

## EXERCISE 29.4

Evaluate the following limits:

$$1. \lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x}$$

**Solution:**

Given:  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x}$

The limit  $\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x}$

We need to find the limit of the given equation when  $x \Rightarrow 0$

Now let us substitute the value of  $x$  as 0, we get an indeterminate form of  $0/0$ .

Let us rationalizing the given equation, we get

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x} = \lim_{x \rightarrow 0} \frac{(\sqrt{1+x+x^2} - 1)(\sqrt{1+x+x^2} + 1)}{x(\sqrt{1+x+x^2} + 1)}$$

[By using the formula:  $(a + b)(a - b) = a^2 - b^2$ ]

$$= \lim_{x \rightarrow 0} \frac{1 + x + x^2 - 1}{x(\sqrt{1+x+x^2} + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{x(1+x)}{x(\sqrt{1+x+x^2} + 1)}$$

$$= \lim_{x \rightarrow 0} \frac{(1+x)}{(\sqrt{1+x+x^2} + 1)}$$

Now we can see that the indeterminate form is removed,

So, now we can substitute the value of  $x$  as 0, we get

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{\sqrt{1+x+x^2} - 1}{x} &= \frac{1}{1+1} \\ &= \frac{1}{2} \end{aligned}$$

∴ The value of the given limit is  $\frac{1}{2}$ .

$$2. \lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}}$$

**Solution:**

Given:  $\lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}}$

The limit  $\lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}}$

We need to find the limit of the given equation when  $x \Rightarrow 0$

Now let us substitute the value of  $x$  as 0, we get an indeterminate form of  $0/0$ .

Let us rationalizing the given equation, we get

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}} &= \lim_{x \rightarrow 0} \frac{2x}{(\sqrt{a+x} - \sqrt{a-x})} \frac{(\sqrt{a+x} + \sqrt{a-x})}{(\sqrt{a+x} + \sqrt{a-x})} \\ \text{[By using the formula: } (a+b)(a-b) &= a^2 - b^2] \\ &= \lim_{x \rightarrow 0} \frac{2x(\sqrt{a+x} + \sqrt{a-x})}{a+x - a+x} \\ &= \lim_{x \rightarrow 0} \frac{2x(\sqrt{a+x} + \sqrt{a-x})}{2x} \\ &= \lim_{x \rightarrow 0} \frac{(\sqrt{a+x} + \sqrt{a-x})}{1} \end{aligned}$$

Now we can see that the indeterminate form is removed,  
So, now we can substitute the value of x as 0, we get

$$\begin{aligned} \lim_{x \rightarrow 0} \frac{2x}{\sqrt{a+x} - \sqrt{a-x}} &= \sqrt{a} + \sqrt{a} \\ &= 2\sqrt{a} \end{aligned}$$

∴ The value of the given limit is  $2\sqrt{a}$

3.  $\lim_{x \rightarrow 0} \frac{\sqrt{a^2 + x^2} - a}{x^2}$

**Solution:**

Given:  $\lim_{x \rightarrow 0} \frac{\sqrt{a^2 + x^2} - a}{x^2}$

The limit

We need to find the limit of the given equation when  $x \Rightarrow 0$

Now let us substitute the value of x as 0, we get an indeterminate form of 0/0.

Let us rationalizing the given equation, we get

$$\lim_{x \rightarrow 0} \frac{\sqrt{a^2 + x^2} - a}{x^2} = \lim_{x \rightarrow 0} \frac{(\sqrt{a^2 + x^2} - a)(\sqrt{a^2 + x^2} + a)}{x^2(\sqrt{a^2 + x^2} + a)}$$

[By using the formula:  $(a+b)(a-b) = a^2 - b^2$ ]

$$\begin{aligned} &= \lim_{x \rightarrow 0} \frac{(a^2 + x^2 - a^2)}{x^2(\sqrt{a^2 + x^2} + a)} \\ &= \lim_{x \rightarrow 0} \frac{x^2}{x^2(\sqrt{a^2 + x^2} + a)} \\ &= \lim_{x \rightarrow 0} \frac{1}{(\sqrt{a^2 + x^2} + a)} \end{aligned}$$

Now we can see that the indeterminate form is removed,

So, now we can substitute the value of  $x$  as 0, we get

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sqrt{a^2+x^2} - a}{x^2} &= \frac{1}{a+a} \\ &= \frac{1}{2a}\end{aligned}$$

∴ The value of the given limit is  $\frac{1}{2a}$ .

4. 
$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{2x}$$

**Solution:**

Given: 
$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{2x}$$

The limit

We need to find the limit of the given equation when  $x \Rightarrow 0$

Now let us substitute the value of  $x$  as 0, we get an indeterminate form of  $0/0$ .

Let us rationalizing the given equation, we get

$$\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{2x} = \lim_{x \rightarrow 0} \frac{(\sqrt{1+x} - \sqrt{1-x})(\sqrt{1+x} + \sqrt{1-x})}{2x(\sqrt{1+x} + \sqrt{1-x})}$$

[By using the formula:  $(a + b)(a - b) = a^2 - b^2$ ]

$$\begin{aligned}&= \lim_{x \rightarrow 0} \frac{1+x - 1+x}{2x(\sqrt{1+x} + \sqrt{1-x})} \\ &= \lim_{x \rightarrow 0} \frac{2x}{2x(\sqrt{1+x} + \sqrt{1-x})} \\ &= \lim_{x \rightarrow 0} \frac{1}{(\sqrt{1+x} + \sqrt{1-x})}\end{aligned}$$

Now we can see that the indeterminate form is removed,

So, now we can substitute the value of  $x$  as 0, we get

$$\begin{aligned}\lim_{x \rightarrow 0} \frac{\sqrt{1+x} - \sqrt{1-x}}{2x} &= \frac{1}{1+1} \\ &= \frac{1}{2}\end{aligned}$$

∴ The value of the given limit is  $\frac{1}{2}$ .

$$5. \lim_{x \rightarrow 2} \frac{\sqrt{3-x} - 1}{2-x}$$

**Solution:**

Given:  $\lim_{x \rightarrow 2} \frac{\sqrt{3-x} - 1}{2-x}$

The limit  $\lim_{x \rightarrow 2} \frac{\sqrt{3-x} - 1}{2-x}$

We need to find the limit of the given equation when  $x \Rightarrow 0$

Now let us substitute the value of  $x$  as 0, we get an indeterminate form of  $0/0$ .

Let us rationalizing the given equation, we get

$$\lim_{x \rightarrow 2} \frac{\sqrt{3-x} - 1}{2-x} = \lim_{x \rightarrow 2} \frac{(\sqrt{3-x} - 1)(\sqrt{3-x} + 1)}{(2-x)(\sqrt{3-x} + 1)}$$

[By using the formula:  $(a + b)(a - b) = a^2 - b^2$ ]

$$= \lim_{x \rightarrow 2} \frac{(3-x-1)}{(2-x)(\sqrt{3-x} + 1)}$$

$$= \lim_{x \rightarrow 2} \frac{(2-x)}{(2-x)(\sqrt{3-x} + 1)}$$

$$= \lim_{x \rightarrow 2} \frac{1}{(\sqrt{3-x} + 1)}$$

Now we can see that the indeterminate form is removed,

So, now we can substitute the value of  $x$  as 0, we get

$$\begin{aligned} \lim_{x \rightarrow 2} \frac{\sqrt{3-x} - 1}{2-x} &= \frac{1}{1+1} \\ &= \frac{1}{2} \end{aligned}$$

$\therefore$  The value of the given limit is  $\frac{1}{2}$ .