$$=\left(\frac{-6+21}{3+7},\frac{21-21}{3+7}\right)$$

$$=\left(\frac{3}{2},0\right)$$

Hence, the required ratio is $3:7$ and the point of division is $\left(\frac{3}{2},0\right)$

**30.** The base QR of a n equilateral triangle PQR lies on x-axis. The coordinates of the point Q are (-4, 0) and origin is the midpoint of the base. Find the coordinates of the points P and R.

**Sol:**

Let $(x,0)$ be the coordinates of $R$. Then

$$0=\frac{-4+x}{2}\Rightarrow x=4$$

Thus, the coordinates of $R$ are $(4,0)$.

Here, $PQ=QR=PR$ and the coordinates of $P$ lies on $y-axis$. Let the coordinates of $P$ be $(0,y)$. Then,

$$PQ=QR\Rightarrow PQ^2=QR^2$$

$$\Rightarrow(0+4)^2+(y-0)^2=8^2$$

$$\Rightarrow y^2=64-16=48$$

$$\Rightarrow y=\pm4\sqrt{3}$$

Hence, the required coordinates are $R(4,0)$ *and* $P\left(0,4\sqrt{3}\right)$ *or* $P\left(0,-4\sqrt{3}\right)$.

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**31.** The base BC of an equilateral triangle ABC lies on y-axis. The coordinates of point C are (0, -3). The origin is the midpoint of the base. Find the coordinates of the points A and B. Also, find the coordinates of another point D such that ABCD is a rhombus.

**Sol:**

Let $(0,y)$ be the coordinates of $B$. Then

$$0=\frac{-3+y}{2}\Rightarrow y=3$$

Thus, the coordinates of $B$ are $(0,3)$

Here. $AB=BC=AC$ and by symmetry the coordinates of A lies on $x$-axis Let the coordinates of A be $(x, 0)$. Then

$$AB=BC\Rightarrow AB^2=BC^2$$

$$\Rightarrow(x-0)^2+(0-3)^2=6^2$$

$\Rightarrow x^2 = 36 - 9 = 27$

$\Rightarrow x = \pm 3\sqrt{3}$

If the coordinates of point $A$ are $\left(3,\sqrt{3},0\right)$. then the coordinates of $D$ are $\left(-3\sqrt{3},0\right)$.

If the coordinates of point $A$ are $\left(-3\sqrt{3},0\right)$. then the coordinates of $D$ are $\left(-3\sqrt{3},0\right)$.

Hence the required coordinates are $A\left(3\sqrt{3},0\right), B\left(0,3\right)$ and $D\left(-3\sqrt{3},0\right)$ or

$A\left(-3\sqrt{3},0\right), B\left(0,3\right)$ *and* $D\left(3\sqrt{3},0\right)$.

**32.** Find the ratio in which the point (-1, y) lying on the line segment joining points A(-3, 10) and (6, -8) divides it. Also, find the value of y.

**Sol:**

Let $k$ be the ratio in which $P\left(-1,y\right)$ divides the line segment joining the points

$A\left(-3,10\right)$ *and* $B\left(6,-8\right)$

Then,

$$\left(-1,y\right) = \left(\frac{k\left(6\right)-3}{k+1}, \frac{k\left(-8\right)+10}{k+1}\right)$$

$$\Rightarrow \frac{k\left(6\right)-3}{k+1} = -1 \text{ and } y = \frac{k\left(-8\right)+10}{k+1}$$

$$\Rightarrow k = \frac{2}{7}$$

Substituting $k = \frac{2}{7}$ in $y = \frac{k\left(-8\right)+10}{k+1}$, we get

$$y = \frac{\frac{-8\times 2}{7}+10}{\frac{2}{7}+1} = \frac{-16+70}{9} = 6$$

Hence, the required ratio is $2:7$ *and* $y = 6$.

**33.** ABCD is rectangle formed by the points A(-1, -1), B(-1, 4), C(5, 4) and D(5, -1). If P,Q,R and S be the midpoints of AB, BC, CD and DA respectively, Show that PQRS is a rhombus.

**Sol:**

Here, the points $P, Q, R$ and $S$ are the midpoint of $AB, BC, CD$ *and* $DA$ respectively. Then

Coordinates of $P = \left(\frac{-1-1}{2}, \frac{-1+4}{2}\right) = \left(-1, \frac{3}{2}\right)$

Coordinates of $Q=\left(\frac{-1+5}{2},\frac{4+4}{2}\right)=(2,4)$

Coordinates of $R=\left(\frac{5+5}{2},\frac{4-1}{2}\right)=\left(5,\frac{3}{2}\right)$

Coordinates of $S=\left(\frac{-1+5}{2},\frac{-1-1}{2}\right)=(2,-1)$

Now,

$$PQ=\sqrt{(2+1)^2+\left(4-\frac{3}{2}\right)^2}=\sqrt{9+\frac{25}{4}}=\frac{\sqrt{61}}{2}$$

$$QR=\sqrt{(5-2)^2+\left(\frac{3}{2}-4\right)^2}=\sqrt{9+\frac{25}{4}}=\frac{\sqrt{61}}{2}$$

$$RS=\sqrt{(5-2)^2+\left(\frac{3}{2}+1\right)^2}=\sqrt{9+\frac{25}{4}}=\frac{\sqrt{61}}{2}$$

$$SP=\sqrt{(2+1)^2+\left(-1-\frac{3}{2}\right)^2}=\sqrt{9+\frac{25}{4}}=\frac{\sqrt{61}}{2}$$

$$PR=\sqrt{(5-1)^2+\left(\frac{3}{2}-\frac{3}{2}\right)^2}=\sqrt{36}=6$$

$$QS=\sqrt{(2-2)^2+(-1-4)^2}=\sqrt{25}=5$$

Thus, $PQ=QR=RS=SP$ and $PR\neq QS$ therefore $PQRS$ is a rhombus.

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**34.** The midpoint P of the line segment joining points A(-10, 4) and B(-2, 0) lies on the line segment joining the points C(-9, -4) and D(-4, y). Find the ratio in which P divides CD. Also, find the value of y.

**Sol:**

The midpoint of $AB$ is $\left(\frac{-10-2}{2},\frac{4+10}{2}\right)=P(-6,2)$.

Let $k$ be the ratio in which $P$ divides $CD$. So

$$(-6,2)=\left(\frac{k(-4)-9}{k+1},\frac{k(y)-4}{k+1}\right)$$

$$\Rightarrow \frac{k(-4)-9}{k+1}=-6 \text{ and } \frac{k(y)-4}{k+1}=2$$

$$\Rightarrow k=\frac{3}{2}$$

Now, substituting $k=\frac{3}{2}$ in $\frac{k(y)-4}{k+1}=2$, we get

$$\frac{y\times\frac{3}{2}-4}{\frac{3}{2}+1}=2$$

$$\Rightarrow \frac{3y-8}{5}=2$$

$$\Rightarrow y=\frac{10+8}{3}=6$$

Hence, the required ratio is $3:2$ and $y=6$.

## Exercise – 16C

**1.** Find the area of $\Delta ABC$ whose vertices are:

(i) $A(1,2), B(-2,3)$ *and* $C(-3,-4)$

(ii) $A(-5,7), B(-4,-5)$ *and* $C(4,5)$

(iii) $A(3,8), B(-4,2)$ *and* $C(5,-1)$

(iv) $A(10,-6), B(2,5)$ *and* $C(-1,-3)$

**Sol:**

(i) $A(1,2), B(-2,3)$ *and* $C(-3,-4)$ are the vertices of $\Delta ABC$. Then,

$(x_1=1, y_1=2), (x_2=-2, y_2=3)$ *and* $(x_3=-3, y_3=-4)$

Area of triangle $ABC$

$$=\frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$$

$$=\frac{1}{2}\{1(3-(-4))+(-2)(-4-2)+(-3)(2-3)\}$$

$$=\frac{1}{2}\{1(3+4)-2(-6)-3(-1)\}$$

$$=\frac{1}{2}\{7+12+3\}$$

$$=\frac{1}{2}(22)$$

$$=11\ sq.\ units$$

(ii) $A(-5,7), B(-4,-5)$ *and* $C(4,5)$ are the vertices of $\Delta ABC$. Then,

$(x_1=-5, y_1=7), (x_2=-4, y_2=-5)$ *and* $(x_3=4, y_3=5)$

Area of triangle $ABC$

$=\frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$

$=\frac{1}{2}\{(-5)(-5-5)+(-4)(5-7)+4(7-(5))\}$

$=\frac{1}{2}\{(-5)(-10)-4(-2)+4(12)\}$

$=\frac{1}{2}\{50+8+48\}$

$=\frac{1}{2}(106)$

$=53\,sq.\ units$

(iii) $A(3,8), B(-4,2)$ *and* $C(5,-1)$ are verticals of $\Delta ABC$. Then,

$(x_1=3, y_1=8),(x_2=-4, y_2=2)$ *and* $(x_3=5, y_3=-1)$

Area of triangle $ABC$

$=\frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$

$=\frac{1}{2}\{3(2-(-1))+(-4)(-1-8)+5(8-2)\}$

$=\frac{1}{2}\{3(2+1)-4(-9)+5(6)\}$

$=\frac{1}{2}\{9+36+30\}$

$\Rightarrow\frac{1}{2}(75)$

$=37.5\,sq.\ units$

(iv) $A(10,-6), B(2,5)$ *and* $C(-1,-3)$ are the vertex of $\Delta ABC$. Then,

$(x_1=10, y_1=-6),(x_2=2, y_2=5)$ *and* $(x_3=-1, y_3=3)$

Area of triangle $ABC$

$=\frac{1}{2}\{x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\}$

$=\frac{1}{2}\{10(5-3)+2(3-(-6))+(-1)(-6-5)\}$

$=\frac{1}{2}\{10(2)+2(9)-1(-11)\}$

$=\frac{1}{2}\{20+18+11\}$

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$$= \frac{1}{2}(49)$$
$$= 24.5\, sq.\ units$$

**2.** Find the area of a quadrilateral ABCD whose vertices area A(3, -1), B(9, -5) C(14, 0) and D(9, 19).

**Sol:**

By joining $A$ and $C$, we get two triangles $ABC$ and $ACD$.

Let

$A(x_1, y_1) = A(3, -1), B(x_2, y_2) = B(9, -5), C(x_3, y_3) = C(14, 0)\ and\ D(x_4, y_4) = D(9, 19)$

Then,

Area of $\Delta ABC = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[3(-5-0) + 9(0+1) + 14(-1+5)]$

$= \frac{1}{2}[-15 + 9 + 56] = 25\, sq.\ units$

Area of $\Delta ACD = \frac{1}{2}[x_1(y_3 - y_4) + x_3(y_4 - y_1) + x_4(y_1 - y_3)]$

$= \frac{1}{2}[3(0-19) + 14(19+1) + 9(-1-0)]$

$= \frac{1}{2}[-57 + 280 - 9] = 107\, sq.\ units$

So, the area of the quadrilateral is $25 + 107 = 132\, sq.\ units.$

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**3.** Find the area of quadrilateral PQRS whose vertices are P(-5, -3), Q(-4,-6),R(2, -3) and S(1,2).

**Sol:**

By joining $P$ and $R$, we get two triangles $PQR$ and $PRS$.

Let $P(x_1, y_1) = P(-5, -3), Q(x_2, y_2) = Q(-4, -6), R(x_3, y_3) = R(2, -3)$ and. Then

$S(x_4, y_4) = S(1, 2)$

Area of $\Delta PQR = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}[-5(-6+3) - 4(-3+3) + 2(-3+6)]$

$= \frac{1}{2}[15 - 0 + 6] = \frac{21}{2}\ sq.\ units$

Area of $\Delta PRS = \frac{1}{2}\left[x_1\left(y_3 - y_4\right) + x_3\left(y_4 - y_1\right) + x_4\left(y_1 - y_3\right)\right]$

$= \frac{1}{2}\left[-5(-3-2) + 2(2+3) + 1(-3+3)\right]$

$= \frac{1}{2}[25 + 10 + 0] = \frac{35}{2}$ *sq. units*

So, the area of the quadrilateral $PQRS$ is $\frac{21}{2} + \frac{35}{2} = 28$ *sq. units sq. units*

**4.** Find the area of quadrilateral ABCD whose vertices are A(-3, -1), B(-2,-4) C(4,-1) and D(3,4)

**Sol:**

By joining $A$ and $C$, we get two triangles $ABC$ and $ACD$.

Let $A(x_1, y_1) = A(-3,-1), B(x_2, y_2) = B(-2,-4), C(x_3, y_3) = C(4,-1)$ and. Then

$D(x_4, y_4) = D(3,4)$

Area of $\Delta ABC = \frac{1}{2}\left[x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right]$

$= \frac{1}{2}\left[-3(-4+1) - 2(-1+1) + 4(-1+4)\right]$

$= \frac{1}{2}[9 - 0 + 12] = \frac{21}{2}$ *sq. units*

Area of $\Delta ACD = \frac{1}{2}\left[x_1\left(y_3 - y_4\right) + x_3\left(y_4 - y_1\right) + x_4\left(y_1 - y_3\right)\right]$

$= \frac{1}{2}\left[-3(-1-4) + 4(4+1) + 3(-1+1)\right]$

$= \frac{1}{2}[15 + 20 + 0] = \frac{35}{2}$ *sq. units*

So, the area of the quadrilateral $ABCD$ is $\frac{21}{2} + \frac{35}{2} = 28$ *sq. units sq. units*

**5.** Find the area of quadrilateral ABCD whose vertices are A(-5, 7), B(-4, -5) C(-1,-6) and D(4,5)

**Sol:**

By joining $A$ and $C$, we get two triangles $ABC$ and $ACD$.

Let $A(x_1, y_1) = A(-5,7), B(x_2, y_2) = B(-4,-5), C(x_3, y_3) = C(-1,-6)$ and.

$D(x_4, y_4) = D(4,5)$

Then

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Area of $\Delta ABC = \frac{1}{2}\left[x_1\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right]$

$= \frac{1}{2}\left[-5(-5+6) - 4(-6-7) - 1(7+5)\right]$

$= \frac{1}{2}[-5+52-12] = \frac{35}{2}\ sq.\ units$

Area of $\Delta ACD = \frac{1}{2}\left[x_1\left(y_3 - y_4\right) + x_3\left(y_4 - y_1\right) + x_4\left(y_1 - y_3\right)\right]$

$= \frac{1}{2}\left[-5(-6-5) - 1(5-7) + 4(7+6)\right]$

$= \frac{1}{2}[55+2+52] = \frac{109}{2}\ sq.\ units$

So, the area of the quadrilateral $ABCD$ is $\frac{35}{2} + \frac{109}{2} = 72\ sq.\ units$

**6.** Find the area of the triangle formed by joining the midpoints of the sides of the triangle whose vertices are A(2,1) B(4,3) and C(2,5)

**Sol:**

The verticals of the triangle are $A(2,1), B(4,3)\ and\ C(2,5)$.

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Coordinates of midpoint of $AB = P\left(x_1, y_1\right) = \left(\frac{2+4}{2}, \frac{1+3}{2}\right) = (3,2)$

Coordinates of midpoint of $BC = Q\left(x_2, y_2\right) = \left(\frac{4+2}{2}, \frac{3+5}{2}\right) = (3,4)$

Coordinates of midpoint of $AC = R\left(x_3, y_3\right) = \left(\frac{2+2}{2}, \frac{1+5}{2}\right) = (2,3)$

Now,

Area of $\Delta PQR = \frac{1}{2}\left[x_2\left(y_2 - y_3\right) + x_2\left(y_3 - y_1\right) + x_3\left(y_1 - y_2\right)\right]$

$= \frac{1}{2}\left[3(4-3) + 3(3-2) + 2(2-4)\right]$

$= \frac{1}{2}[3+3-4] = 1\ sq.\ unit$

Hence, the area of the quadrilateral triangle is 1 sq. unit.

**7.** A(7, -3), B(5,3) and C(3,-1) are the vertices of a $\Delta ABC$ and AD is its median. Prove that the median AD divides $\Delta ABC$ into two triangles of equal areas.

**Sol:**

The vertices of the triangle are $A(7,-3), B(5,3), C(3,-1)$.

Coordinates of $D=\left(\frac{5+3}{2},\frac{3-1}{2}\right)=(4,1)$

For the area of the triangle *ADC*, let

$A(x_1,y_1)=A(7,-3),D(x_2,y_2)=D(4,1)$ *and* $C(x_3,y_3)=C(3,-1)$. Then

Area of $\Delta ADC=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

$=\frac{1}{2}\left[7(1+1)+4(-1+3)+3(-3-1)\right]$

$=\frac{1}{2}[14+8-12]=5\ sq.\ unit$

Now, for the area of triangle *ABD*, let

$A(x_1,y_1)=A(7,-3),B(x_2,y_2)=B(5,3)$ *and* $D(x_3,y_3)=D(4,1)$. Then

Area of $\Delta ADC=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

$=\frac{1}{2}\left[7(3-1)+5(1+3)+4(-3-3)\right]$

$=\frac{1}{2}[14+20-24]=5\ sq.\ unit$

Thus, Area $(\Delta ADC)=$ Area$(\Delta ABD)=5\ sq.\ units$

Hence, *AD* divides $\Delta ABC$ into two triangles of equal areas.

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**8.** Find the area of $\Delta ABC$ with A(1, -4) and midpoints of sides through A being (2, -1) and (0, -1).

**Sol:**

Let $(x_2,y_2)$ *and* $(x_3,y_3)$ be the coordinates of *B* and *C* respectively. Since, the coordinates of *A* are $(1,-4)$, therefore

$\frac{1+x_2}{2}=2\Rightarrow x_2=3$

$\frac{-4+y_2}{2}=-1\Rightarrow y_2=2$

$\frac{1+x_2}{2}=0\Rightarrow x_3=-1$

$\frac{-4+y_3}{2}=-1\Rightarrow y_3=2$

Let $A(x_1,y_1)=A(1,-4),B(x_2,y_2)=B(3,2)$ *and* $C(x_3,y_3)=C(-1,2)$ Now

Area $(\Delta ABC)=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

$= \frac{1}{2}[1(2-2)+3(2+4)-1(-4-2)]$

$= \frac{1}{2}[0+18+6]$

$= 12\ sq.\ units$

Hence, the area of the triangle $\Delta ABC$ is 12 sq. units

**9.** A(6,1) , B(8,2) and C(9,4) are the vertices of a parallelogram ABCD. If E is the midpoint of DC, find the area of $\Delta ADE$

**Sol:**

Let $(x, y)$ be the coordinates of $D$ and $(x', y')$ be thee coordinates of E. since, the diagonals of a parallelogram bisect each other at the same point, therefore

$\frac{x+8}{2} = \frac{6+9}{2} \Rightarrow x = 7$

$\frac{y+2}{2} = \frac{1+4}{2} \Rightarrow y = 3$

Thus, the coordinates of $D$ are $(7,3)$

$E$ is the midpoint of $DC$, therefore

$x' = \frac{7+9}{2} \Rightarrow x' = 8$

$y' = \frac{3+4}{2} \Rightarrow y' = \frac{7}{2}$

Thus, the coordinates of $E$ are $\left(8, \frac{7}{2}\right)$

Let $A(x_1, y_1) = A(6,1), E(x_2, y_2) = E\left(8, \frac{7}{2}\right)$ *and* $D(x_3, y_3) = D(7,3)$ Now

Area $(\Delta ABC) = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$= \frac{1}{2}\left[6\left(\frac{7}{2} - 3\right) + 8(3-1) + 7\left(1 - \frac{7}{2}\right)\right]$

$= \frac{1}{2}\left[\frac{3}{2}\right]$

$= \frac{3}{4} sq.\ unit$

Hence, the area of the triangle $\Delta ADE$ is $\frac{3}{4}$ sq. units

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**10.** If the vertices of $\Delta ABC$ be A(1, -3) B(4, p) and C(-9, 7) and its area is 15 square units, find the values of p.

**Sol:**

Let $A(x_1, y_1) = A(1, -3), B(x_2, y_2) = B(4, p)$ *and* $C(x_3, y_3) = C(-9, 7)$ Now

Area $(\Delta ABC) = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$\Rightarrow 15 = \frac{1}{2}[1(p-7) + 4(7+3) - 9(-3-p)]$

$\Rightarrow 15 = \frac{1}{2}[10p + 60]$

$\Rightarrow |10p + 60| = 30$

Therefore

$\Rightarrow 10p + 60 = -30$ or $30$

$\Rightarrow 10p = -90$ or $-30$

$\Rightarrow p = -9$ or $-3$

Hence, $p = -9$ or $p = -3$.

**11.** Find the value of k so that the area of the triangle with vertices A (k+1, 1), B(4, -3) and C(7, -k) is 6 square units.

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**Sol:**

Let $A(x_1, y_1) = A(k+1, 1), B(x_2, y_2) = B(4, -3)$ and $C(x_3, y_3) = C(7, -k)$ now

Area $(\Delta ABC) = \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$

$\Rightarrow 6 = \frac{1}{2}[(k+1)(-3+k) + 4(-k-1) + 7(1+3)]$

$\Rightarrow 6 = \frac{1}{2}[k^2 - 2k - 3 - 4k - 4 + 28]$

$\Rightarrow k^2 - 6k + 9 = 0$

$\Rightarrow (k-3)^2 = 0 \Rightarrow k = 3$

Hence, $k = 3$.

**12.** For what value of k(k>0) is the area of the triangle with vertices (-2, 5), (k, -4) and (2k+1, 10) equal to 53 square units?

**Sol:**

Let $A(x_1 = -2, y_1 = 5), B(x_2 = k, y_2 = -4)$ and $C(x_3 = 2k+1, y_3 = 10)$ be the vertices of the triangle, So

Area $(\Delta ABC)=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

$\Rightarrow 53=\frac{1}{2}\left[(-2)(-4-10)+k(10-5)+(2k+1)(5+4)\right]$

$\Rightarrow 53=\frac{1}{2}\left[28+5k+9(2k+1)\right]$

$\Rightarrow 28+5k+18k+9=106$

$\Rightarrow 37+23k=106$

$\Rightarrow 23k=106-37=69$

$\Rightarrow k=\frac{69}{23}=3$

Hence, $k=3$.

**13.** Show that the following points are collinear:

(i) A(2,-2), B(-3, 8) and C(-1, 4)
(ii) A(-5,1), B(5, 5) and C(10, 7)
(iii) A(5,1), B(1, -1) and C(11, 4)
(iv) A(8,1), B(3, -4) and C(2, -5)

**Sol:**

(i) Let $A(x_1=2, y_1=-2), B(x_2=-3, y_2=8)$ and $C(x_3=-1, y_3=4)$ be the given points.

Now $x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)$

$=2(8-4)+(-3)(4+2)+(-1)(-2-8)$

$=8-18+10$

$=0$

Hence, the given points are collinear.

(ii) Let $A(x_1=-5, y_1=1), B(x_2=5, y_2=5)$ and $C(x_3=10, y_3=7)$ be the given points.

Now $x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)$

$=(-5)(5-7)+5(7-1)+10(1-5)$

$=-5(-2)+5(6)+10(-4)$

$=10+30-40$

$=0$

Hence, the given points are collinear.

(iii) Let $A(x_1=5, y_1=1), B(x_2=1, y_2=-1)$ and $C(x_3=11, y_3=4)$ be the given points.

Now $x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)$

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$= 5(-1-4)+1(4-1)+11(1+1)$
$= -25+3+22$
$= 0$
Hence, the given points are collinear.

(iv) Let $A(x_1 = 8, y_1 = 1), B(x_2 = 3, y_2 = -4)$ and $C(x_3 = 2, y_3 = -5)$ be the given points.
Now $x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$
$= 8(-4+5)+3(-5-1)+2(1+4)$
$= 8-18+10$
$= 0$
Hence, the given points are collinear.

**14.** Find the value of x for which points A(x, 2), B(-3, -4) and C(7, -5) are collinear.
**Sol:**
Let $A(x_1, y_1) = A(x, 2), B(x_2, y_2) = B(-3, -4)$ *and* $C(x_3, y_3) = C(7, -5)$. So the condition for three collinear points is
$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$
$\Rightarrow x(-4+5) - 3(-5-2) + 7(2+4) = 0$
$\Rightarrow x + 21 + 42 = 0$
$\Rightarrow x = -63$
Hence, $x = -63$.

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**15.** For what value of x are the points A(-3, 12), B(7, 6) and C(x, 9) collinear.
**Sol:**
$A(-3, 12), B(7, 6)$ *and* $C(x, 9)$ are the given points. Then:
$(x_1 = -3, y_1 = 12), (x_2 = 7, y_2 = 6)$ and $(x_3 = x, y_3 = 9)$
It is given that points *A, B* and *C* are collinear. Therefore,
$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$
$\Rightarrow (-3)(6-9) + 7(9-12) + x(12-6) = 0$
$\Rightarrow (-3)(-3) + 7(-3) + x(6) = 0$
$\Rightarrow 9 - 21 + 6x = 0$
$\Rightarrow 6x - 12 = 0$
$\Rightarrow 6x = 12$
$\Rightarrow x = \frac{12}{6} = 12$
Therefore, when $x = 2$, the given points are collinear

**16.** For what value of y, are the points P(1, 4), Q(3,y) and R(-3, 16) are collinear?

**Sol:**

$P(1,4), Q(3,y)$ *and* $R(-3,16)$ are the given points. Then:

$(x_1 = 1, y_1 = 4), (x_2 = 3, y_2 = y)$ *and* $(x_3 = -3, y_3 = 16)$

It is given that the points *P*, *Q* and *R* are collinear.

Therefore,

$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$\Rightarrow 1(y-16) + 3(16-4) + (-3)(4-y) = 0$

$\Rightarrow 1(y-16) + 3(12) - 3(4-y) = 0$

$\Rightarrow y - 16 + 36 - 12 + 3y = 0$

$\Rightarrow 8 + 4y = 0$

$\Rightarrow 4y = -8$

$\Rightarrow y = -\frac{8}{4} = -2$

When, $y = -2$, the given points are collinear.

**17.** Find the value of y for which the points A(-3, 9), B(2,y) and C(4,-5) are collinear.

**Sol:**

Let $A(x_1 = -3, y_1 = 9), B(x_2 = 2, y_2 = y)$ and $C(x_3 = 4, y_3 = -5)$ be the given points

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The given points are collinear if

$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_2 - y_2) = 0$

$\Rightarrow (-3)(y+5) + 2(-5-9) + 4(9-y) = 0$

$\Rightarrow -3y - 15 - 28 + 36 - 4y = 0$

$\Rightarrow 7y = 36 - 43$

$\Rightarrow y = -1$

**18.** For what values of k are the points A(8, 1) B(3, -2k) and C(k, -5) collinear.

**Sol:**

Let $A(x_1 = 8, y_1 = 1), B(x_2 = 3, y_2 = -2k)$ and $C(x_3 = k, y_3 = -5)$ be the given points

The given points are collinear if

$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$\Rightarrow 8(-2k+5) + 3(-5-1) + k(1+2k) = 0$

$\Rightarrow -16k + 40 - 18 + k + 2k^2 = 0$

$\Rightarrow 2k^2 - 15k + 22 = 0$

$\Rightarrow 2k^2 - 11k - 4k + 22 = 0$

$\Rightarrow k(2k-11) - 2(2k-11) = 0$

$\Rightarrow (k-2)(2k-11)=0$

$\Rightarrow k=2 \text{ or } k=\frac{11}{22}$

Hence, $k=2 \text{ or } k=\frac{11}{22}$.

**19.** Find a relation between x and y, if the points A(2, 1), B(x, y) and C(7,5) are collinear.

**Sol:**

Let $A(x_1=2, y_1=1), B(x_2=x, y_2=y) \text{ and } C(x_3=7, y_3=5)$ be the given points

The given points are collinear if

$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$

$\Rightarrow 2(y-5)+x(5-1)+7(1-y)=0$

$\Rightarrow 2y-10+4x+7-7y=0$

$\Rightarrow 4x-5y-3=0$

Hence, the required relation is $4x-5y-3=0$.

**20.** Find a relation between x and y, if the points A(x, y), B(-5, 7) and C(-4, 5) are collinear.

**Sol:**

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Let $A(x_1=x, y_1=y), B(x_2=-5, y_2=7) \text{ and } C(x_3=-4, y_3=5)$ be the given points

The given points are collinear if

$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$

$\Rightarrow x(7-5)+(-5)(5-y)+(-4)(y-7)=0$

$\Rightarrow 7x-5x-25+5y-4y+28=0$

$\Rightarrow 2x+y+3=0$

Hence, the required relation is $2x+y+3=0$

**21.** Prove that the points $A(a,0), B(0,b) \text{ and } C(1,1)$ are collinear, if $\left(\frac{1}{a}+\frac{1}{b}\right)=1$.

**Sol:**

Consider the points $A(a,0), B(0,b) \text{ and } C(1,1)$.

Here, $(x_1=a, y_1=0).(x_2=0, y_2=b) \text{ and } (x_3=1, y_3=1)$.

It is given that the points are collinear. So,

$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$

$\Rightarrow a(b-1)+0(1-0)+1(0-b)=0$

$\Rightarrow ab-a-b=0$

Dividing the equation by *ab*:

$\Rightarrow 1-\frac{1}{b}-\frac{1}{a}=0$

$\Rightarrow 1-\left(\frac{1}{a}+\frac{1}{b}\right)=0$

$\Rightarrow \left(\frac{1}{a}+\frac{1}{b}\right)=1$

Therefore, the given points are collinear if $\left(\frac{1}{a}+\frac{1}{b}\right)=1$.

**22.** If the points P(-3, 9), Q(a, b) and R(4, -5) are collinear and a+b=1, find the value of a and b.

**Sol:**

Let $A(x_1=3,\ y_1=9)$, $B(x_2=a,\ y_3=b)$ and $C(x_3=4,\ y=-5)$ be the given points.

The given pots are collinear if

$x_1(y_2-y_3)+x_3(y_3-y_1)+x_3(y_1-y_2)=0$

$\Rightarrow (-3)(b+5)+a(-5-9)+4(9-b)=0$

$\Rightarrow -3b-15-14a+36-4b=0$

$\Rightarrow 2a+b=3$

Now solving $a+b=1$ and $2a+b=3$, we get $a=2$ and $b=-1$.

Hence, $a=2$ and $b=-1$.

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**23.** Find the area of $\Delta ABC$ with vertices A(0, -1), B(2,1) and C(0, 3). Also, find the area of the triangle formed by joining the midpoints of its sides. Show that the ratio of the areas of two triangles is 4:1.

**Sol:**

Let $A(x_1=0, y_1=-1), B(x_2=2, y_2=1)$ and $C(x_3=0, y_3=3)$ be the given points. Then

$Area(\Delta ABC)=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$

$=\frac{1}{2}\left[0(1-3)+2(3+1)+0(-1-1)\right]$

$=\frac{1}{2}\times 8=4\ sq.\ units$

So, the area of the triangle $\Delta ABC\ is\ 4\ sq.\ units$

Let $D(a_1,b_1), E(a_2,b_2)$ and $F(a_3,b_3)$ be the midpoints of AB, BC and AC respectively

Then

$a_1=\frac{0+2}{2}=1 \quad b_1=\frac{-1+1}{2}=0$

$a_2 = \frac{2+0}{2} = 1 \quad b_2 = \frac{1+3}{2} = 2$

$a_3 = \frac{0+0}{2} = 0 \quad b_3 = \frac{-1+3}{2} = 1$

Thus, the coordinates of $D$, $E$ and $F$are $D(a_1 = 1, b_1 = 0)$, $E(a_2 = 1, b_2 = 2)$ and $F(a_3 = 0,\ b_3 = 1)$.Now

Area $(\Delta DEF) = \frac{1}{2}\left[a_1(b_2 - b_2) + a_2(b_3 - b_1) + a_3(b_1 - b_2)\right]$

$= \frac{1}{2}\left[1(2-1) + 1(1-0) + 0(0-2)\right]$

$= \frac{1}{2}[1+1+0] = 1\ sq.\ unit$

So, the area of the triangle $\Delta DEF$ is 1 sq. unit.

Hence, $\Delta ABC : \Delta DEF = 4:1$.

## Exercise – 16D

**1.** Points A(-1, y) and B(5,7) lie on the circle with centre O(2, -3y).Find the value of y.

**Sol:**

The given points are $A(-1, y)$, $8(5,7)$ and $O(2, -3y)$.

Here, $AO$ and $BO$ are the radii of the circle. So

$AO = BO \Rightarrow AO^2 = BO^2$

$\Rightarrow (2+1)^2 + (-3y - y)^2 = (2-5)^2 + (-3y-7)^2$

$\Rightarrow 9 + (4y)^2 = (-3)^2 + (3y+7)^2$

$\Rightarrow 9 + 16y^2 = 9 + 9y^2 + 49 + 42y$

$\Rightarrow 7y^2 - 42y - 49 = 0$

$\Rightarrow y^2 - 6y - 7 = 0$

$\Rightarrow y^2 - 7y + y - 7 = 0$

$\Rightarrow y(y-7) + 1(y-7) = 0$

$\Rightarrow (y-7)(y+1) = 0$

$\Rightarrow y = -1\ or\ y = 7$

Hence, $y = 7\ or\ y = -1$.

**2.** If the point A(0,2) is equidistant from the points B(3,p) and C(p, 5), find p.

**Sol:**

The given ports are $A(0,2), B(3,p)$ and $C(p,5)$.

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$AB = AC \Rightarrow AB^2 = AC^2$

$\Rightarrow (3-0)^2 + (p-2)^2 = (p-0)^2 + (5-2)^2$

$\Rightarrow 9 + p^2 - 4p + 4 = p^2 + 9$

$\Rightarrow 4p = 4 \Rightarrow p = 1$

Hence, $p = 1$.

**3.** ABCD is a rectangle whose three vertices are A(4,0), C(4,3) and D(0,3). Find the length of one its diagonal.

**Sol:**

The given vertices are $B(4, 0)$, $C(4, 3)$ and $D(0, 3)$ Here, $BD$ one of the diagonals So

$BD = \sqrt{(4-0)^2 + (0-3)^2}$

$= \sqrt{(4)^2 + (-3)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

$= 5$

Hence, the length of the diagonal is 5 units.

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**4.** If the point P(k-1, 2) is equidistant from the points A(3,k) and B(k,5), find the value of k.

**Sol:**

The given points are $P(k-1, 2), A(3, k)$ *and* $B(k, 5)$.

$\because AP = BP$

$\therefore AP^2 = BP^2$

$\Rightarrow (k-1-3)^2 + (2-k)^2 = (k-1-k)^2 + (2-5)^2$

$\Rightarrow (k-4)^2 + (2-k)^2 = (-1)^2 + (-3)^2$

$\Rightarrow k^2 - 8y + 16 + 4 + k^2 - 4k = 1 + 9$

$\Rightarrow k^2 - 6y + 5 = 0$

$\Rightarrow (k-1)(k-5) = 0$

$\Rightarrow k = 1$ *or* $k = 5$

Hence, $k = 1$ *or* $k = 5$

**5.** Find the ratio in which the point P(x,2) divides the join of A(12, 5) and B(4, -3).

**Sol:**

Let $k$ be the ratio in which the point $P(x, 2)$ divides the line joining the points

$A(x_1 = 12,\ y_1 = 5)$ *and* $B(x_2 = 4, y_2 = -3)$. Then

$x = \frac{k \times 4 + 12}{k+1}$ *and* $2 = \frac{k \times (-3) + 5}{k+1}$

Now,

$2 = \frac{k \times (-3) + 5}{k+1} \Rightarrow 2k + 2 = -3k + 5 \Rightarrow k = \frac{3}{5}$

Hence, the required ratio is $3:5$.

**6.** Prove that the diagonals of a rectangle ABCD with vertices A(2,-1), B(5,-1) C(5,6) and D(2,6) are equal and bisect each other.

**Sol:**

The vertices of the rectangle $ABCD$ are $A(2,-1), B(5,-1), C(5,6)$ and $D(2,6)$. Now

Coordinates of midpoint of $AC = \left(\frac{2+5}{2}, \frac{-1+6}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right)$

Coordinates of midpoint of $BD = \left(\frac{5+2}{2}, \frac{-1+6}{2}\right) = \left(\frac{7}{2}, \frac{5}{2}\right)$

Since, the midpoints of $AC$ and $BD$ coincide, therefore the diagonals of rectangle $ABCD$ bisect each other

**7.** Find the lengths of the medians AD and BE of $\Delta ABC$ whose vertices are A(7,-3), B(5,3) and C(3,-1)

**Sol:**

The given vertices are $A(7,-3), B(5,3)$ *and* $C(3,-1)$.

Since $D$ and $E$ are the midpoints of $BC$ and $AC$ respectively. therefore

Coordinates of $D = \left(\frac{5+3}{2}, \frac{3-1}{2}\right) = (4,1)$

Coordinates of $E = \left(\frac{7+3}{2}, \frac{-3-1}{2}\right) = (5,-2)$

Now

$AD = \sqrt{(7-4)^2 + (-3-1)^2} = \sqrt{9+16} = 5$

$BE = \sqrt{(5-5)^2 + (3+2)^2} = \sqrt{0+25} = 5$

Hence, $AD = BE = 5$ units.

**8.** If the point C(k,4) divides the join of A(2,6) and B(5,1) in the ratio 2:3 then find the value of k.

**Sol:**

Here, the point $C(k,4)$ divides the join of $A(2,6)$ and $B(5,1)$ in ratio $2:3$. So

$k = \frac{2 \times 5 + 3 \times 2}{2+3}$

$= \frac{10+6}{5}$

$= \frac{16}{5}$

Hence, $k = \frac{16}{5}$.

**9.** Find the point on x-axis which is equidistant from points A(-1,0) and B(5,0)

**Sol:**

Let $P(x,0)$ be the point on $x-axis$. Then

$AP = BP \Rightarrow AP^2 = BP^2$

$\Rightarrow (x+1)^2 + (0-0)^2 = (x-5)^2 + (0-0)^2$

$\Rightarrow x^2 + 2x + 1 = x^2 - 10x + 25$

$\Rightarrow 12x = 24 \Rightarrow x = 2$

Hence, $x = 2$

**10.** Find the distance between the points $A\left(\frac{-8}{5}, 2\right)$ and $B\left(\frac{2}{5}, 2\right)$

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**Sol:**

The given points are $A\left(\frac{-8}{5}, 2\right)$ and $B\left(\frac{2}{5}, 2\right)$

Then, $\left(x_1 = \frac{-8}{5}, y_1 = 2\right)$ *and* $\left(x_2 = \frac{2}{5}, y_2 = 2\right)$

Therefore,

$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{\left\{\frac{2}{5} - \left(\frac{-8}{5}\right)\right\}^2 + (2-2)^2}$

$= \sqrt{(2)^2 + (0)^2}$

$= \sqrt{4+0}$

$= \sqrt{4}$

$= 2$ *units.*

**11.** Find the value of a, so that the point $(3,a)$ lies on the line represented by $2x-3y=5$.

**Sol:**

The points $(3,a)$ lies on the line $2x-3y=5$.

If point $(3,a)$ lies on the line $2x-3y=5$, then $2x-3y=5$

$\Rightarrow (2\times 3)-(3\times a)=5$

$\Rightarrow 6-3a=5$

$\Rightarrow 3a=1$

$\Rightarrow a=\frac{1}{3}$

Hence, the value of $a$ is $\frac{1}{3}$.

**12.** If the points $A(4,3)$ *and* $B(x,5)$ lie on the circle with center $O(2,3)$, find the value of $x$.

**Sol:**

The given points $A(4, 3)$ and $B(x, 5)$ lie on the circle with center $O(2, 3)$.

Then, $OA = OB$

$\Rightarrow \sqrt{(x-2)^2+(5-3)^2}=\sqrt{(4-2)^2+(3-3)^2}$

$\Rightarrow (x-2)^2+2^2=2^2+0^2$

$\Rightarrow (x-2)^2=(2^2-2^2)$

$\Rightarrow (x-2)^2=0$

$\Rightarrow x-2=0$

$\Rightarrow x=2$

Hence, the value of $x=2$

**13.** If $P(x,y)$ is equidistant from the points $A(7,1)$ *and* $B(3,5)$, find the relation between $x$ and $y$.

**Sol:**

Let the point $P(x, y)$ be equidistant from the points $A(7, 1)$ and $B(3, 5)$

Then,

$PA=PB$

$\Rightarrow PA^2=PB^2$

$\Rightarrow (x-7)^2+(y-1)^2=(x-3)^2+(y-5)^2$

$\Rightarrow x^2+y^2-14x-2y+50=x^2+y^2-6x-10y+34$

$\Rightarrow 8x-8y=16$

$\Rightarrow x-y=2$

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**14.** If the centroid of $\Delta ABC$ having vertices $A(a,\ b)$, $B(b,\ c)$ and $C(c,\ a)$ is the origin, then find the value of $(a+b+c)$.

**Sol:**

The given points are $A(a,b), B(b,c)\ \ and\ \ C(c,a)$

Here,

$(x_1 = a, y_1 = b), (x_2 = b, y_2 = c)\ and\ (x_3 = c, y_3 = a)$

Let the centroid be $(x, y)$.

Then,

$x = \frac{1}{3}(x_1 + x_2 + x_3)$

$= \frac{1}{3}(a+b+c)$

$= \frac{a+b+c}{3}$

$y = \frac{1}{3}(y_1 + y_2 + y_3)$

$= \frac{1}{3}(b+c+a)$

$= \frac{a+b+c}{3}$

But it is given that the centroid of the triangle is the origin.

Then, we have

$\frac{a+b+c}{3} = 0$

$\Rightarrow a+b+c = 0$

**15.** Find the centroid of $\Delta ABC$ whose vertices are $A(2,2), B(-4,-4)\ \ and\ \ C(5,-8)$.

**Sol:**

The given points are $A(2,2), B(-4,-4)\ \ and\ \ C(5,-8)$.

Here, $(x_1 = 2, y_1 = 2), (x_2 = -4, y_2 = -4)\ and\ (x_3 = 5, y_3 = -8)$

Let $G(x, y)$ be the centroid of $\Delta ABC$ Then,

$x = \frac{1}{3}(x_1 + x_2 + x_3)$

$= \frac{1}{3}(2 - 4 + 5)$

$= 1$

$y = \frac{1}{3}(y_1 + y_2 + y_3)$

$= \frac{1}{3}(2 - 4 - 8)$

$= \frac{-10}{3}$

Hence, the centroid of $\Delta ABC$ is $G\left(1, \frac{-10}{3}\right)$.

**16.** In what ratio does the point $C(4,5)$ divides the join of $A(2,3)$ *and* $B(7,8)$?

**Sol:**

Let the required ratio be $k : 1$

Then, by section formula, the coordinates of C are

$C\left(\frac{7k+2}{k+1}, \frac{8k+3}{k+1}\right)$

Therefore,

$\frac{7k+2}{k+1} = 4$ *and* $\frac{8k+3}{k+1} = 5$ $\quad [\because C(4,5) \text{ is given}]$

$\Rightarrow 7k + 2 = 4k + 4$ and $8k + 3 = 5k + 5 \Rightarrow 3k = 2$

$\Rightarrow k = \frac{2}{3}$ in each case

So, the required ratio is $\frac{2}{3} : 1$, which is same as $2:3$.

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**17.** If the points $A(2,3)$, $B(4,k)$ *and* $C(6,-3)$ are collinear, find the value of $k$.

**Sol:**

The given points are $A(2,3)$, $B(4,k)$ *and* $C(6,-3)$

Here, $(x_1 = 2, y_1 = 3), (x_2 = 4, y_2 = k)$ and $(x_3 = 6, y_3 = -3)$

It is given that the points *A, B* and *C* are collinear. Then,

$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2) = 0$

$\Rightarrow 2(k+3) + 4(-3-3) + 6(3-k) = 0$

$\Rightarrow 2k + 6 - 24 + 18 - 6k = 0$

$\Rightarrow -4k = 0$

$\Rightarrow k = 0$

## Exercise – Multiple Choice Questions

**1.** The distance of the point P(-6,8) from the origin is

(a) 8 (b) $2\sqrt{7}$ (c) 6 (d) 10

**Answer:** (d) 10

**Sol:**

The distance of a point $(x, y)$ from the origin $O(0,0)$ is $\sqrt{x^2+y^2}$

Let $P(x=-6, y=8)$ be the gen point. Then

$OP=\sqrt{x^2+y^2}$

$=\sqrt{(-6)^2+8^2}$

$=\sqrt{36+64}$

$=\sqrt{100}=10$

**2.** The distance of the point (-3, 4) from x-axis is

(a) 3 (b) -3 (c) 4 (d) 5

**Answer:** (c) 4

**Sol:**

The distance of a point $(x, y)$ from $x-axis$ is $|y|$.

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Here, the point is $(-3,4)$. So, its distance from $x$-axis is $|4|=4$

**3.** The point on x-axis which is equidistant from the points A(-1, 0) and B(5,0) is

(a) (0,2) (b) (2,0) (c) (3,0) (d) (0,3)

**Answer:** (b) (2,0)

**Sol:**

Let $P(x,0)$ the point on $x$-axis, then

$AP=BP \Rightarrow AP^2=BP^2$

$\Rightarrow (x+1)^2+(0-0)^2=(x-5)^2+(0-0)^2$

$\Rightarrow x^2+2x+1=x^2-10x+25$

$\Rightarrow 12x=24 \Rightarrow x=2$

Thus, the required point is (2, 0).

**4.** If R(5,6) is the midpoint of the line segment AB joining the points A(6,5) and B(4,4) then y equals

(a) 5 (b) 7 (c) 12 (d) 6

**Answer:** (b) 7

**Sol:**

Since $R(5,6)$ is the midpoint of the line segment $AB$ joining the points $A(6,5)$ *and* $B(4,y)$, therefore

$$\frac{5+y}{2}=6$$
$$\Rightarrow 5+y=12$$
$$\Rightarrow y=12-5=7$$

**5.** If the point C(k,4) divides the join of the points A(2,6) and B(5,1) in the ratio 2:3 then the value of k is

(a) 16 (b) $\frac{28}{5}$ (c) $\frac{16}{5}$ (d) $\frac{8}{5}$

**Answer:** (c) $\frac{16}{5}$

**Sol:**

The point $C(k,4)$ dives the join of the points $A(2,6)$ *and* $B(5,1)$ in the ratio $2:3$. So

$$k=\frac{2\times5+3\times2}{2+3}=\frac{10+6}{5}=\frac{16}{5}$$

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**6.** The perimeter of the triangle with vertices (0,4), (0,0) and (3,0) is

(a) $(7+\sqrt{5})$ (b) 5 (c) 10 (d) 12

**Answer:** (d) 12

**Sol:**

Let $A(0,4)$, $B(0,0)$ and $C(3,0)$ be the given vertices. So

$$AB=\sqrt{(0-0)^2+(4-0)^2}=\sqrt{16}=4$$
$$BC=\sqrt{(0-3)^2+(0-0)^2}=\sqrt{9}=3$$
$$AC=\sqrt{(0-3)^2+(4-0)^2}=\sqrt{9+16}=5$$

Therefore

$$AB+BC+AC=4+3+5=12.$$

**7.** If A(1,3) B(-1,2) C(2,5) and D(x,4) are the vertices of a ||gm ABCD then the value of x is

(a) 3 (b) 4 (c) 0 (d) $\frac{3}{2}$

**Answer:** (b) 4

**Sol:**

The diagonals of a parallelogram bisect each other. The vertices of the 11gm $ABCD$ are $A(1,3), B(-1,2)$ and $C(2,5)$ $and$ $D(x,4)$

Here, $AC$ and $BD$ are the diagonals. So

$$\frac{1+2}{2}=\frac{-1+x}{2}$$

$$\Rightarrow x-1=3$$

$$\Rightarrow x=1+3=4$$

**8.** If the points A(x,2), B(-3, -4) and C(7, -5) are collinear then the value of x is

(a) -63 (b) 63 (c) 60 (d) -60

**Answer:** (a) -63

**Sol:**

Let $A(x_1=x,\ y_1=2), B(x_2=-3,\ y_2=-4)$ and $C(x_3=7, y_3=-5)$ be collinear points. Then

$$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$$

$$\Rightarrow x(-4+5)+(-3)(-5-2)+7(2+4)=0$$

$$\Rightarrow x+21+42=0$$

$$\Rightarrow x=-63$$

**9.** The area of a triangle with vertices A(5,0), B(8,0) and C(8,4) in square units is

(a) 20 (b) 12 (c) 6 (d) 16

**Answer:** (c) 6

**Sol:**

Let $A(x_1=5, y_1=0), B(x_2=8, y_2=0)$ and $C(x_3=8, y_3=4)$ be the vertices of the triangle.

Then,

$$Area(\Delta ABC)=\frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$$

$$=\frac{1}{2}[5(0-4)+8(4-0)+8(0-0)]$$

$$=\frac{1}{2}[-20+32+0]$$

$$=6\ sq.\ units$$

**10.** The area of $\Delta ABC$ with vertices A(a,0), O(0,0) and B(0,b) in square units is

(a) ab (b) $\frac{1}{2}ab$ (c) $\frac{1}{2}a^2b^2$ (d) $\frac{1}{2}b^2$

**Answer:** (b) $\frac{1}{2}ab$

**Sol:**

Let $A(x_1 = a, y_1 = 0), O(x_2 = 0, y_2 = 0)$ *and* $B(x_3 = 0, y_3 = b)$ be the given vertices. So

$$Area(\Delta ABO) = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$$

$$= \frac{1}{2}|a(0-b) + 0(b-0) + 0(0-0)|$$

$$= \frac{1}{2}|-ab|$$

$$= \frac{1}{2}ab$$

**11.** If $P\left(\frac{a}{2}, 4\right)$ is the midpoint of the line segment joining the points A(-6, 5) and B(-2,3) then the value of a is

(a) -8 (b) 3 (c) -4 (d) 4

**Answer:** (a) -8

**Sol:**

The point $P\left(\frac{a}{2}, 4\right)$ is the midpoint of the line segment joining the points $A(-6,5)$ and $B(-2,3)$.

So $\frac{a}{2} = \frac{-6-2}{2}$

$$\Rightarrow \frac{a}{2} = -4$$

$$\Rightarrow a = -8$$

**12.** ABCD is a rectangle whose three vertices are B(4,0), C(4,3) and D(0,3) The length of one of its diagonals is

(a) 5 (b) 4 (c) 3 (d) 245

**Answer:** (a) 5

**Sol:**

Here, *AC* and *BD* are two diagonals of the rectangle *ABCD*. So

$$BD = \sqrt{(4-0)^2 + (0-3)^2}$$

$$= \sqrt{(4)^2 + (-3)^2}$$

$$= \sqrt{16+9}$$

$$= \sqrt{25}$$

= 5 units

**13.** The coordinates of the point P dividing the line segment joining the points A(1,3), and B(4,6) in the ratio 2:1 is
(a) (2,4) (b) (3,5) (c) (4,2) (d) (5,3)
**Answer:** (b) (3,5)
**Sol:**
Here, the point $P$ divides the me segment pining the points $A(1,3)$ and $B(4,6)$ in the ratio $2:1$. Then,

Coordinates of $P=\left(\frac{2\times4+1\times1}{2+1},\frac{2\times6+1\times3}{2+1}\right)$

$=\left(\frac{8+1}{3},\frac{12+3}{3}\right)$

$=\left(\frac{9}{3},\frac{15}{3}\right)$

$=(3,5)$

**14.** If the coordinates of one end of a diameter of a circle are (2,3) and the coordinates of its centre are (-2,5), then the coordinates of the other end of the diameter are
(a) (-6,7) (b) (6.-7) (c) (4,2) (d) (5,3)
**Answer:** (a) (-6,7)
**Sol:**
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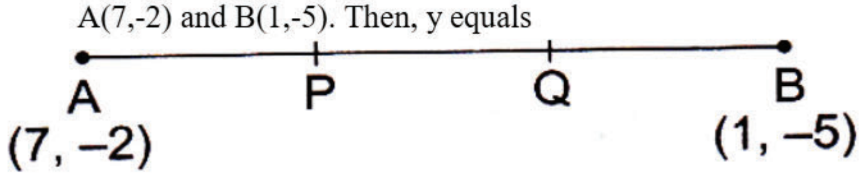
Let $(x,y)$ be the coordinates of the other end of the diameter. Then

$-2=\frac{2+x}{2}\Rightarrow x=-6$

$5=\frac{3+y}{2}\Rightarrow y=7$

**15.** In the given figure P(5,-3) and Q(3,y) are the points of trisection of the line segment joining A(7,-2) and B(1,-5). Then, y equals




(a) 2 (b) 4 (c) -4 (d) $\frac{-5}{2}$

**Answer:** (c) -4
**Sol:**
Here, $AQ:BQ=2:1$. Then,

$y=\frac{2\times(-5)+1\times(-2)}{2+1}$

$= \frac{-10-2}{3}$

$= -4$

**16.** The midpoint of segment AB is P(0,4). If the coordinates of B are (-2, 3), then the coordinates of A are
(a) (2,5) (b) (-2,-5) (c) (2,9) (d) (-2,11)
**Answer:** (a) (2,5)
**Sol:**
Let $(x, y)$ be the coordinates of $A$. then,

$0 = \frac{-2+x}{2} \Rightarrow x = 2$

$4 = \frac{3+y}{2} \Rightarrow y = 8-3 = 5$

Thus, the coordinates of $A$ are $(2,5)$.

**17.** The point P which divides the line segment joining the points A(2,-5) and B(5,2) in the ratio 2:3 lies in the quadrant
(a) I (b) II (c) III (d) IV
**Answer:** (d) IV
**Sol:**
Let $(x, y)$ be the coordinates of $P$. Then,

$x = \frac{2\times5+3\times2}{2+3} = \frac{10+6}{5} = \frac{16}{5}$

$y = \frac{2\times2+3\times(-5)}{2+3} = \frac{4-15}{5} = \frac{-11}{5}$

Thus, the coordinates of point $P$ are $\left(\frac{16}{5}, \frac{-11}{5}\right)$ and so it lies in the fourth quadrant

**18.** If A(-6,7) and B(-1,-5) are two given points then the distance 2AB is
(a) 13 (b) 26 (c) 169 (d) 238
**Answer:** (b) 26
**Sol:**
The given points are $A(-6,7)$ *and* $B(-1,-5)$.So

$AB = \sqrt{(-6+1)^2 + (7+5)^2}$

$= \sqrt{(-5)^2 + (12)^2}$

$= \sqrt{25+144}$
$= \sqrt{169}$
$= 13$
Thus, $2AB = 26$.

**19.** Which point on x-axis is equidistant from the points A(7,6) and B(-3,4)
(a) (0,4) (b) (-4,0) (c) (3,0) (d) (0,3)
**Answer:** (c) (3,0)
**Sol:**
Let $p(x,0)$ be the point on $x-axis$. Then as per the question

$AP = BP \Rightarrow AP^2 = BP^2$
$\Rightarrow (x-7)^2 + (0-6)^2 = (x-3)^2 + (0-4)^2$
$\Rightarrow x^2 - 14x + 49 + 36 = x^2 + 6x + 9 + 16$
$\Rightarrow 60 = 20x$
$\Rightarrow x = \frac{60}{20} = 3$
Thus, the required point is $(3,0)$.

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**20.** The distance of P(3,4) from the x-axis is
(a) 3 units (b) 4 units (c) 5 units (d) 1 unit
**Answer:** (b) 4 units
**Sol:**
The *y*-coordinate the distance of the point from the *x*-axis
Here, the *y*-coordinate is 4.

**21.** In what ratio does the x-axis divide the join of A(2, -3) and B(5,6)?
(a) 2:3 (b) 3:5 (c) 1:2 (d) 2:1
**Answer:** (c) 1 :2
**Sol:**
Let $AB$ be divided by the *x*-axis in the ratio $k:1$ at the point $P$.
Then, by section formula, the coordinates of $P$ are

$P\left(\frac{5k+2}{k+1}, \frac{6k-3}{k+1}\right)$

But $P$ lies on the *x*-axes so, its ordinate is 0.

$\frac{6k-3}{k+1} = 0$
$\Rightarrow 6k - 3 = 0$

$\Rightarrow 6k = 3$

$\Rightarrow k = \frac{1}{2}$

Hence, the required ratio is $\frac{1}{2}:1$ which is same as $1:2$.

**22.** In what ratio does the y-axis divide the join of P(-4,2) and Q(8,3)?
(a) 3:1 (b) 1:3 (c) 2:1 (d) 1:2
**Answer:** (d) 1:2
**Sol:**
Let AB be divided by the $y$-axis in the ratio $k:1$ at the point $P$.
Then, by section formula, the coordinates of $P$ are

$P\left(\frac{8k-4}{k+1}, \frac{3k+2}{k+1}\right)$

But, $P$ lies on the $y$-axis, so, its abscissa is 0.

$\Rightarrow \frac{8k-4}{k+1} = 0$

$\Rightarrow 8k - 4 = 0$

$\Rightarrow 8k = 4$

$\Rightarrow k = \frac{1}{2}$

Hence, the required ratio is $\frac{1}{2}:1$, which is same as $1:2$.

**23.** If P(-1,1) is the midpoint of the line segment joining A(-3,b) and B(1, b+4) then b=?
(a) 1 (b) -1 (c) 2 (d) 0
**Answer:** (b) -1
**Sol:**
The given ports are $A(-3,b)$ *and* $B(1,b+4)$.

Then, $(x_1 = -3, y_1 = b)$ *and* $(x_2 = 1, y_2 = b+4)$

Therefore,

$x = \frac{[(-3)+1]}{2}$

$= \frac{-2}{2}$

$= -1$

And

$y = \frac{[b + (b+4)]}{2}$

$= \frac{2b+4}{2}$

$= b + 2$

But the midpoint is $P(-1,1)$.

Therefore,

$b + 2 = 1$

$\Rightarrow b = -1$

**24.** The line 2x+y-4=0 divide the line segment joining A(2,-2) and B(3,7) in the ratio
(a) 2:5 (b) 2:9 (c) 2:7 (d) 2:3

**Answer:** (b) 2:9

**Sol:**

Let the line $2x + y - 4 = 0$ divide the line segment in the ratio $k:1$ at the point *P*.

Then, by section formula the coordinates of *P* are

$P\left(\frac{3k+2}{k+1}, \frac{7k-2}{k+1}\right)$

Since *P* lies on the line $2x + y - 4 = 0$, we have

$\frac{2(3k+2)}{k+1} + \frac{7k-2}{k+1} - 4 = 0$

$\Rightarrow (6k+4) + (7k-2) - (4k+4) = 0$

$\Rightarrow 9k = 2$

$\Rightarrow k = \frac{2}{9}$

Hence, the required ratio is $\frac{2}{9}:1$ which is same as $2:9$.

**25.** If A(4,2), B(6,5) and C(1,4) be the vertices of $\Delta ABC$ and AD is a median, then the coordinates of D are

(a) $\left(\frac{5}{2}, 3\right)$ (b) $\left(5, \frac{7}{2}\right)$ (c) $\left(\frac{7}{2}, \frac{9}{2}\right)$ (d) none of these

**Answer:** (c) $\left(\frac{7}{2}, \frac{9}{2}\right)$

**Sol:**

*D* is the midpoint of *BC*

So, the coordinates of *D* are

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$D\left(\frac{6+1}{2}.\frac{5+4}{2}\right)\left[B(6,5)\ and\ C(1,4) \Rightarrow (x_1=6, y_1=5)\ and\ (x_2=1, y_2=4)\right]$

i.e., $D\left(\frac{7}{2},\frac{9}{2}\right)$

**26.** If A(-1,0), B(5,-2) and C(8,2) are the vertices of $\Delta ABC$ then its centroid is

(a) (12,0) (b) (6,0) (c) (0,6) (d) (4,0)

**Answer:** (d) (4,0)

**Sol:**

The given point are $A(-1,0), B(5,-2)\ and\ C(8,2)$.

Here, $(x_1=-1, y=0), (x_2=5, y=-2)\ and\ (x_3=8, y_3=2)$

Let $G(x,y)$ be the centroid of $\Delta ABC$. Then,

$x=\frac{1}{3}(x_1+x_2+x_3)$

$=\frac{1}{3}(-1+5+8)$

$=4$

and

$y=\frac{1}{3}(y_1+y_2+y_3)$

$=\frac{1}{3}(0-2+2)$

$=0$

Hence, the centroid of $\Delta ABC$ is $G(4,0)$.

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**27.** Two vertices of $\Delta ABC$ are A(-1,4) and B(5,2) and its centroid is G(0,-3). Then the coordinates of C are

(a) (4,3) (b) (4,15) (c) (-4,-15) (d) (-15, -4)

**Answer:** (c) (-4,-15)

**Sol:**

Two vertices of $\Delta ABC$ are $A(-1,4)\ and\ B(5,2)$.

Let the third vertex be $C(a,b)$.

Then, the coordinates of its centroid are

$G\left(\frac{-1+5+a}{3},\frac{4+2+b}{3}\right)$

$i.e., G\left(\frac{4+a}{3},\frac{6+b}{3}\right)$

But it is given that the centroid is $G(0,-3)$.

Therefore,

$\frac{4+a}{3}=0 \text{ and } \frac{6+b}{3}=-3$

$\Rightarrow 4+a=0 \text{ and } 6+b=-9$

$\Rightarrow a=-4 \text{ and } b=-15$

Hence, the third vertex of $\Delta ABC$ is $C(-4,-15)$.

**28.** The points A(-4,0), B(4,0) and C(0,3) are the vertices of a triangle, which is
(a) isosceles (b) equilateral (c) scalene (d) right-angled

**Answer:** (a) isosceles

**Sol:**

Let $A(-4,0), 8(4,0)$ and $C(0,3)$ be the given points. Then,

$AB=\sqrt{(4+4)^2+(0-0)^2}$

$=\sqrt{(8)^2+(0)^2}$

$=\sqrt{64+0}$

$=\sqrt{64}$

$=8 \text{ units}$

$BC=\sqrt{(0-4)^2+(3-0)^2}$

$=\sqrt{(-4)^2+(3)^2}$

$=\sqrt{16+9}$

$=\sqrt{25}$

$=5 \text{ units}$

$AC=\sqrt{(0+4)^2+(3-0)^2}$

$=\sqrt{(4)^2+(3)^2}$

$=\sqrt{16+9}$

$=\sqrt{25}$

$=5 \text{ units}$

$BC=AC=5 \text{ units}$

Therefore, $\Delta ABC$ is isosceles

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**29.** The points P(0,6), Q(-5,3) and R(3,1) are the vertices of a triangle, which is
(a) equilateral (b) isosceles (c) scalene (d) right-angled
**Ans:** (d) right - angled
**Sol:**
Let $P(0,6), Q(-5,3)$ *and* $R(3,1)$ be the given points. Then,

$PQ = \sqrt{(-5-0)^2 + (3-6)^2}$
$= \sqrt{(-5)^2 + (-3)^2}$
$= \sqrt{25+9}$
$= \sqrt{34}$ *units*

$QR = \sqrt{(3+5)^2 + (1-3)^2}$
$= \sqrt{(8)^2 + (-2)^2}$
$= \sqrt{64+4}$
$= \sqrt{68}$
$= 2\sqrt{17}$ *units*

$PR = \sqrt{(3-0)^2 + (1-6)^2}$
$= \sqrt{(3)^2 + (-5)^2}$
$= \sqrt{9+25}$
$= \sqrt{34}$ *units*

$PQ^2 + PR^2 \Rightarrow \left\{\left(\sqrt{34}\right)^2 + \left(\sqrt{34}\right)^2\right\} = 68$

$QR^2 \Rightarrow \left(2\sqrt{17}\right)^2 = 68$

Thus, $PQ^2 + PR^2 = QR^2$
Therefore, $\Delta PQR$ is right-angled.

**30.** If the points A(2,3), B(5,k) and C(6,7) are collinear then
(a) k =4 (b) k = 6 (c) k = $\frac{-3}{2}$ (d) k = $\frac{11}{4}$
**Ans:** (b) k = 6
**Sol:**
The given points are $A(2,3), B(5,k)$ *and* $C(6,7)$.
Here, $(x_1 = 2, y_1 = 3), (x_2 = 5, y_2 = k)$ *and* $(x_3 = 6, y_3 = 7)$.

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Points *A, B* and *C* are collinear. Then,

$x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$

$\Rightarrow 2(k-7)+5(7-3)+6(3-k)=0$

$\Rightarrow 2k-14+20+18-6k=0$

$\Rightarrow -4k=-24$

$\Rightarrow k=6$

**31.** If the point $A(1,2), O(0,0)\ and\ C(a,b)$ are collinear, then

(a) a = b (b) a = 2b (c) 2a = b (d) a + b = 0

**Ans:** (c) 2a = b

**Sol:**

The given points are $A(1,2), O(0,0)\ and\ C(a,b)$

Here, $(x_1=1, y_1=2), (x_2=0, y_2=0)\ and\ (x_3=a, y_3=b)$.

Point *A*, *O* and *C* are collinear

$\Rightarrow x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)=0$

$\Rightarrow 1(0-b)+0(b-2)+a(2-0)=0$

$\Rightarrow -b+2a=0$

$\Rightarrow 2a=b$

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**32.** The area of $\Delta ABC$ with vertices A(3,0) , B(7,0) and C(8,4) is

(a) 14 sq units (b) 28 sq units (c) 8 sq units (d) 6 sq units

**Ans:** (c) 8 sq units

**Sol:**

The given points are $A(3,0), B(7,0)\ and\ C(8,4)$.

Here, $(x_1=3, y_1=0), (x_2=7, y_2=0)\ and\ (x_3=8, y_3=4)$

Therefore,

$$Area\ of\ \Delta ABC=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

$$=\frac{1}{2}\left[3(0-4)+7(4-0)+8(0-0)\right]$$

$$=\frac{1}{2}\left[-12+28+0\right]$$

$$=\left(\frac{1}{2}\times 16\right)$$

$$=8\ sq.\ units$$

**33.** AOBC is rectangle whose three vertices are A(0,3), O(0,0) and B(5,0). The length of each of its diagonals is

(a) 5 units (b) 3 units (c) 4 units (d) $\sqrt{34}$ units

**Ans:** (c) 4 units

**Sol:**

$A(0,3), O(0,0)$ *and* $B(5,0)$ are the three vertices of a rectangle; let C be the fourth vertex

Then, the length of the diagonal,

$AB = \sqrt{(5-0)^2 + (0-3)^2}$

$= \sqrt{(5)^2 + (-3)^2}$

$= \sqrt{25+9}$

$= \sqrt{34}$ *units*

Since, the diagonals al rectangle is equal.

Hence, the length of its diagonals is $\sqrt{34}$ units.

**34.** If the distance between the points A(4,p) and B(1,0) is 5 then

(a) p = 4 only (b) p = -4 only (c) p = $\pm 4$ (d) p = 0

**Ans:** (c) p = $\pm 4$

**Sol:**

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The given points are $A(4, p)$ *and* $B(1,0)$ *and* $AB = 5$.

Then, $(x_1 = 4, y_1 = p)$ *and* $(x_2 = 1, y_2 = 0)$

Therefore,

$AB = 5$

$\Rightarrow \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 5$

$\Rightarrow \sqrt{(1-4)^2 + (0-p)^2} = 5$

$\Rightarrow (-3)^2 + (-p)^2 = 25$

$\Rightarrow 9 + p^2 = 25$

$\Rightarrow p^2 = 16$

$\Rightarrow p = \pm\sqrt{16}$

$\Rightarrow p = \pm 4$