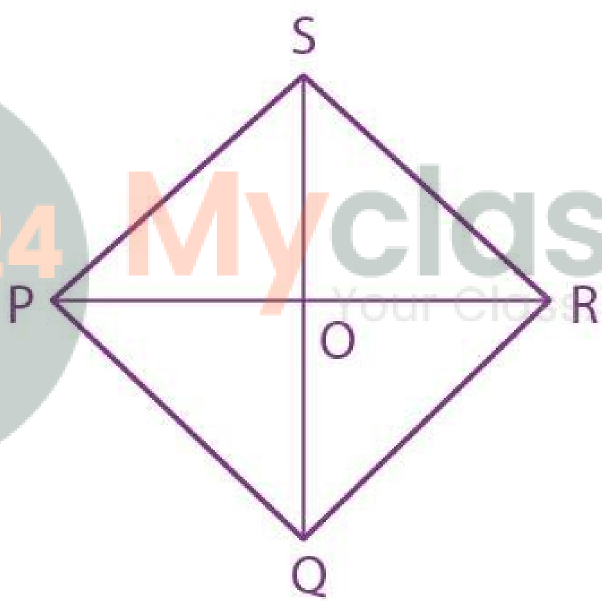
720 + 20CD = 36CD
36CD – 20CD = 720
16CD = 720
CD = 720/16
CD = 45

Therefore, the length of CD is 45 m.

21. The perimeter of a rhombus is 96 cm and obtuse angle of it is 120°. Find the lengths of its diagonals.
Solution:

As rhombus has all sides equal

Let's consider the diagram as below:



Hence, PQ = 96/4 = 24 cm
And, ∠PQR = 120°
We also know that,
In a rhombus, the diagonals bisect each other perpendicularly and the diagonal bisect the angle at vertex
Thus, ∠POR is a right-angle triangle
∠PQR = ½ (∠PQR)
= 60°
sin 60° = perpendicular/hypotenuse
√3/2 = PO/PQ
√3/2 = PO/24
PO = 24√3/2
= 12√3
= 20/784 cm

Hence,

PR = 2PO
= 2 x 20.784
= 41.568 cm
Also, we have
cos 60^{o} = base/hypotenuse
½ = OQ/24
OQ = 24/2
= 12 cm
Hence, SQ = 2 x OQ
= 2 x 12
= 24 cm

Therefore, the length of the diagonals PR is 41.568 cm and of SQ is 24 cm