EXERCISE 28.1

Name the octants in which the following points lie:
(i) (5, 2, 3)
(ii) (-5, 4, 3)
(iii) (4, -3, 5)
(iv) (7, 4, -3)
(v) (-5, -4, 7)
(vi) (-5, -3, -2)
(vii) (2, -5, -7)
(viii) (-7, 2, -5)
Solution:
(i) (5, 2, 3)
In this case, since x, y and z all three are positive then octant will be XOYZ

(ii) (-5, 4, 3)
In this case, since x is negative and y and z are positive then the octant will be X′OYZ

(iii) (4, -3, 5)
In this case, since y is negative and x and z are positive then the octant will be XOY′Z

(iv) (7, 4, -3)
In this case, since z is negative and x and y are positive then the octant will be XOYZ′

(v) (-5, -4, 7)
In this case, since x and y are negative and z is positive then the octant will be X′OY′Z

(vi) (-5, -3, -2)
In this case, since x, y and z all three are negative then octant will be X′OY′Z′

(vii) (2, -5, -7)
In this case, since z and y are negative and x is positive then the octant will be XOY′Z′

(viii) (-7, 2, -5)
In this case, since x and z are negative and x is positive then the octant will be X′OYZ′

2. Find the image of:
(i) (-2, 3, 4) in the yz-plane
(ii) (-5, 4, -3) in the xz-plane

(iii) (5, 2, -7) in the xy-plane
(iv) (-5, 0, 3) in the xz-plane
(v) (-4, 0, 0) in the xy-plane
Solution:
(i) (-2, 3, 4)
Since we need to find its image in yz-plane, a sign of its x-coordinate will change
So, Image of point (-2, 3, 4) is (2, 3, 4)

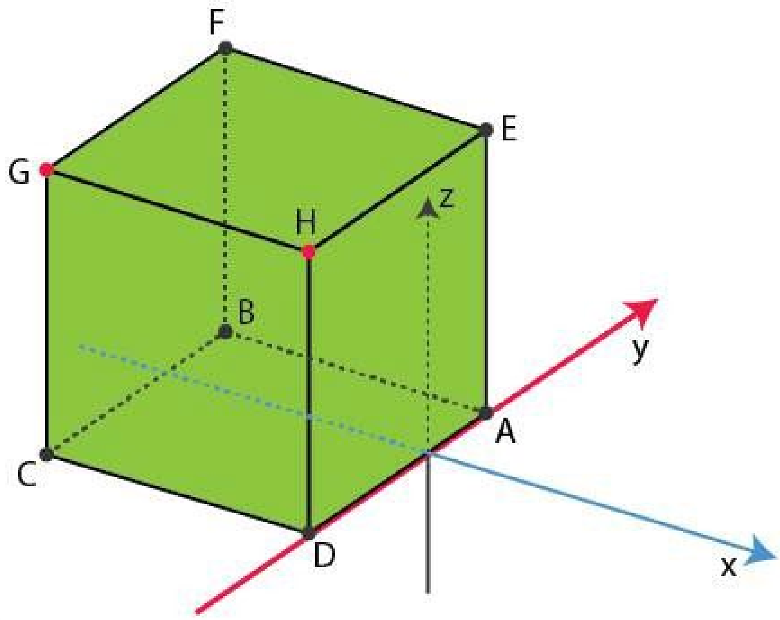
(ii)(-5, 4, -3)
Since we need to find its image in xz-plane, sign of its y-coordinate will change
So, Image of point (-5, 4, -3) is (-5, -4, -3)

(iii) (5, 2, -7)
Since we need to find its image in xy-plane, a sign of its z-coordinate will change
So, Image of point (5, 2, -7) is (5, 2, 7)

(iv) (-5, 0, 3)
Since we need to find its image in xz-plane, sign of its y-coordinate will change
So, Image of point (-5, 0, 3) is (-5, 0, 3)

(v) (-4, 0, 0)
Since we need to find its image in xy-plane, sign of its z-coordinate will change
So, Image of point (-4, 0, 0) is (-4, 0, 0)

3. A cube of side 5 has one vertex at the point (1, 0, 1), and the three edges from this vertex are, respectively, parallel to the negative x and y-axes and positive z-axis. Find the coordinates of the other vertices of the cube.
Solution:
Given: A cube has side 4 having one vertex at (1, 0, 1)
Side of cube = 5
We need to find the coordinates of the other vertices of the cube.
So let the Point A(1, 0, 1) and AB, AD and AE is parallel to –ve x-axis, -ve y-axis and +ve z-axis respectively.



Since side of cube = 5
Point B is (-4, 0, 1)
Point D is (1, -5, 1)
Point E is (1, 0, 6)

Now, EH is parallel to –ve y-axis
Point H is (1, -5, 6)

HG is parallel to –ve x-axis
Point G is (-4, -5, 6)

Now, again GC and GF is parallel to –ve z-axis and +ve y-axis respectively
Point C is (-4, -5, 1)
Point F is (-4, 0, 6)

4. Planes are drawn parallel to the coordinates planes through the points (3, 0, -1) and (-2, 5, 4). Find the lengths of the edges of the parallelepiped so formed.
Solution:
Given:
Points are (3, 0, -1) and (-2, 5, 4)
We need to find the lengths of the edges of the parallelepiped formed.

For point (3, 0, -1)
$x_1 = 3$, $y_1 = 0$ and $z_1 = -1$

For point (-2, 5, 4)
$x_2 = -2$, $y_2 = 5$ and $z_2 = 4$

Plane parallel to coordinate planes of x_1 and x_2 is yz-plane

Plane parallel to coordinate planes of y_1 and y_2 is xz-plane

Plane parallel to coordinate planes of z_1 and z_2 is xy-plane

Distance between planes $x_1 = 3$ and $x_2 = -2$ is $3 – (-2) = 3 + 2 = 5$
Distance between planes $x_1 = 0$ and $y_2 = 5$ is $5 – 0 = 5$
Distance between planes $z_1 = -1$ and $z_2 = 4$ is $4 – (-1) = 4 + 1 = 5$

∴Theedges of parallelepiped is 5, 5, 5

5. Planes are drawn through the points (5, 0, 2) and (3, -2, 5) parallel to the coordinate planes. Find the lengths of the edges of the rectangular parallelepiped so formed.

Solution:
Given:
Points are (5, 0, 2) and (3, -2, 5)
We need to find the lengths of the edges of the parallelepiped formed

For point (5, 0, 2)
$x_1 = 5$, $y_1 = 0$ and $z_1 = 2$

For point (3, -2, 5)
$x_2 = 3$, $y_2 = -2$ and $z_2 = 5$

Plane parallel to coordinate planes of x_1 and x_2 is yz-plane

Plane parallel to coordinate planes of y_1 and y_2 is xz-plane

Plane parallel to coordinate planes of z_1 and z_2 is xy-plane

Distance between planes $x_1 = 5$ and $x_2 = 3$ is $5 – 3 = 2$

Distance between planes $x_1 = 0$ and $y_2 = -2$ is 0 – (-2) = 0 + 2 = 2
Distance between planes $z_1 = 2$ and $z_2 = 5$ is 5 – 2 = 3

∴Theedges of parallelepiped is 2, 2, 3

6. Find the distances of the point P (-4, 3, 5) from the coordinate axes.
Solution:
Given:
The point P (-4, 3, 5)
The distance of the point from x-axis is given as:

$$\text{Distance} = \sqrt{y^2 + z^2}$$
$$= \sqrt{3^2 + 5^2}$$
$$= \sqrt{9 + 25}$$
$$= \sqrt{34}$$

The distance of the point from y-axis is given as:

$$\text{Distance} = \sqrt{x^2 + z^2}$$
$$= \sqrt{(-4)^2 + 5^2}$$
$$= \sqrt{16 + 25}$$
$$= \sqrt{41}$$

The distance of the point from z-axis is given as:

$$\text{Distance} = \sqrt{x^2 + y^2}$$
$$= \sqrt{(-4)^2 + 3^2}$$
$$= \sqrt{16 + 9}$$
$$= \sqrt{25}$$
$$= 5$$

7. The coordinates of a point are (3, -2, 5). Write down the coordinates of seven points such that the absolute values of their coordinates are the same as those of the coordinates of the given point.
Solution:
Given:

Point (3, -2, 5)
The Absolute value of any point(x, y, z) is given by,
$\sqrt{(x^2 + y^2 + z^2)}$
We need to make sure that absolute value to be the same for all points.
So let the point A(3, -2, 5)
Remaining 7 points are:
Point B(3, 2, 5) (By changing the sign of y coordinate)
Point C(-3, -2, 5) (By changing the sign of x coordinate)
Point D(3, -2, -5) (By changing the sign of z coordinate)
Point E(-3, 2, 5) (By changing the sign of x and y coordinate)
Point F(3, 2, -5) (By changing the sign of y and z coordinate)
Point G(-3, -2, -5) (By changing the sign of x and z coordinate)
Point H(-3, 2, -5) (By changing the sign of x, y and z coordinate)