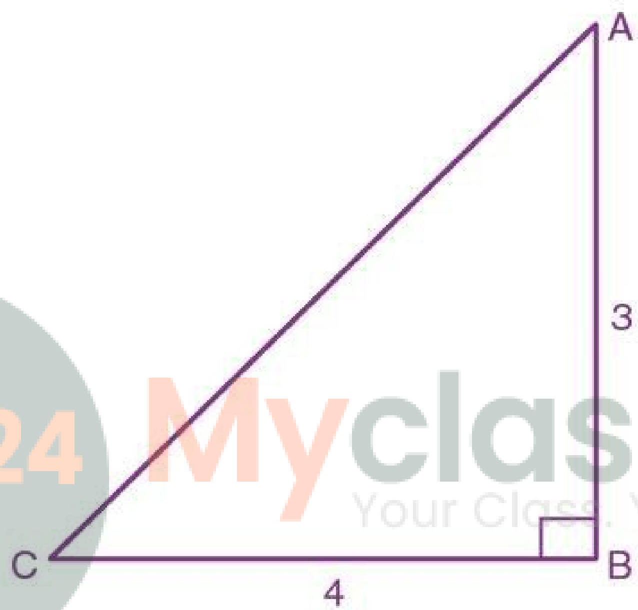
Exercise 22(A)

1. From the following figure, find the values of:
(i) sin A
(ii) cos A
(iii) cot A
(iv) sec C
(v) cosec C
(vi)
(vii)tan C



Solution:

Given, $\angle ABC = 90^o$
$AC^2 = AB^2 + BC^2$ (AC is hypotenuse)
$AC^2 = 3^2 + 4^2$
$= 9 + 16$
$= 25$
Taking square root on both sides, we get
AC = 5 cm

(i) sin A = perpendicular/hypotenuse
= BC/AC
= 4/5

(ii) cos A = base/hypotenuse
= AB/AC
= 3/5

(iii) cot A = base/perpendicular

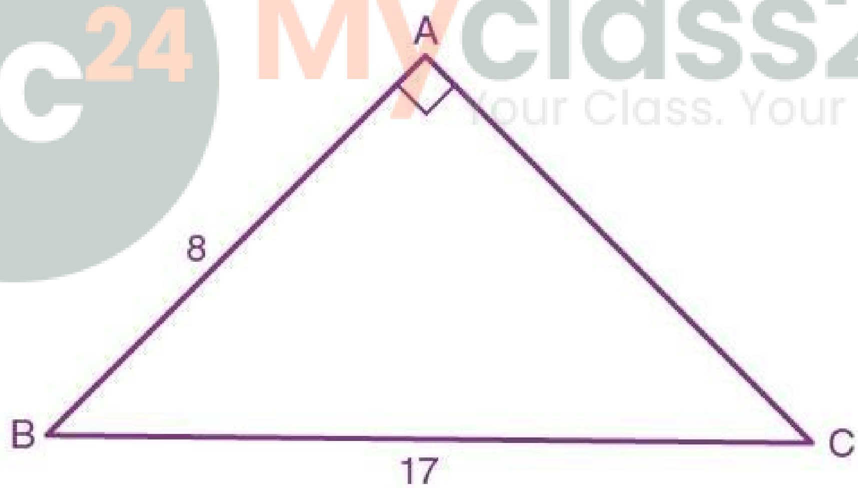
= AB/BC
= 3/4

(iv) sec C = hypotenuse/base
= AC/BC
= 5/4

(v) cosec C = hypotenuse/perpendicular
= AC/AB
= 5/3

(vi) tan C = perpendicular/base
= AB/BC
= 3/4

2. Form the following figure, find the values of:
(i) cos B
(ii) tan C
(iii) $\sin^2 B + \cos^2 B$
(iv)
(v) sin B. cos C + cos B. sin C



Solution:

Given, ∠BAC = 90°
$BC^2 = AB^2 + AC^2$ (As BC is the hypotenuse)
$17^2 = 8^2 + AC^2$
$AC^2 = 289 - 64$
= 225
Taking square root on both sides, we get
AC = 15 cm

(i) cos B = base/hypotenuse
= AB/BC

= 8/17

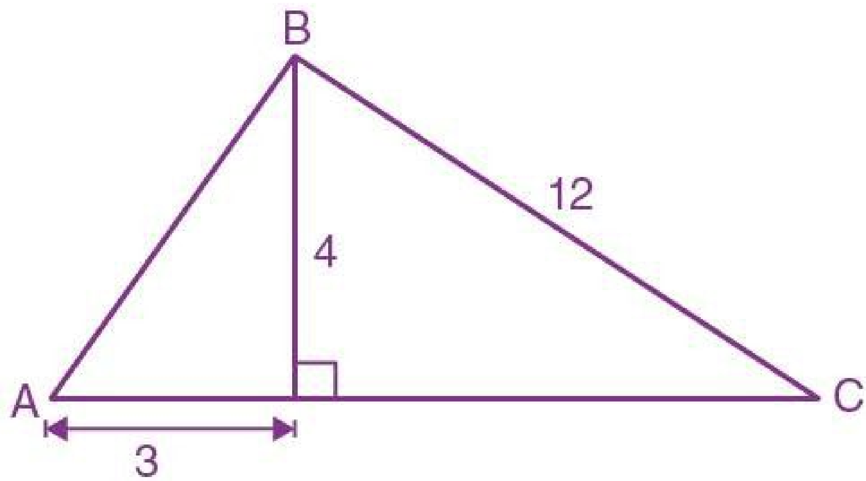
(ii) tan C = perpendicular/base
= AB/AC
= 8/15

(iii) sin B = perpendicular/hypotenuse
= AC/BC
= 15/17
cos B = base/hypotenuse
= AB/BC
= 8/17
Now,
$\sin^2 B + \cos^2 B = (15/17)^2 + (8/17)^2$
= (225 + 64)/289
= 289/289
= 1

(iv) sin B = perpendicular/hypotenuse
= AC/BC
= 15/17
cos B = base/hypotenuse
= AB/BC
= 8/17
sin C = perpendicular/hypotenuse
= AB/BC
= 8/17
cos C = base/hypotenuse
= AC/BC
= 15/17

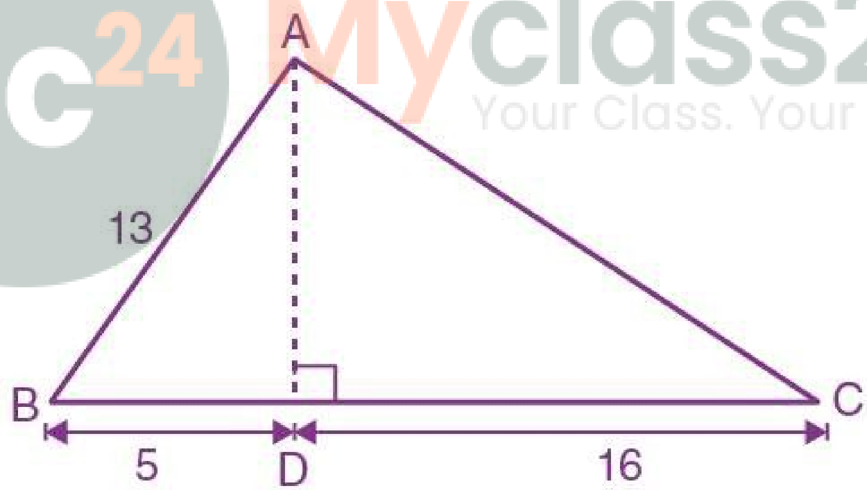
Now,
sin B. cos C + cos B. sin C = 15/17 x 15/17 + 8/17 x 8/17
= (225 + 64)/289
= 289/289
= 1

3. From the following figure, find the values of:
(i) cos A
(ii) cosec A
(iii) $\tan^2 A - \sec^2 A$
(iv) sin C
(v) sec C
(vi) $\cot^2 C - 1/\sin^2 C$



Solution:

Considering the given diagram, we have



$\angle ADB = 90^o$ and $\angle BDC = 90^o$
So, by Pythagoras theorem
$AB^2 = AD^2 + BD^2$ (As AB is the hypotenuse in ΔABD)
$= 3^2 + 4^2$
$= 9 + 16$
$= 25$
Taking square root on both sides, we get
AB = 5
Also,
$BC^2 = BD^2 + DC^2$ (As BC is the hypotenuse in ΔBDC)
$DC^2 = BC^2 - BD^2$

$= 12^2 - 4^2$
$= 144 – 16$
$= 128$

Taking square root on both sides, we get
DC = $8\sqrt{2}$

Now,
(i) cos A = base/hypotenuse
= AD/AB
= 3/5

(ii) cosec A = hypotenuse/perpendicular
= AB/BD
= 5/4

(iii) tan A = perpendicular/base
= BD/AD
= 4/3
sec A = hypotenuse/base
= AB/AD
= 5/3
$\tan^2 A – \sec^2 A = (4/3)^2 – (5/3)^2$
$= 16/9 – 25/9$
$= -9/9$
$= -1$

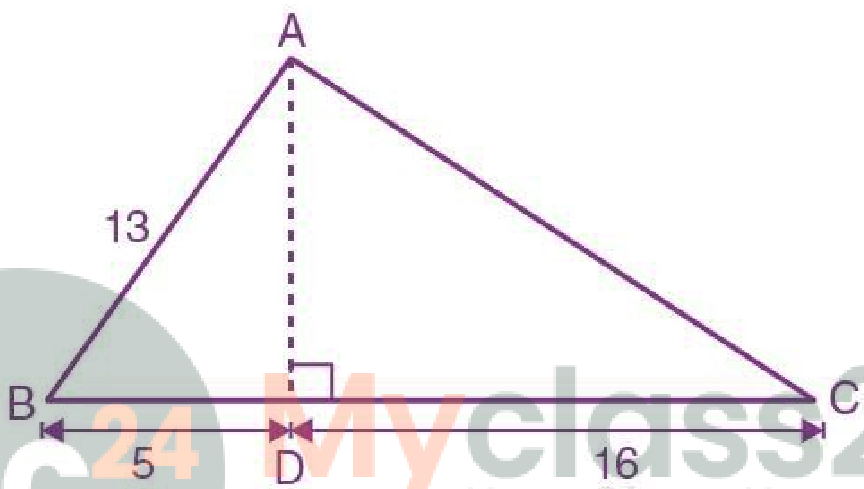
(iv) sin C = perpendicular/hypotenuse
= BD/BC
= 4/12
= 1/3

(v) sec C = hypotenuse/base
= BC/DC
= $12/8\sqrt{2}$
= $3/2\sqrt{2}$
= $3\sqrt{2}/4$

(vi) cot C = base/perpendicular
= DC/BD
= $8\sqrt{2}/4$
= $2\sqrt{2}$
sin C = perpendicular/hypotenuse
= BD/BC
= 4/12
= 1/3
Now,
$\cot^2 C – 1/\sin^2 C = (2\sqrt{2})^2 – 1/(1/3)^2$

= 8 – 1/(1/9)
= 8 – 9
= -1

4. From the following figure, find the values of:
(i) sin B
(ii) tan C
(iii) $\sec^2 B - \tan^2 B$
(iv)
(v) $\sin^2 C + \cos^2 C$



Solution:

From the figure, we have
∠ADB = 90° and ∠ADC = 90°
So, by Pythagoras theorem
$AB^2 = AD^2 + BD^2$ (As AB is the hypotenuse in ΔABD)
$13^2 = AD^2 + 5^2$
$AD^2 = 13^2 - 5^2$
= 169 – 25
= 144
Taking square root on both sides, we get
AD = 12
Also,
$AC^2 = AD^2 + DC^2$ (As AC is the hypotenuse in ΔADC)
$AC^2 = 12^2 + 16^2$
= 144 + 256
= 400
Taking square root on both sides, we get
AC = 20

Now,
(i) sin B = perpendicular/hypotenuse

= AD/AB
= 12/13

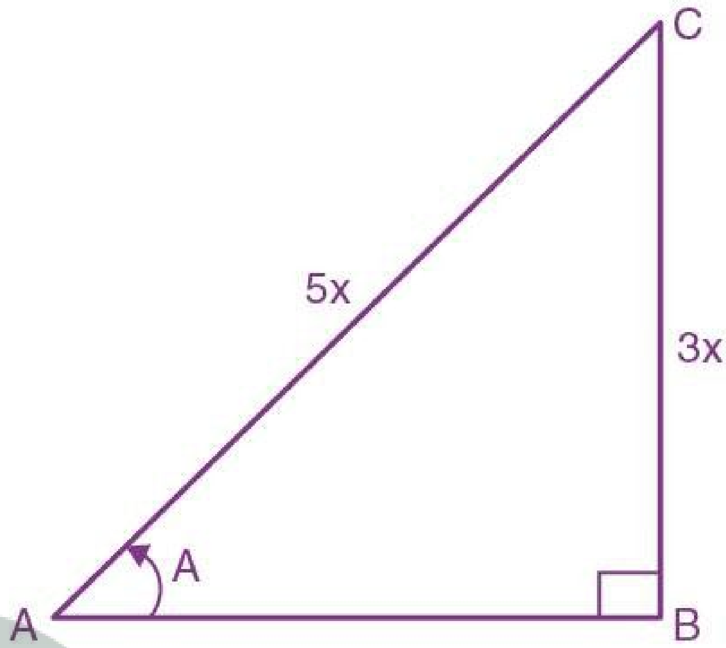
(ii) tan C = perpendicular/base
= 12/16
= ¾

(iii) sec B = hypotenuse/base
= AB/BD
= 13/5
tan B = perpendicular/base
= AD/BD
= 12/5
Hence,
$\sec^2 B - \tan^2 B = (13/5)^2 - (12/5)^2$
= (169 - 144)/25
= 25/25
= 1

(iv) sin C = perpendicular/hypotenuse
= AD/AC
= 12/20
= 3/5
cos C = base/hypotenuse
= DC/AC
= 16/20
= 4/5
Hence,
$\sin^2 C + \cos^2 C = (3/5)^2 + (4/5)^2$
= (9 + 16)/25
= 25/25
= 1

5. Given: sin A = 3/5, find:
(i) tan A
(ii) cos A
Solution:

Let's consider the diagram below:

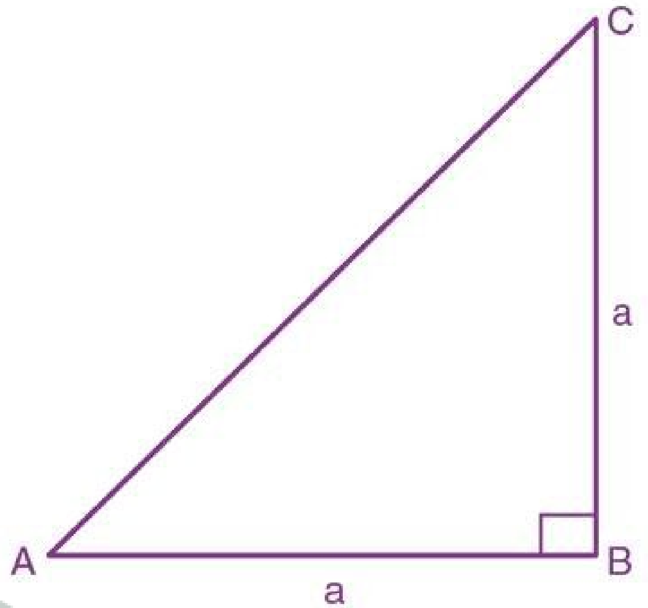


Given, sin A = 3/5
⇒ perpendicular/hypotenuse = 3/5
BC/AC = 3/5
Hence,
If the length of BC is 3x, the length of AC is 5x
We have,
$AB^2 + BC^2 = AC^2$ [By Pythagoras Theorem]
$AB^2 + (3x)^2 = (5x)^2$
$AB^2 = 25x^2 – 9x^2$
$= 16x^2$
Taking square root on both sides, we get
AB = 4x, which is the base

Now,
(i) tan A = perpendicular/base
= 3x/4x
= 3/4

(ii) cos A = base/hypotenuse
= 4x/5x
= 4/5

6. From the following figure, find the values of:
(i) sin A
(ii) sec A
(iii) $\cos^2 A + \sin^2 A$



Solution:

From the given figure, we have
∠ABC = 90° and AC is the hypotenuse ΔABC
So, by Pythagoras Theorem
$AC^2 = AB^2 + BC^2$
$= a^2 + a^2$
$= 2a^2$
Taking square root on both sides, we get
AC = √2a

Now,
(i) sin A = perpendicular/hypotenuse
= BC/AB
= a/√2a
= 1/√2

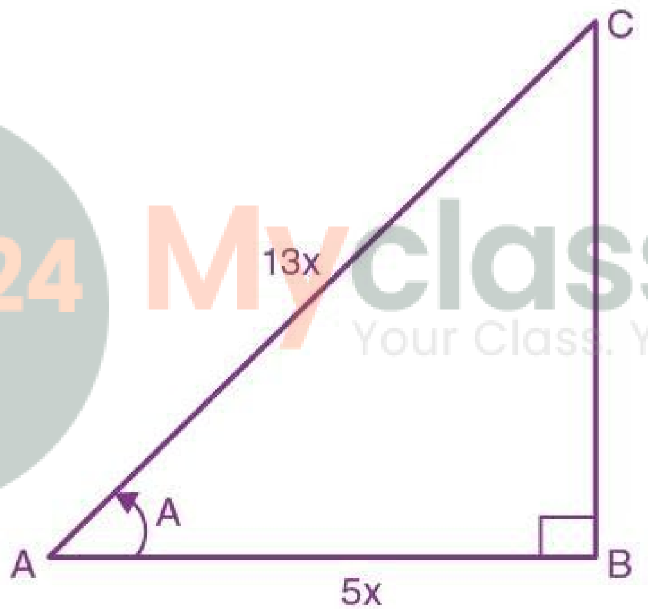
(ii) sec A = hypotenuse/base
= AC/AB
= √2a/a
= √2

(iii) sin A = perpendicular/hypotenuse
= BC/AC
= a/√2a
= 1/√2
cos A = base/hypotenuse

= AB/AC
= $a/\sqrt{2}a$
= $1/\sqrt{2}$
Hence,
$\cos^2 A + \sin^2 A = (1/\sqrt{2})^2 + (1/\sqrt{2})^2$
= ½ + ½
= 1

7. Given: cos A = 5/13
Evaluate: (i) (sin A – cot A)/2tan A (ii) cot A + 1/cos A
Solution:

Let's consider the following diagram:



Given, cos A = 5/13
⇒ base/hypotenuse = 5/13
AB/AC = 5/13
Hence,
If length of AB = 5x, the length of AC = 13x
So, by Pythagoras Theorem
$AB^2 + BC^2 = AC^2$
$(5x)^2 + BC^2 = (13x)^2$
$BC^2 = 169x^2 - 25x^2$
$= 144x^2$
Taking square root on both sides, we get
BC = 12x, which is the perpendicular

Now,

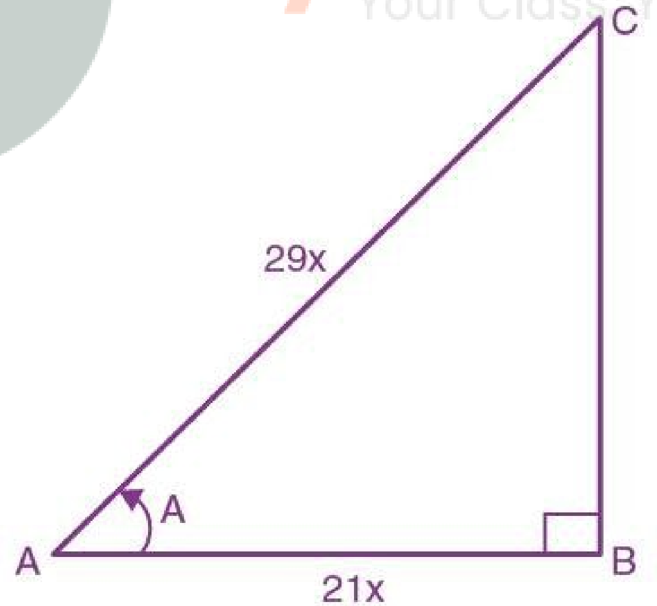
tan A = perpendicular/base
= 12x/5x
= 12/5
sin A = perpendicular/base
= 12x/13x
= 12/13
cot A = base/perpendicular
= 5x/12x
= 5/12

(i) (sin A – cot A)/2tan A = [(12/13) – (5/12)]/ 2(12/5)
= 79/156 x 5/24
= 395/3744

(ii) cot A + 1/cos A = 5/12 + 1/(5/13)
= 5/12 + 13/5
= 181/60

8. Given: sec A = 29/21, evaluate: sin A – 1/tan A
Solution:

Let's consider the diagram below:



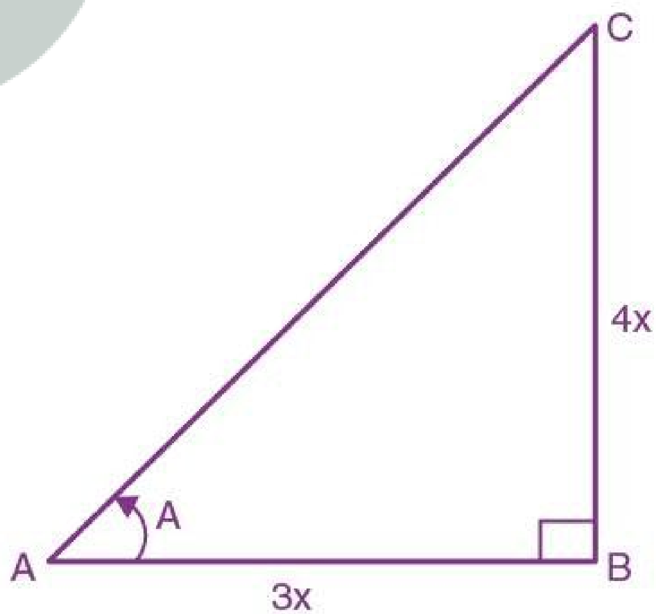
Given, sec A = 29/21
⇒ hypotenuse/base = 29/21
AC/AB = 29/21
Hence,
If length of AB = 21x, the length of AC = 29x

So, by Pythagoras Theorem
$AB^2 + BC^2 = AC^2$
$(21x)^2 + BC^2 = (29x)^2$
$BC^2 = 841x^2 – 441x^2$
$= 400x^2$
Taking square root on both sides, we get
BC = 20x, which is the perpendicular

Now,
sin A = perpendicular/hypotenuse
= 20x/29x
= 20/29
tan A = perpendicular/base
= 20x/21x
= 20/21
Therefore,
sin A – 1/tan A = 20/29 – 1/(20/21)
= 20/29 – 21/20
= - 209/580

9. Given: tan A = 4/3, find: cosec A/(cot A – sec A)
Solution:

Let's consider the diagram below:



Given, tan A = 4/3
⇒ perpendicular/base = 4/3
BC/AB = 4/3

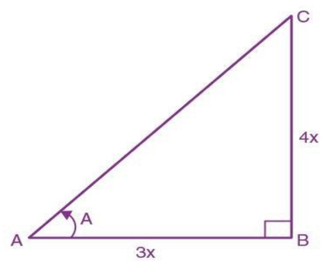
Hence,
If length of AB = 3x, the length of BC = 4x
So, by Pythagoras Theorem
$AB^2 + BC^2 = AC^2$
$(3x)^2 + (4x)^2 = AC^2$
$AC^2 = 9x^2 + 16x^2$
$= 25x^2$
Taking square root on both sides, we get
AC = 5x, which is the hypotenuse

Now,
sec A = hypotenuse/base
= AC/AB
= 5x/3x
= 5/3
cot A = base/perpendicular
= AB/BC
= 3x/4x
= ¾
cosec A = hypotenuse/perpendicular
= AC/BC
= 5x/4x
= 5/x

Therefore,
cosec A/(cot A – sec A) = (5/4)/(3/4 – 5/3)
= (5/4)/(-11/12)
= - 60/44
= - 15/11

10. Given: 4 cot A = 3, find;
(i) sin A
(ii) sec A
(iii) $cosec^2 A - cot^2 A$.
Solution:

Let's consider the diagram below:



Given, 4 cot A = 3
cot A = 3/4
⇒ base/perpendicular = 4/3
AB/BC = 3/4
Hence,
If length of AB = 3x, the length of BC = 4x
So, by Pythagoras Theorem
$AB^2 + BC^2 = AC^2$
$(3x)^2 + (4x)^2 = AC^2$
$AC^2 = 9x^2 + 16x^2$
$= 25x^2$
Taking square root on both sides, we get
AC = 5x, which is the hypotenuse

Now,
(i) sin A = perpendicular/hypotenuse
= 4x/5x
= 4/5

(ii) sec A = hypotenuse/base
= AC/AB
= 5x/3x
= 5/3

(iii) cosec A = hypotenuse/perpendicular
= AC/BC
= 5x/4x
= 5/4
cot A = 3/4
Hence,
$\text{cosec}^2 A – \cot^2 A = (5/4)^2 – (3/4)^2$
= (25 - 9)/16
= 16/16
= 1