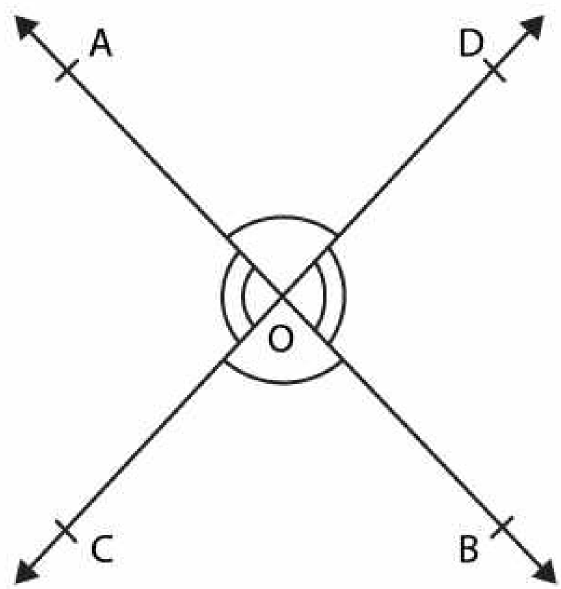
EXERCISE 6.4

If two lines intersect, prove that the vertically opposite angles are equal. Solution:



From the figure, we know that,

AB and CD intersect each other at point O.

Let the two pairs of vertically opposite angles be,

1st pair – ∠AOC and ∠BOD

2nd pair – ∠AOD and ∠BOC

To prove:

Vertically opposite angles are equal,

i.e., ∠AOC = ∠BOD, and ∠AOD = ∠BOC

From the figure,

The ray AO stands on the line CD.

We know that,

If a ray lies on a line, then the sum of the adjacent angles is equal to 180°.

⇒ ∠AOC + ∠AOD = 180° (By linear pair axiom) … (i)

Similarly, the ray DO lies on line AOB.

⇒ ∠AOD + ∠BOD = 180° (By linear pair axiom) … (ii)

From equations (i) and (ii),

We have,

∠AOC + ∠AOD = ∠AOD + ∠BOD

⇒ ∠AOC = ∠BOD – – – – (iii)

Similarly, the ray BO lies on the line COD.

⇒ ∠DOB + ∠COB = 180° (By linear pair axiom) – – – – (iv)

Also, the ray CO lies on line AOB.

⇒ ∠COB + ∠AOC = 180° (By linear pair axiom) – – – – (v)

From equations (iv) and (v),

We have,

∠DOB + ∠COB = ∠COB + ∠AOC

⇒ ∠DOB = ∠AOC – – – – (vi)

Thus, from equation (iii) and equation (vi),

We have,

∠AOC = ∠BOD, and ∠DOB = ∠AOC

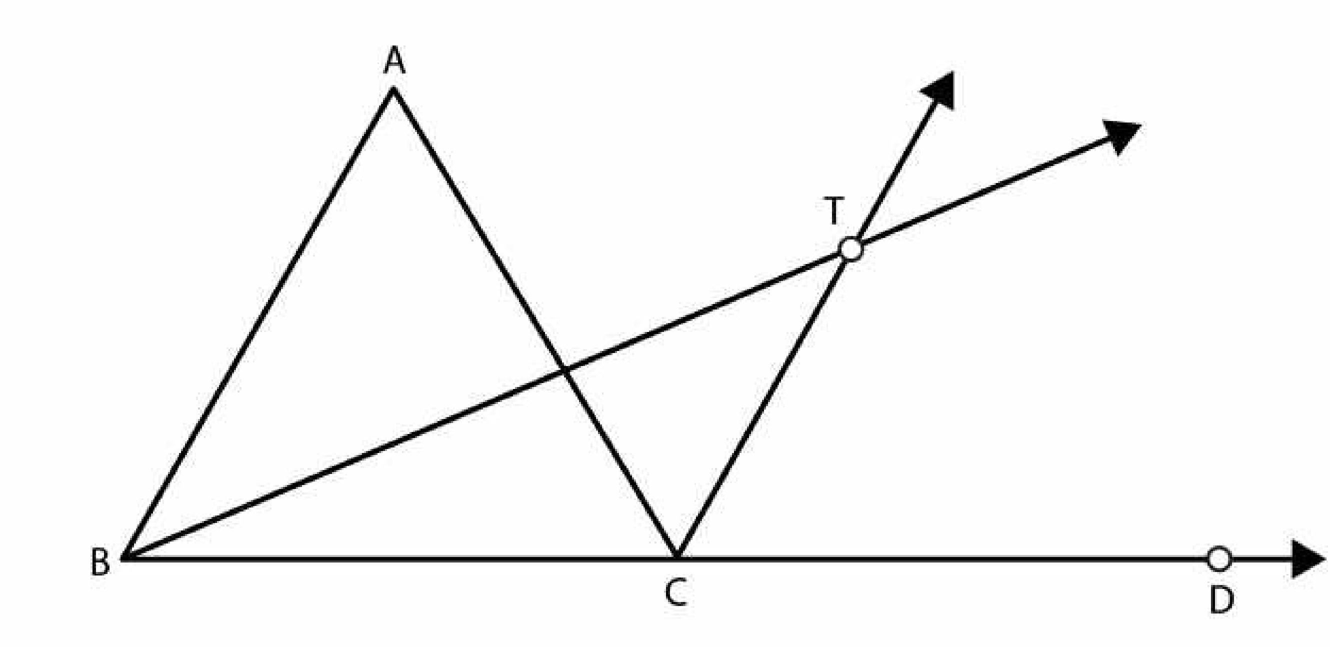
Therefore, we get vertically opposite angles are equal.

Hence Proved.

1. Bisectors of interior ∠B and exterior ∠ACD of a Δ ABC intersect at point T.
Prove that ∠ BTC = ½ ∠ BAC.

Solution:

Given: △ ABC, produce BC to D, and the bisectors of ∠ABC and ∠ACD meet at point T.



To prove:

∠BTC = ½ ∠BAC

Proof:

In △ABC,∠ACD is an exterior angle.

We know that,

The exterior angle of a triangle is equal to the sum of two opposite angles,

Then,

∠ACD = ∠ABC + ∠CAB

Dividing L.H.S and R.H.S by 2,

⇒ ½ ∠ACD = ½ ∠CAB + ½ ∠ABC

⇒ ∠TCD = ½ ∠CAB + ½ ∠ABC …(1)

[∵CT is a bisector of ∠ACD⇒ ½ ∠ACD = ∠TCD]
We know that,

The exterior angle of a triangle is equal to the sum of two opposite angles,

Then in △ BTC,

∠TCD = ∠BTC +∠CBT

⇒ ∠TCD = ∠BTC + ½ ∠ABC …(2)

[∵BT is the bisector of △ ABC ⇒∠CBT = ½ ∠ABC]
From equations (1) and (2),

We get,

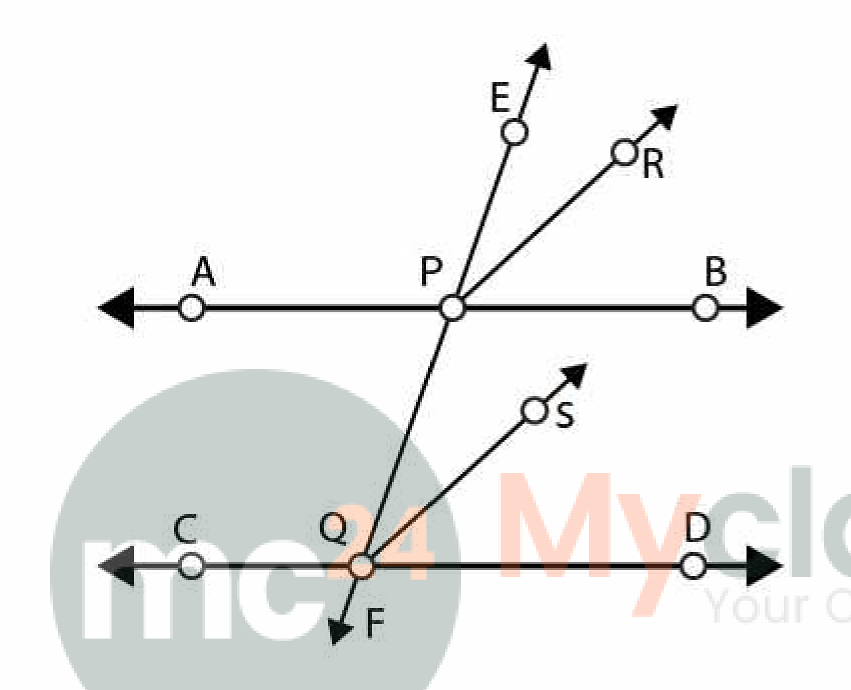
$\frac{1}{2}\angle CAB + \frac{1}{2}\angle ABC = \angle BTC + \frac{1}{2}\angle ABC$

$\Rightarrow \frac{1}{2}\angle CAB = \angle BTC$ or $\frac{1}{2}\angle BAC = \angle BTC$

Hence, proved.

2. A transversal intersects two parallel lines. Prove that the bisectors of any pair of corresponding angles so formed are parallel.

Solution:



Let,

$AB \parallel CD$

EF be the transversal passing through the two parallel lines at P and Q, respectively.

PR and QS are the bisectors of $\angle EPB$ and $\angle PQD$.

We know that the corresponding angles of parallel lines are equal,

So, $\angle EPB = \angle PQD$

$\frac{1}{2}\angle EPB = \frac{1}{2}\angle PQD$

$\angle EPR = \angle PQS$

But, we also know that they are corresponding angles of PR and QS

Since the corresponding angles are equal,

We have,

$PR \parallel QS$

Hence Proved.