Exercise 3.9

1. A father is three times as old as his son. After twelve years, his age will be twice as that of his son then. Find their present ages.
Solution:

Let's assume the present ages of the father as x years and that of his son's age as y years.
From the question it's given that,
Father is 3 times as old as his son. (Present)
So, the equation formed is

$x = 3y$
$\Rightarrow x - 3y = 0$……. (i)

Also again from the question it's given as,
After 12 years, father's age will be (x+12) years and son's age will be (y+12) years.
Furthermore, the relation between their ages after 12 years is given below

$x + 12 = 2(y + 12)$
$\Rightarrow x + 12 = 2y + 24$
$\Rightarrow x - 2y - 12 = 0$…… (ii)

Solving (i) and (ii), we get the solution
By using cross-multiplication, we have

$$\frac{x}{(-3)\times(-12)-(-2)\times 0} = \frac{-y}{1\times(-12)-1\times 0} = \frac{1}{1\times(-2)-1\times(-3)}$$

$$\Rightarrow \frac{x}{36-0} = \frac{-y}{-12-0} \quad \frac{1}{-2+3}$$

$$\frac{x}{36} = \frac{-y}{-12} = \frac{1}{1}$$

$$\frac{x}{36} = \frac{y}{12} = 1$$

$x = 36, y = 12$

Hence, the present age of father is 36 years and the present age of son is 12 years.

2. Ten years later, A will be twice as old as B and five years ago, A was three times as old as B. What are the present ages of A and B.
Solution:

Let the present ages of A be x years and that of B be y years
From the question it's given that,
After 10 years, A's age will be (x +10) years and B's age will be (y + 10) years.
Furthermore, the relation between their ages after 10 years is given below

$x + 10 = 2(y + 10)$
$\Rightarrow x + 10 = 2y + 20$
$\Rightarrow x - 2y - 10 = 0$.....(i)

Also again from the question it's given as,
Before 5 years, the age of A was (x – 5) years and the age of B was (y – 5) years.
So, the equation formed is

x – 5 = 3(y-5)
⇒ x – 5 = 3y – 15
⇒ x – 3y + 10 = 0..... (ii)

Thus, by solving (i) and (ii), we get the required solution

Using cross-multiplication, we get,

$$\frac{x}{(-2)\times 10-(-3)\times(-10)} = \frac{-y}{1\times 10-1\times(-10)} = \frac{1}{1\times(-3)-1\times(-2)}$$

$$\Rightarrow \frac{x}{-20-3-} = \frac{-y}{10+10} = \frac{1}{-3+2}$$

$$\Rightarrow \frac{x}{-50} = \frac{-y}{20} = \frac{1}{-1}$$

$$\Rightarrow \frac{x}{50} = \frac{y}{20} = 1$$

⇒x = 50, y = 20

Hence, the present age of A is 50 years and the present age if B is 20 years.

3. A is elder to B by 2 years. A's father F is twice as old as A and B is twice as old as his sister S. If the age of the father and sister differ by 40 years, find the age of A.
Solution:

Assuming that, the present age of A = x
the present age of B = y
the present age of F = z
the present age of S = t

It's understood from the question that,
A is elder to b by 2 years. ⇒ x = y + 2
F is twice as old as A. ⇒ z = 2x
B is twice as old as S. ⇒ y = 2t

Also given that the ages of F and S is differing by 40 years. ⇒ z – t = 40.
So, the four equations are:

x = y + 2 … (i)
z = 2x … (ii)
y = 2t … (iii)
z – t = 40 …(iv)

It's clearly seen from the equations obtained that x, y, z and t are unknowns.
And we have to find the value of x.

So, by using equation (iii) in (i),
(i) Becomes x = 2t + 2

From (iv), we have t = z – 40
Hence, we get

$x = 2(z - 40) + 2$
$= 2z - 80 + 2$
$= 2z - 78$

Using the equation (ii), we have

$x = 2 \times 2x - 78$
$\Rightarrow x = 4x - 78$
$\Rightarrow 4x - x = 78$
$\Rightarrow 3x = 78$
$\Rightarrow x = 78/3$
$\Rightarrow x = 26$

Hence, the age of A is 26 years.

4. Six year hence a man's age will be three times age of his son and three years ago, he was nine times as old as his son. Find their present ages.
Solution:

Let's assume the present ages of the father as x years and that of his son's age as y years.
From the question it's given that,
After 6 years, the man's age will be (x + 6) years and son's age will be (y + 6) years.
So, the equation formed is

$x + 6 = 3(y + 6)$
$x + 6 = 3y + 18$
$x - 3y - 12 = 0$……. (i)

Also again from the question it's given as,
Before 3 years, the age of the man was (x – 3) years and the age of son's was (y - 3) years.
Furthermore, the relation between their 3 years ago is given below

$x - 3 = 9(y - 3)$
$x - 3 = 9y - 27$
$x - 9y + 24 = 0$……. (ii)

Thus, by solving (i) and (ii), we get the required solution

Using cross-multiplication, we get

$$\frac{x}{(-3)\times 24-(-9)\times(-12)} = \frac{-y}{1\times 24-1\times(-12)} = \frac{1}{1\times(-9)-1\times(-3)}$$

$$\Rightarrow \frac{x}{-72-108} = \frac{-y}{24+12} = \frac{1}{9+3}$$

$$\Rightarrow \frac{x}{-180} = \frac{-y}{36} = \frac{1}{-9+3}$$

$$\Rightarrow \frac{x}{180} = \frac{y}{36} = \frac{1}{6}$$

$$\Rightarrow x = \frac{180}{6}, y = \frac{36}{6}$$

⇒x = 30, y = 6

Hence, the present age of the man is 30 years and the present age of son is 6 years.

5. Ten years ago, a father was twelve times as old as his son and ten years hence, he will be twice as old as his son will be then. Find their present ages.
Solution:

Let's assume the present ages of the father as x years and that of his son's age as y years.
From the question it's given that,
After 10 years, father's age will be (x+10) years and son's age will be (y + 10) years.
So, the equation formed is

x + 10 = 2(y + 10)
x - 10 = 2y + 20
x - 2y – 10 = 0…… (i)

Also again from the question it's given as,
Before 10 years, the age of father was (x - 10) years and the age of son was (y - 10) years.
Furthermore, the relation between their 10 years ago is given below

x - 10 = 12(y - 10)
x - 10 = 12y - 120
x - 12y + 110 = 0…… (ii)

Thus, by solving (i) and (ii), we get the required solution

Using cross-multiplication, we have

$$\frac{x}{(-2)\times 110-(-12)\times(-10)} = \frac{-y}{1\times 110-1\times(-10)} = \frac{1}{1\times(-12)-1\times(-12)}$$

$$\Rightarrow \frac{x}{-220-120} = \frac{-y}{110+10} = \frac{1}{-12+2}$$

$$\Rightarrow \frac{x}{-340} = \frac{-y}{120} = \frac{1}{-10}$$

$$\Rightarrow \frac{x}{340} = \frac{y}{120} = \frac{1}{10}$$

$$\Rightarrow x = \frac{340}{10}, y = \frac{120}{10}$$

⇒x = 34, y = 12

Hence, the present age of father is 34 years and the present age of the son is 12 years.

6. The present age of father is 3 years more than three times of the age of the son. Three years hence, father's age will be 10 years more than twice the age of the son. Determine their present age.

Solution:

Let's assume the present ages of the father as x years and that of his son's age as y years.
From the question it's given that,
The present age of father is three years more than three times the age of the son.
So, the equation formed is

x = 3y + 3

x - 3y -3 = 0..... (i)

Also again from the question it's given as,
After 3 years, father's age will be (x + 3) years and son's age will be (y + 3) years.
Furthermore, the relation between their ages after 3 years is given below

x + 3 = 2(y + 3) + 10

x - 2y - 13 = 0..... (ii)

Thus, by solving (i) and (ii), we get the required solution

Using cross-multiplication, we have

$$\frac{x}{(-3)\times(-13)-(-2)\times(-3)} = \frac{-y}{1\times(-13)-1\times(-3)} = \frac{1}{1\times(-2)-1\times(-3)}$$

$$\Rightarrow \frac{x}{39-6} = \frac{-y}{-13+3} = \frac{1}{-2+3}$$

$$\Rightarrow \frac{x}{33} = \frac{-y}{-10} = \frac{1}{1}$$

$$\Rightarrow \frac{x}{33} = \frac{y}{10} = 1$$

⇒x = 33, y = 10

Hence,
The present age of father = 33 years and the present age of his son = 10 years.