NCERT Solutions for Class 9 Maths Chapter 2 Polynomials Ex 2.4

Question 1.

Determine which of the following polynomials has (x + 1) a factor:

(i) $x^3 + x^2 + x + 1$

(ii) $x^4 + x^3 + x^2 + x + 1$

(iii) $x^4 + 3x^3 + 3x^2 + x + 1$

(iv) $x^3 - x^2 - \left(2 + \sqrt{2}\right)x + \sqrt{2}$

Solution.

(i) If (x + 1) is a factor of p (x) = $x^3 + x^2 + x + 1$, p (-1) must be zero

Here, p (x) = $x^3 + x^2 + x + 1$

p (-1) = $(-1)^3 + (-1)^2 + (-1) + 1$

= -1 + 1 – 1 + 1 = 0

Therefore, x + 1 is a factor of this polynomial

(ii) If (x + 1) is a factor of p (x) = $x^4 + x^3 + x^2 + x + 1$, p (-1) must be zero

Here, p (x) = $x^4 + x^3 + x^2 + x + 1$

p (-1) = $(-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

= 1 - 1 + 1 – 1 + 1 ≠ 0

Therefore, x + 1 is not a factor of this polynomial

(iii) If (x + 1) is a factor of p (x) = $x^4 + 3x^3 + 3x^2 + x + 1$, p (-1) must be zero

Here, $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$

$p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 \neq 0$

Therefore, x + 1 is not a factor of this polynomial

(iv) If (x + 1) is a factor of polynomial

$p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$, p(-1) must be zero

$p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2}$

$= -1 - 1 + 2 + \sqrt{2} + \sqrt{2}$

$= 2\sqrt{2}$

As, $p(-1) \neq 0$

Therefore, x + 1 is not a factor of this polynomial

Question 2.

Use the Factor Theorem to determine whether g(x) is a factor of p(x) in each of the following cases:

(i) $p(x) = 2x^3 + x^2 - 2x - 1, g(x) = x + 1$

(ii) $p(x) = x^3 + 3x^2 + 3x + 1, g(x) = x + 2$

(iii) $p(x) = x^3 - 4x^2 + x + 6, g(x) = x - 3$

Solution:

(i) If g (x) = x + 1 is a factor of given polynomial p (x), p (-1) must be zero

$p(x) = 2x^3 + x^2 - 2x - 1$

$p(-1) = 2(-1)^3 + (-1)^2 - 2(-1) - 1$

$= 2(-1) + 1 + 2 - 1 = 0$

Hence, $g(x) = x + 1$ is a factor of given polynomial

(ii) If $g(x) = x + 2$ is a factor of given polynomial $p(x)$, $p(-2)$ must be zero

$p(x) = x^3 + 3x^2 + 3x + 1$

$p(-2) = (-2)^3 + 3(-2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1 = -1$

As, $p(-2) \neq 0$

Hence, $g(x) = x + 2$ is not a factor of given polynomial

(iii) If $g(x) = x - 3$ is a factor of given polynomial $p(x)$, $p(3)$ must be zero

$p(x) = x^3 - 4x^2 + x + 6$

$p(3) = (3)^3 - 4(3)^2 + 3 + 6$

$= 27 - 36 + 9 = 0$

Therefore, $g(x) = x - 3$ is a factor of the given polynomial

Question 3.

Find the value of k, if $x - 1$ is a factor of $p(x)$ in each of the following cases:

(i) $p(x) = x^2 + x + k$

(ii) $p(x) = 2x^2 + kx + \sqrt{2}$

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$

(iv) $p(x) = kx^2 - 3x + k$

Solution.

(i) If $x - 1$ is a factor of polynomial $p(x) = x^2 + x + k$, then

$p(1) = 0$

$(1)^2 + 1 + k = 0$

$2 + k = 0$

$k = -2$

Therefore, value of k is -2

(ii) If $x - 1$ is a factor of polynomial $p(x) = 2x^2 + kx + \sqrt{2}$, then

$p(1) = 0$

$2(1)^2 + k(1) + \sqrt{2} = 0$

$2 + k + \sqrt{2} = 0$

$k = -2 - \sqrt{2}$

$= -(2 + \sqrt{2})$

Therefore, value of k is $-(2 + \sqrt{2})$

(iii) If $x - 1$ is a factor of given polynomial $p(x) = kx^2 - \sqrt{2}x + 1$, then

$p(1) = 0$

$k(1)^2 - \sqrt{2}(1) + 1 = 0$

$k - \sqrt{2} + 1 = 0$

$k = \sqrt{2} - 1$

Therefore, value of k is $\sqrt{2} - 1$

(iv) If $x - 1$ is a factor of the given polynomial $p(x) = kx^2 - 3x + k$, then

$p(1) = 0$

$k(1)^2 + 3(1) + k = 0$

$k – 3 + k = 0$

$2k – 3 = 0$

$k = 3/2$

Therefore, value of k is 3/2

Question 4.

Factorize:

(i) $12x^2 - 7x + 1$

(ii) $2x^2 + 7x + 3$

(iii) $6x^2 + +5x - 6$

(iv) $3x^2 - x - 4$

Solution:

(i) $12x^2 - 7x + 1$

$= 12x^2 – 4x – 3x + 1$

$= 4x (3x – 1) – 1 (3x – 1)$

$= (3x – 1) (4x – 1)$

(ii) $2x^2 + 6x + x + 3$

$= 2x (x + 3) + 1 (x + 3)$

$= (x + 3) (2x + 1)$

(iii) $6x^2 + 5x – 6$

$= 6x^2 + 9x - 4x - 6$

$= 3x (2x + 3) - 2 (2x + 3)$

$= (2x + 3) (3x - 2)$

(iv) $3x^2 - 4x + 3x - 4$

$= x (3x - 4) + 1 (3x - 4)$

$= (3x - 4) (x + 1)$

Question 5.

Factorize:

(i) $x^3 - 2x^2 - x + +2$

(ii) $x^3 - 3x^2 - 9x - 5$

(iii) $x^3 + 13x^2 + +32x + 20$

(iv) $2y^3 + y^2 - 2y - 1$

Solution:

(i) Let p (x) = $x^3 - 2x^2 - x + 2$

Factors of 2 are ±1 and ±2

By trial method, we find that

p (1) = 0

So, (x + 1) is a factor of p (x)

Now,

p (x) = $x^3 - 2x^2 - x + 2$

$p(-1) = (-1)^3 - 2(-1)^2 - (-1) + 2$

$= -1 - 2 + 1 + 2 = 0$

Therefore, $(x + 1)$ is the factor of $p(x)$

$$
\begin{array}{r|l}
 & x^2 - 3x + 2 \\
\hline
x+1 & x^3 - 2x^2 - x + 2 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & -3x^2 - x + 2 \\
 & -3x^2 - 3x \\
 & + \quad + \\
\hline
 & 2x + 2 \\
 & 2x + 2 \\
 & - \quad - \\
\hline
 & 0
\end{array}
$$

Now, Dividend = Divisor * Quotient + Remainder

$(x + 1)(x^2 - 3x + 2)$

$= (x + 1)(x^2 - x - 2x + 2)$

$= (x + 1)\{x(x - 1) - 2(x - 1)\}$

$= (x + 1)(x - 1)(x + 2)$

(ii) Let $p(x) = x^3 - 3x^2 - 9x - 5$

Factors of 5 are ±1 and ±5

By trial method, we find that

$p(5) = 0$

So, $(x - 5)$ is a factor of $p(x)$

Now,

p (x) = $x^3 - 2x^2 - x + 2$

p (5) = $(5)^3 - 3\,(5)^2 - 9\,(5) - 5$

= 125 – 75 – 45 – 5 = 0

Therefore, (x – 5) is the factor of p (x)

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
x-5 & x^3 - 3x^2 - 9x - 5 \\
 & x^3 - 5x^2 \\
 & -\quad + \\
\hline
 & \quad 2x^2 - 9x - 5 \\
 & \quad 2x^2 - 10x \\
 & \quad -\quad + \\
\hline
 & \qquad x - 5 \\
 & \qquad x - 5 \\
 & \qquad -\quad + \\
\hline
 & \qquad\quad 0
\end{array}
$$

Now, Dividend = Divisor * Quotient + Remainder

$(x - 5)\,(x^2 + 2x + 1)$

$= (x - 5)\,(x^2 + x + x + 1)$

$= (x - 5)\,\{x\,(x + 1) + 1\,(x + 1)\}$\= $(x - 5)\,(x + 1)\,(x + 1)$

(iii) Let p (x) = $x^3 + 13x^2 + 32x + 20$

Factors of 20 are ±1, ±2, ±4, ±5, ±10 and ±20

By trial method, we find that

p (-1) = 0

So, (x + 1) is factor of p (x)

Now,

p (x) = $x^3 + 13x^2 + 32x + 20$

p (-1) = $(-1)^3 + 13\ (-1)^2 + 32\ (-1) + 20$

= -1 + 13 – 32 + 20 = 0

Therefore, (x + 1) is the factor of p (x)

$$
\begin{array}{r|l}
 & x^2 + 12x + 20 \\
\hline
x+1 & x^3 + 13x^2 + 32x + 20 \\
 & x^3 + x^2 \\
 & - \quad - \\
\hline
 & \quad 12x^2 + 32x + 20 \\
 & \quad 12x^2 + 12x \\
 & \quad - \quad\quad - \\
\hline
 & \quad\quad 20x + 20 \\
 & \quad\quad 20x + 20 \\
 & \quad\quad - \quad - \\
\hline
 & \quad\quad\quad 0
\end{array}
$$

Now, Dividend = Divisor * Quotient + Remainder

(x + 1) (x^2 + 2x + 20)

= (x + 1) (x^2 + 2x + 10x + 20)

= (x – 5) {x (x + 2) + 10 (x + 2)}

= (x – 5) (x + 2) (x + 10)

(iv) Let, p(y) = $2y^3 + y^2 – 2y – 1$

Factors of ab = 2 * (-1) = -2 are ±1 and ±2

By trial method, we find that

p (1) = 0

So, (y – 1) is a factor of p (y)

Now,

p (y) = $2y^3 + y^2 - 2y - 1$

p (1) = $2(1)^3 + (1)^2 - 2(1) - 1$

= 2 + 1 -1 -1 = 0

Therefore, (y – 1) is the factor if p (y)

$$
\begin{array}{r|l}
 & 2y^2 + 3y + 1 \\
\hline
y-1 & 2y^3 + y^2 - 2y - 1 \\
 & 2y^3 - 2y^2 \\
 & - \quad + \\
\hline
 & \quad 3y^2 - 2y - 1 \\
 & \quad 3y^2 - 3y \\
 & \quad - \quad + \\
\hline
 & \qquad y - 1 \\
 & \qquad y - 1 \\
 & \qquad - \quad + \\
\hline
 & \qquad\quad 0
\end{array}
$$

Now, Dividend = Divisor * Quotient + Remainder

$(y - 1)(2y^2 + 3y + 1)$

$= (y - 1)(2y^2 + 2y + y + 1)$

$= (y - 1)\{2y(y + 1) + 1(y + 1)\}$

$= (y - 1)(2y + 1)(y + 1)$