EXERCISE 28.4

Q1.
Solution:

Let us consider,

The foot of the perpendicular drawn from P(3, -1, 11) to the line $\frac{x}{2}=\frac{y-2}{-3}=\frac{z-3}{4}$ is Q.
So let us find the length of PQ.

Q is the general point on the line $\frac{x}{2}=\frac{y-2}{-3}=\frac{z-3}{4}$

Let us consider $\frac{x}{2}=\frac{y-2}{-3}=\frac{z-3}{4}=\lambda$

Co-ordinate of Q = $(2\lambda, -3\lambda+2, 4\lambda+3)$
The direction ratios of the given line = 2, -3, 4

Since PQ is perpendicular to the given line,
$a_1a_2 + b_1b_2 + c_1c_2 = 0$
$\Rightarrow 2(2\lambda-3)+(-3)(-3\lambda+3)+4(4\lambda-8)=0$
$\Rightarrow 4\lambda-6+9\lambda-9+16\lambda-32=0$
$\Rightarrow 29\lambda-47=0$
$\Rightarrow \lambda=\frac{47}{29}$

So, the Co-ordinates of Q are
$=2\left(\frac{47}{29}\right),-3\left(\frac{47}{29}\right)+2,4\left(\frac{47}{29}\right)+3$
$=\frac{94}{29},\frac{-83}{29},\frac{275}{29}$

Distance between P and Q is given as

$$=\sqrt{\left(\frac{94}{29}-3\right)^2+\left(\frac{-83}{29}+1\right)^2+\left(\frac{275}{29}-11\right)^2}$$

$$=\sqrt{\left(\frac{94-87}{2}\right)^2+\left(\frac{-83+29}{2}\right)^2+\left(\frac{275-319}{2}\right)^2}$$

$$=\sqrt{\left(\frac{7}{29}\right)^2+\left(\frac{-54}{29}\right)^2+\left(\frac{-44}{29}\right)^2}$$

$$=\sqrt{\frac{49}{841}+\frac{2916}{841}+\frac{1936}{841}}$$

$$=\sqrt{\frac{4901}{841}}$$

Hence, the required distance is √(4901/841) units.

Q2.

Solution:

Let us consider,

The foot of the perpendicular drawn from P (1, 0, 0) to the line

$\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ is Q.

So let us find the length of PQ.

Q is the general point on the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$

Let us consider $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}=\lambda$

Co-ordinate of Q = (2λ+1, -3λ-1, 8λ-10)

The direction ratios of the given line = (2λ+1-1), (-3λ-1-1), (8λ-10-0)

= (2λ), (-3λ-1), (8λ-10)

Since PQ is perpendicular to the given line,

$$a_1a_2+b_1b_2+c_1c_2=0$$

$$(2)(2\lambda)+(-3)(-3\lambda-1)+8(8\lambda-10)=0$$

$$4\lambda+9\lambda+3+64\lambda-80=0$$

$$77\lambda-77=0$$

$$\lambda=1$$

So, the Co-ordinates of Q are

(2λ+1, -3λ-1, 8λ-10) = [2(1)+1, -3(1)-1, 8(1)-10]

= [3, 4, -2]

Distance between P and Q is given as

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$
$$= \sqrt{(1-3)^2 + (0+4)^2 + (0+2)^2}$$
$$= \sqrt{4+16+4}$$
$$= \sqrt{24}$$
$$= 2\sqrt{6}$$

Hence,
The foot of perpendicular = (3, -4, -2)
Length of perpendicular = 2√6 units.

Q3.
Solution:
Let us consider,
The foot of the perpendicular drawn from A(1, 0, 3) to the line joining the points B(4,7,1) and C(3,5,3) be D.
The equation of line passing through points B(4,7,1) and C(3,5,3) is

$$\frac{x-x1}{x2-x1} = \frac{y-y1}{y2-y1} = \frac{z-z1}{z2-z1}$$
$$\Rightarrow \frac{x-4}{3-4} = \frac{y-7}{5-7} = \frac{z-1}{3-1}$$
$$\Rightarrow \frac{x-4}{-1} = \frac{y-7}{-2} = \frac{z-1}{2}$$

Let $\frac{x-4}{-1} = \frac{y-7}{-2} = \frac{z-1}{2} = \lambda$

So, the direction ratio of AD is (-λ+4-1), (-2λ+7-0), (2λ+1-3)
= (-λ+3), (-2λ+7), (2λ-2)

Line AD is perpendicular to BC so,
$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$\Rightarrow (-1)(-\lambda+3) + (-2)(-2\lambda+7) + 2(2\lambda-2) = 0$$
$$\Rightarrow \lambda - 3 + 4\lambda - 14 + 4\lambda - 4 = 0$$
$$\Rightarrow 9\lambda - 21 = 0$$
$$\Rightarrow \lambda = \frac{21}{9}$$

Hence,
Coordinates of D are:

$$= \left(-\frac{21}{9} + 4, (-2)\left(\frac{21}{9} + 7\right), 2\left(\frac{21}{9} + 1\right)\right)$$
$$= \left(\frac{15}{9}, \frac{21}{9}, \frac{51}{9}\right)$$
$$= \left(\frac{5}{3}, \frac{7}{3}, \frac{17}{3}\right)$$

Q4.
Solution:
Given:
D is the foot of perpendicular from A(1, 0, 4) on BC.
So,
Equation of line passing through B, C is

$$\frac{x - x_1}{x_2 - x_1} = \frac{y - y_1}{y_2 - y_1} = \frac{z - z_1}{z_2 - z_1}$$

$$\frac{x - 0}{2 - 0} = \frac{y + 11}{-3 + 11} = \frac{z - 3}{1 - 3}$$

$$\frac{x}{2} = \frac{y + 11}{8} = \frac{z - 3}{-2}$$

Let $\frac{x}{2} = \frac{y + 11}{8} = \frac{z - 3}{-2} = \lambda$

Coordinates of D = $(2\lambda, 8\lambda-11, -2\lambda+3)$

Direction ratios of AD is $(2\lambda-1), (8\lambda-11-0), (-2\lambda+3-4)$
$= (2\lambda-1), (8\lambda-11), (-2\lambda-1)$

Line AD is perpendicular to BC so,
$a_1a_2 + b_1b_2 + c_1c_2 = 0$

$$(2)(2\lambda - 1) + (8)(8\lambda - 11) + (-2)(-2\lambda - 1) = 0$$

$$4\lambda - 2 + 64\lambda - 88 + 4\lambda + 2 = 0$$

$$72\lambda - 88 = 0$$

$$\lambda = \frac{88}{72}$$

$$\lambda = \frac{11}{9}$$

Hence,
Coordinates of D = $(2\lambda, 8\lambda-11, -2\lambda+3)$

$$= \left(2\left(\frac{11}{9}\right), 8\left(\frac{11}{9}\right) - 11, -2\left(\frac{11}{9}\right) + 3\right)$$

$$= \left(\frac{22}{9}, \frac{-11}{9}, \frac{5}{9}\right)$$

Q5.
Solution:
Let us consider,

The foot of the perpendicular drawn from P(2, 3, 4) to the line $\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$ is θ.
Equation of given line is,

$$\frac{4-x}{2}=\frac{y}{6}=\frac{1-z}{3}$$

Let $\frac{x-4}{-2}=\frac{y}{6}=\frac{z-1}{-3}=\lambda$

Coordinates of Q = (-2λ+4, 6λ, -3λ+1)

Direction ratios of PQ = (-2λ+4-2), (6λ-3), (-3λ+1-4)
= (-2λ+2), (6λ-3), (-3λ-3)

Since PQ is perpendicular to the given line,

$$a_1a_2+b_1b_2+c_1c_2=0$$
$$(-2)(-2\lambda+2)+(6)(6\lambda-3)+(-3)(-3\lambda-3)=0$$
$$4\lambda-4+36\lambda-18+9\lambda+9=0$$
$$49\lambda-13=0$$
$$\lambda=\frac{13}{49}$$

Coordinates of Q = (-2λ+4, 6λ, -3λ+1)

$$=\left(-2\left(\frac{13}{49}\right)+4,\ 6\left(\frac{13}{49}\right),\ -3\left(\frac{13}{49}\right)+1\right)$$
$$=\left(\frac{-26+196}{49},\ \frac{78}{49},\ \frac{-39+49}{49}\right)$$
$$=\left(\frac{170}{49},\ \frac{78}{49},\ \frac{10}{49}\right)$$

Distance between P and Q is given as

$$PQ = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$
$$= \sqrt{\left(\frac{170}{49} - 2\right)^2 + \left(\frac{78}{49} - 3\right)^2 + \left(\frac{10}{49} - 4\right)^2}$$
$$= \sqrt{\left(\frac{72}{49}\right)^2 + \left(\frac{69}{49}\right)^2 + \left(-\frac{168}{49}\right)^2}$$
$$= \sqrt{\frac{5184 + 4761 + 34596}{2401}}$$
$$= \sqrt{\frac{44541}{2401}}$$
$$= \sqrt{\frac{909}{49}}$$
$$= \frac{3\sqrt{101}}{49}$$

Hence,
Perpendicular distance from (2,3,4) to given line is (3√101)/49 units.