

NCERT Solutions for Class 9 Maths Chapter 9 Circles Ex 9.4

Question 1.

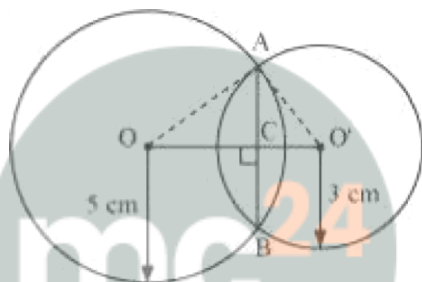
Two circles of radii 5 cm and 3 cm intersect at two points and the distance between their centers is 4 cm. Find the length of the common chord.

Solution.

Let the radius of the circle centered at O and O' be 5 cm and 3 cm respectively

$$OA = OB = 5 \text{ cm}$$

$$O'A = O'B = 3 \text{ cm}$$



OO' will be the perpendicular bisector of chord AB

$$AC = CB$$

It is given that, $OO' = 4 \text{ cm}$

Let OC be x. Therefore, O'C will be $4 - x$

In $\triangle OAC$,

$$OA^2 = AC^2 + OC^2$$

$$5^2 = AC^2 + x^2$$

$$25 - x^2 = AC^2 \text{ (i)}$$

In $\triangle O'AC$,

$$O'A^2 = AC^2 + O'C^2$$

$$3^2 = AC^2 + (4 - x)^2$$

$$9 = AC^2 + 16 + x^2 - 8x$$

$$AC^2 = -x^2 - 7 + 8x \text{ (ii)}$$

From (i) and (ii), we get

$$25 - x^2 = -x^2 - 7 + 8x$$

$$8x = 32$$

$$x = 4$$

Therefore, the common chord will pass through the centre of the smaller circle i.e., O' and hence, it will be the diameter of the smaller circle

$$AC^2 = 25 - x^2$$

$$= 25 - 4^2$$

$$= 25 - 16$$

$$= 9$$

$$AC = 3 \text{ m}$$

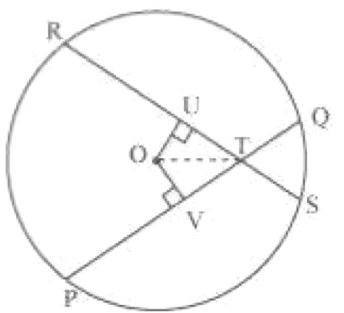
$$\text{Length of the common chord } AB = 2 AC = (2 \times 3) \text{ m} = 6 \text{ m}$$

Question 2.

If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.

Solution.

Let PQ and RS be two equal chords of a given circle and they are intersecting each other at point T



Draw perpendiculars OV and OU on these chords.

In $\triangle OVT$ and $\triangle OUT$,

$OV = OU$ (Equal chords of a circle are equidistant from the centre)

$\angle OVT = \angle OUT$ (Each 90°)

$OT = OT$ (Common)

$\triangle OVT \cong \triangle OUT$ (RHS congruence rule)

$VT = UT$ (By CPCT) (i)

It is given that,

$PQ = RS$ (ii)

$$\frac{1}{2} PQ = \frac{1}{2} RS$$

$PV = RU$ (iii)

On adding (i) and (iii), we get

$$PV + VT = RU + UT$$

$PT = RT$ (iv)

On subtracting equation (iv) from equation (ii), we obtain

$$PQ - PT = RS - RT$$

$$QT = ST \text{ (v)}$$

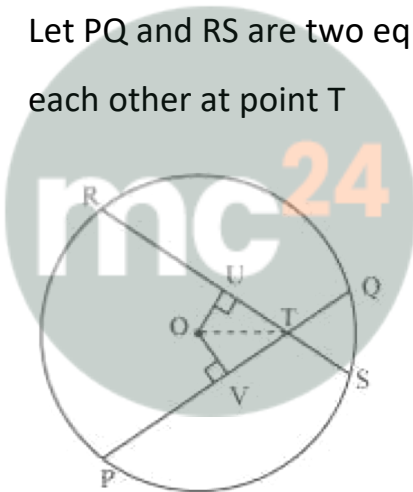
Equations (iv) and (v) indicate that the corresponding segments of chords PQ and RS are congruent to each other

Question 3.

If two equal chords of a circle intersect within the circle, prove that the line joining the point of intersection to the centre makes equal angles with the chords.

Solution.

Let PQ and RS are two equal chords of a given circle and they are intersecting each other at point T



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Draw perpendiculars OV and OU on these chords.

In $\triangle OVT$ and $\triangle OUT$,

$OV = OU$ (Equal chords of a circle are equidistant from the centre)

$\angle OVT = \angle OUT$ (Each 90°)

$OT = OT$ (Common)

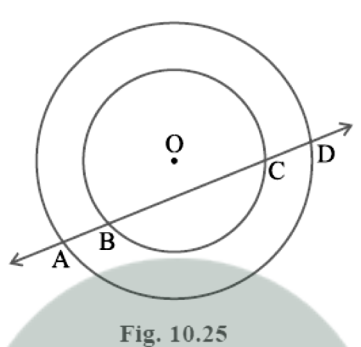
$\triangle OVT \cong \triangle OUT$ (RHS congruence rule)

$$\angle OTV = \angle OTU \text{ (By CPCT)}$$

Therefore, it is proved that the line joining the point of intersection to the centre makes equal angles with the chords

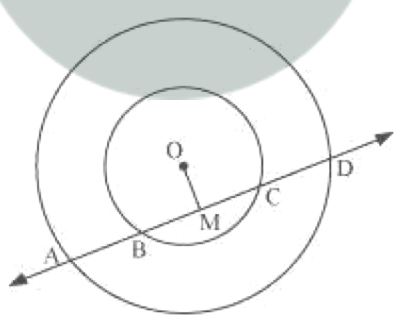
Question 4.

If a line intersects two concentric circles (circles with the same centre) with centre O at A, B, C and D, prove that $AB = CD$ (see Fig. 10.25).



Solution.

Let us draw a perpendicular OM on line AD



It can be observed that BC is the chord of the smaller circle and AD is the chord of the bigger circle

We know that perpendicular drawn from the centre of the circle bisects the chord

$BM = MC$ (i) And,

$$AM = MD \text{ (ii)}$$

On subtracting (ii) from (i), we get

$$AM - BM = MD - MC$$

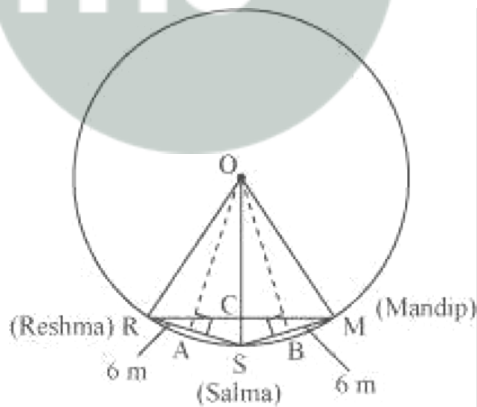
$$AB = CD$$

Question 5.

Three girls Reshma, Salma and Mandip are playing a game by standing on a circle of radius 5m drawn in a park. Reshma throws a ball to Salma, Salma to Mandip, Mandip to Reshma. If the distance between Reshma and Salma and between Salma and Mandip is 6m each, what is the distance between Reshma and Mandip?

Solution.

$$AR = AS = \frac{6}{2} = 3 \text{ cm}$$



$$OR = OS = OM = 5 \text{ m (Radii of the circle)}$$

In $\triangle OAR$,

$$OA^2 + AR^2 = OR^2$$

$$OA^2 + (3\text{m})^2 = (5\text{m})^2$$

$$OA^2 = (25 - 9) \text{ m}^2 = 16\text{m}^2$$

$$OA = 4 \text{ m}$$

ORSM will be a kite (OR = OM and RS = SM)

We know that the diagonals of a kite are perpendicular and the diagonal common to both the isosceles triangles is bisected by another diagonal

RCS will be of 90° and

$$RC = CM$$

$$\text{Area of } \triangle ORS = \frac{1}{2} * OA * RS$$

$$\frac{1}{2} * RC * OS = \frac{1}{2} * 4 * 6$$

$$RC * 5 = 24$$

$$RC = 4.8$$

$$RM = 2 (RC)$$

$$= 2 * 4.8$$

$$= 9.6$$

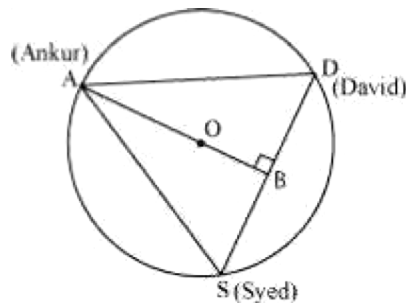
Therefore, the distance between Reshma and Mandip is 9.6 m

Question 6.

A circular park of radius 20m is situated in a colony. Three boys Ankur, Syed and David are sitting at equal distance on its boundary each having a toy telephone in his hands to talk each other. Find the length of the string of each phone.

Solution.

It is given that $AS = SD = DA$



Therefore, $\triangle ASD$ is an equilateral triangle

OA (radius) = 20 m

Medians of equilateral triangle pass through the circum centre (O) of the equilateral triangle ASD. We also know that medians intersect each other in the ratio 2: 1.

As, AB is the median of equilateral triangle ASD

$$\frac{OA}{OB} = \frac{2}{1}$$

$$\frac{20}{OB} = \frac{2}{1}$$

$$OB = \left(\frac{20}{2}\right)m$$

$$= 10m$$

$$AB = OA + OB$$

$$= (20 + 10) m$$

$$= 30 m$$

In $\triangle ABD$,

$$AD^2 = AB^2 + BD^2$$

$$AD^2 = (30)^2 + \left(\frac{AD}{2}\right)^2$$

$$AD^2 = 900 + \frac{1}{4} AD^2$$

$$\frac{3}{4} AD^2 = 900$$

$$AD^2 = 1200$$

$$AD = 20\sqrt{3}$$

Therefore, the length of the string of each phone will be $20\sqrt{3}$ m

