10. Differentiation

Exercise 10A

1. Question

Differentiate each of the following w.r.t. x:

sin 4x

Answer

Formulae :

• $\frac{d}{dx}(\sin x) = \cos x$

• $\frac{d}{dx}(kx) = k$

Let,

y = sin 4x

and u = 4x

therefore, y = sin u

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\sin u).\frac{d}{dx}(4x)$

$= \cos u . 4$ .... $\left(\because \frac{d}{dx}(\sin x) = \cos x \;\&\; \frac{d}{dx}(kx) = k\right)$

= cos 4x . 4

= 4 cos 4x

2. Question

Differentiate each of the following w.r.t. x:

cos 5x

Answer

Formulae :

• $\frac{d}{dx}(\cos x) = -\sin x$

• $\frac{d}{dx}(kx) = k$

Let,

y = cos 5x

and u = 5x

therefore, y= cos u

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ ………… By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\cos u) \cdot \frac{d}{dx}(5x)$

$= -\sin u \cdot 5$ ………… $\left(\because \frac{d}{dx}(\cos x) = -\sin x \;\&\; \frac{d}{dx}(kx) = k\right)$

= - sin 5x . 5

= - 5 sin 5x

3. Question

Differentiate each of the following w.r.t. x:

tan 3x

Answer

Formulae :

• $\frac{d}{dx}(\tan x) = \sec^2 x$

• $\frac{d}{dx}(kx) = k$

Let,

y = tan 3x

and u = 3x

therefore, y= tan u

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ ………… By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\tan u).\frac{d}{dx}(3x)$

$= \sec^2 u\ .3$ .... $\left(\because \frac{d}{dx}(\tan x) = \sec^2 x \ \&\ \frac{d}{dx}(kx) = k\right)$

$= \sec^2 3x\ .\ 3$

$= 3\sec^2 3x$

4. Question

Differentiate each of the following w.r.t. x:

$\cos x^3$

Answer

Formulae :

• $\frac{d}{dx}(\cos x) = -\sin x$

• $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

$y = \cos x^3$

and $u = x^3$

therefore, $y = \cos u$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\cos u).\frac{d}{dx}(x^3)$

$= -\sin u\ .3x^2$ .... $\left(\because \frac{d}{dx}(\cos x) = -\sin x \ \&\ \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$= -\sin x^3\ .\ 3x^2$

$= -3x^2 \sin x^3$

5. Question

Differentiate each of the following w.r.t. x:

$\cot^2 x$

Answer

Formulae :

• $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

• $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

$y = \cot^2 x$

and $u = \cot x$

therefore, $y = u^2$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(u^2).\frac{d}{dx}(\cot x)$

$= 2u.(-\operatorname{cosec}^2 x)$ .... $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x\right)$

$= 2\cot x.(-\operatorname{cosec}^2 x)$

$= -2\cot x.\operatorname{cosec}^2 x$

6. Question

Differentiate each of the following w.r.t. x:

$\tan^3 x$

Answer

Formulae :

• $\frac{d}{dx}(\tan x) = \sec^2 x$

• $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

$y = \tan^3 x$

and $u = \tan x$

therefore, $y = u^3$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(u^3) . \frac{d}{dx}(\tan x)$

$= 3\ u^2\ .\sec^2 x$ .... $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1}\ \&\ \frac{d}{dx}(\tan x) = \sec^2 x\right)$

$= 3\ \tan^2 x\ .(\sec^2 x)$

$= 3\ \tan^2 x\ .\ \sec^2 x$

7. Question

Differentiate each of the following w.r.t. x:

$\cot\sqrt{x}$

Answer

<u>Formulae</u> :

- $\frac{d}{dx}(\cot x) = -\ \mathrm{cosec}^2 x$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

Let,

$y = \cot\sqrt{x}$

and $u = \sqrt{x}$

therefore, y= cot u

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} . \frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\cot u) . \frac{d}{dx}(\sqrt{x})$

$= -\ \mathrm{cosec}^2 u\ . \frac{1}{2\sqrt{x}}$ .... $\left(\because \frac{d}{dx}(\cot x) = -\ \mathrm{cosec}^2 x\ \&\ \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}\right)$

$= -\ \mathrm{cosec}^2\sqrt{x} . \frac{1}{2\sqrt{x}}$

$= \frac{-1}{2\sqrt{x}}\ \mathrm{cosec}^2\sqrt{x}$

8. Question

Differentiate each of the following w.r.t. x:

$\sqrt{\tan x}$

Answer

Formulae :

- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

Let,

$y = \sqrt{\tan x}$

and u = tan x

therefore, $y = \sqrt{u}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ ………… By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\sqrt{u}).\frac{d}{dx}(\tan x)$$

$$= \frac{1}{2\sqrt{u}}.\sec^2 x \text{ .... } \left(\because \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} \ \& \ \frac{d}{dx}(\tan x) = \sec^2 x\right)$$

$$= \frac{1}{2\sqrt{\tan x}}.\sec^2 x$$

$$= \frac{\sec^2 x}{2\sqrt{\tan x}}$$

9. Question

Differentiate each of the following w.r.t. x:

$(5 + 7x)^6$

Answer

Formulae :

- $\frac{d}{dx}(x^n) = n.x^{n-1}$
- $\frac{d}{dx}(kx) = k$
- $\frac{d}{dx}(k) = 0$

- $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = (5+7x)^6$

and u = (5+7x)

therefore, $y = u^6$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(u^6).\frac{d}{dx}(5+7x)$$

$= 6.(u)^5 .\left(\frac{d}{dx}(5) + \frac{d}{dx}(7x)\right)$ .... $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$= 6.(5+7x)^5.(0+7)$ .... $\left(\because \frac{d}{dx}(k) = 0 \;\&\; \frac{d}{dx}(kx) = k\right)$

$= 42.(5+7x)^5$

10. Question

Differentiate each of the following w.r.t. x:

$(3 - 4x)^5$

Answer

Formulae :

- $\frac{d}{dx}(x^n) = n.x^{n-1}$

- $\frac{d}{dx}(kx) = k$

- $\frac{d}{dx}(k) = 0$

- $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = (3-4x)^5$

and u = (3-4x)

therefore, $y = u^5$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ ………… By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(u^5).\frac{d}{dx}(3-4x)$

$= 5.(u)^4.\left(\frac{d}{dx}(3) + \frac{d}{dx}(-4x)\right)$ ………… $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$= 5.(3-4x)^4.(0-4)$ ………… $\left(\because \frac{d}{dx}(k) = 0 \;\&\; \frac{d}{dx}(kx) = k\right)$

$= -20(3-4x)^4$

11. Question

Differentiate each of the following w.r.t. x:

$(2x^2 - 3x + 4)^5$

Answer

Formulae :

- $\frac{d}{dx}(x^n) = n.x^{n-1}$
- $\frac{d}{dx}(kx) = k$
- $\frac{d}{dx}(k) = 0$
- $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = (2x^2 - 3x + 4)^5$

and $u = (2x^2 - 3x + 4)$

therefore, $y = u^5$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ ………… By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(u^5).\frac{d}{dx}(2x^2-3x+4)$

$= 5.(u)^4.\left(\frac{d}{dx}(2x^2) + \frac{d}{dx}(-3x) + \frac{d}{dx}(4)\right)$ …………
$\left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$= 5. (2x^2 - 3x + 4)^4. (4x-3+0)$ .... $\left(\because \frac{d}{dx}(kx) = k \;\&\; \frac{d}{dx}(k) = 0\right)$

$= 5. (2x^2 - 3x + 4)^4 (4x-3)$

12. Question

Differentiate each of the following w.r.t. x:

$(ax^2 + bx + c)^6$

Answer

Formulae :

• $\frac{d}{dx}(x^n) = n.x^{n-1}$

• $\frac{d}{dx}(kx) = k$

• $\frac{d}{dx}(k) = 0$

• $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = (ax^2 + bx + c)^6$

and $u = (ax^2 + bx + c)$

therefore, $y = u^6$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(u^6).\frac{d}{dx}(ax^2 + bx + c)$

$= 6.(u)^5 . \left(\frac{d}{dx}(ax^2) + \frac{d}{dx}(bx) + \frac{d}{dx}(c)\right)$

$= 6.(ax^2 + bx + c)^5 . \frac{d}{dx}(ax^2 + bx + c)$ .... $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$= 6. (ax^2 + bx + c)^5. (2ax+b+0)$ .... $\left(\because \frac{d}{dx}(kx) = k \;\&\; \frac{d}{dx}(k) = 0\right)$

13. Question

Differentiate each of the following w.r.t. x:

$$\frac{1}{(x^2-3x+5)^3}$$

Answer

Formulae :

- $\frac{d}{dx}\left(\frac{1}{x}\right)=\frac{-1}{x^2}$
- $\frac{d}{dx}(x^n)=n.x^{n-1}$
- $\frac{d}{dx}(kx)=k$
- $\frac{d}{dx}(k)=0$
- $\frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}$

Let,

$$y=\frac{1}{(x^2-3x+5)^3}$$

Let, $u=(x^2-3x+5)^3$

Therefore, $y=\frac{1}{u}$

For $u=(x^2-3x+5)^3$

Let, $v=(x^2-3x+5)$

Therefore, $u=(v)^3$

Therefore, $y=\frac{1}{v^3}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dv}.\frac{dv}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx}=\frac{d}{du}\left(\frac{1}{u}\right).\frac{d}{dv}(v)^3.\frac{d}{dx}(x^2-3x+5)$$

$$=\frac{-1}{u^2}.3v^2.\left(\frac{d}{dx}(x^2)+\frac{d}{dx}(-3x)+\frac{d}{dx}(5)\right)$$

.... $\left(\because \frac{d}{dx}\left(\frac{1}{x}\right)=\frac{-1}{x^2}, \frac{d}{dx}(x^n)=n.x^{n-1} \;\&\; \frac{d}{dx}(u+v)=\frac{du}{dx}+\frac{dv}{dx}\right)$

$= \frac{-1}{(x^2-3x+5)^6}.3(x^2-3x+5)^2.(2x-3+0)$ .... $\left(\because \frac{d}{dx}(kx) = k \ \& \ \frac{d}{dx}(k) = 0\right)$

$= \frac{-3}{(x^2-3x+5)^4}.(2x-3)$

$= \frac{-3(2x-3)}{(x^2-3x+5)^4}$

14. Question

Differentiate each of the following w.r.t. x:

$\sqrt{\frac{a^2-x^2}{a^2+x^2}}$

Answer

Formulae :

- $\frac{d}{dx}(x^n) = n.x^{n-1}$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx}(k) = 0$
- $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u.\frac{d}{dx}(v)}{(v)^2}$

Let,

$y = \sqrt{\frac{a^2-x^2}{a^2+x^2}}$

and $u = \frac{a^2-x^2}{a^2+x^2}$

$\therefore y = \sqrt{u}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\sqrt{u}).\frac{d}{dx}\left(\frac{a^2-x^2}{a^2+x^2}\right)$

$$= \frac{1}{2\sqrt{u}} \left(\frac{(a^2+x^2).\frac{d}{dx}(a^2-x^2) - (a^2-x^2).\frac{d}{dx}(a^2+x^2)}{(a^2+x^2)^2} \right) \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u.\frac{d}{dx}(v)}{(v)^2} \;\&\; \frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}} \right)$$

$$= \frac{1}{2\sqrt{\frac{a^2-x^2}{a^2+x^2}}} \left(\frac{(a^2+x^2).(-2x) - (a^2-x^2).(2x)}{(a^2+x^2)^2} \right) \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(k) = 0 \right)$$

$$= \frac{\sqrt{a^2+x^2}}{2\sqrt{a^2-x^2}}.(2x)\left(\frac{-a^2-x^2-\ a^2+x^2}{(a^2+x^2)^2} \right)$$

$$= \frac{(a^2+x^2)^{1/2}}{2(a^2-x^2)^{1/2}}.(2x).\frac{-2a^2}{(a^2+x^2)^2}$$

$$= \frac{-2a^2x}{(a^2-x^2)^{1/2}\,(a^2+x^2)^{2-\frac{1}{2}}}$$

$$= \frac{-2a^2x}{(a^2-x^2)^{1/2}.(a^2+x^2)^{3/2}}$$

15. Question

Differentiate each of the following w.r.t. x:

$$\sqrt{\frac{1+\sin x}{1-\sin x}}$$

Answer

Formulae :

- $1 - \sin^2 x = \cos^2 x$
- $\frac{d}{dx}(\sec x) = \sec x.\tan x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$

Let,

$$y = \sqrt{\frac{1+\sin x}{1-\sin x}}$$

Multiplying numerator and denominator by (1+sin x),

$$\therefore y = \sqrt{\frac{1+\sin x}{1-\sin x}.\frac{1+\sin x}{1+\sin x}}$$

$$= \sqrt{\frac{(1+\sin x)^2}{1-\sin^2 x}}$$

$$= \sqrt{\frac{(1+\sin x)^2}{\cos^2 x}}$$ .... $(1 - \sin^2 x = \cos^2 x)$

$$= \frac{1+\sin x}{\cos x}$$

$$= \frac{1}{\cos x} + \frac{\sin x}{\cos x}$$

y = sec x + tan x

Differentiating above equation w.r.t. x,

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sec x + \tan x)$$

$$\therefore \frac{dy}{dx} = \frac{d}{dx}(\sec x) + \frac{d}{dx}(\tan x)$$

$$= \sec x.\tan x + \sec^2 x \text{ .... } \left(\because \frac{d}{dx}(\sec x) = \sec x.\tan x \text{ \& } \frac{d}{dx}(\tan x) = \sec^2 x\right)$$

$$= \sec x\ (\tan x + \sec x)$$

16. Question

Differentiate each of the following w.r.t. x:

$\cos^2 x^3$

Answer

<u>Formulae</u> :

• $\frac{d}{dx}(\cos x) = -\sin x$

• $\frac{d}{dx}(x^n) = n.x^{n-1}$

• 2 sin x. cos x = sin 2x

Let,

$y = \cos^2 x^3$

and $u = x^3$

therefore, $y = \cos^2 u$

let, v = cos u

therefore, y= v^2

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(v^2).\frac{d}{du}(\cos u).\frac{d}{dx}(x^3)$$

$$= 2\,v\,.(-\sin u).3x^2 \text{ .... } \left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(\cos x) = -\sin x\right)$$

$$= -\,2\cos u\,.\sin u\,.3x^2$$

$$= -\,\sin 2u\,.3x^2 \text{ .... } (\because 2\sin x.\cos x = \sin 2x)$$

$$= -\,\sin 2x^3.3x^2$$

17. Question

Differentiate each of the following w.r.t. x:

$\sec^3(x^2+1)$

Answer

__**Formulae**__ :

- $\frac{d}{dx}(\sec x) = \sec x.\tan x$
- $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

y = $\sec^3(x^2+1)$

and u = x^2+1

therefore, y= $\sec^3 u$

let, v = sec u

therefore, y= v^3

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(v^3).\frac{d}{du}(\sec u).\frac{d}{dx}(x^2+1)$$

$$= 3\,v^2\,.(\sec u.\tan u).2x \text{ .... } \left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(\sec x) = \sec x.\tan x\right)$$

$= 3\sec^2 u\,.(\sec u.\tan u)\,.\,2x$

$= 6x.\sec^3 u\,.\tan u$

$= 6x.\sec^3(x^2+1)\ .\tan(x^2+1)$

18. Question

Differentiate each of the following w.r.t. x:

$\sqrt{\cos 3x}$

Answer

__**Formulae**__ **:**

- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx}(kx) = k$

Let,

$y = \sqrt{\cos 3x}$

and u = 3x

therefore, $y = \sqrt{\cos u}$

let, v = cos u

therefore, $y = \sqrt{v}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{dv}(\sqrt{v}).\frac{d}{du}(\cos u).\frac{d}{dx}(3x)$

$= \frac{1}{2\sqrt{v}}.(-\sin u).3$ .... $\left(\because \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}, \frac{d}{dx}(\cos x) = -\sin x \;\&\; \frac{d}{dx}(kx) = k\right)$

$= \frac{-3}{2}.\frac{\sin u}{\sqrt{\cos u}}$

$= \frac{-3}{2}.\frac{\sin 3x}{\sqrt{\cos 3x}}$

19. Question

Differentiate each of the following w.r.t. x:

$\sqrt[3]{\sin 2x}$

Answer

<u>**Formulae**</u> **:**

- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(x^n) = n.x^{n-1}$
- $\frac{d}{dx}(kx) = k$

Let,

$y = \sqrt[3]{\sin 2x}$

and u = 2x

therefore, $y = \sqrt[3]{\sin u}$

let, v = sin u

therefore, $y = \sqrt[3]{v} = v^{3/2}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ ………… By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(v^{1/3}).\frac{d}{du}(\sin u).\frac{d}{dx}(2x)$$

$= \frac{1}{3}v^{-2/3}.(\cos u).2$ ………… $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1}, \frac{d}{dx}(\sin x) = \cos x \ \& \ \frac{d}{dx}(kx) = k\right)$

$$= \frac{2}{3}\frac{\cos u}{v^{2/3}}.$$

$$= \frac{2}{3}\frac{\cos u}{(\sin u)^{2/3}}$$

$$= \frac{2}{3}\frac{\cos 2x}{(\sin 2x)^{2/3}}$$

20. Question

Differentiate each of the following w.r.t. x:

$\sqrt{1+\cot x}$

Answer

Formulae :

- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx}(k) = 0$
- $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = \sqrt{1+\cot x}$

and u = 1+ cot x

therefore, $y = \sqrt{u}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ ………… By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{du}(\sqrt{u}) \cdot \frac{d}{dx}(1+\cot x)$

$= \frac{1}{2\sqrt{u}} \cdot \left(\frac{d}{dx}(1) + \frac{d}{dx}(\cot x)\right)$ ………… $\left(\because \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} \;\&\; \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$= \frac{1}{2\sqrt{1+\cot x}} \cdot (0 - \operatorname{cosec}^2 x)$.

$= \frac{-1}{2} \frac{\operatorname{cosec}^2 x}{\sqrt{1+\cot x}}$

21. Question

Differentiate each of the following w.r.t. x:

$\operatorname{cosec}^3 \frac{1}{x^2}$

Answer

Formulae :

• $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x.\cot x$

• $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

$$y = \operatorname{cosec}^3 \frac{1}{x^2}$$

and $u = \frac{1}{x^2}$

therefore, y= cosec^3 u

let, v = cosec u

therefore, y= v^3

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ ………… By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(v^3).\frac{d}{du}(\operatorname{cosec} u).\frac{d}{dx}\left(\frac{1}{x^2}\right)$$

$$= 3v^2 .(-\operatorname{cosec} u.\cot u).\frac{d}{dx}(x^{-2})$$

………… $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \,\&\, \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x.\cot x\right)$

$$= 3\operatorname{cosec}^2 u .(-\operatorname{cosec} u.\cot u).(-2x^{-3})$$

$$= 3\operatorname{cosec}^3 u.\cot u \,(2\frac{1}{x^3})$$

$$= \frac{6}{x^3}.\operatorname{cosec}^3\left(\frac{1}{x^2}\right).\cot\left(\frac{1}{x^2}\right)$$

22. Question

Differentiate each of the following w.r.t. x:

$$\sqrt{\sin x^3}$$

Answer

<u>**Formulae**</u> **:**

• $\frac{d}{dx}(\sin x) = \cos x$

• $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

• $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

$y = \sqrt{\sin x^3}$

and $u = x^3$

therefore, $y = \sqrt{\sin u}$

let, v = sin u

therefore, $y = \sqrt{v}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$ .... By chain rule

$\therefore \frac{dy}{dx} = \frac{d}{dv}(\sqrt{v}) \cdot \frac{d}{du}(\sin u) \cdot \frac{d}{dx}(x^3)$

$= \frac{1}{2\sqrt{v}} \cdot (\cos u) \cdot 3x^2$

.... $\left(\because \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}, \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(\sin x) = \cos x\right)$

$= \frac{1}{2\sqrt{\sin u}} \cdot (\cos u) \cdot 3x^2$

$= \frac{3}{2}x^2 \cdot \frac{\cos x^3}{\sqrt{\sin x^3}}$

23. Question

Differentiate each of the following w.r.t. x:

$\sqrt{x \sin x}$

Answer

Formulae :

• $\frac{d}{dx}(\sin x) = \cos x$

• $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

• $\frac{d}{dx}(kx) = k$

• $\frac{d}{dx}(u.v) = u.\frac{d}{dx}(v) + v.\frac{d}{dx}(u)$

Let,

$y = \sqrt{x.\sin x}$

and u = x. sin x

therefore, $y = \sqrt{u}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{du}(\sqrt{u}).\frac{d}{dx}(x.\sin x)$$

$$= \frac{1}{2\sqrt{u}}.\left(x.\frac{d}{dx}(\sin x) + \sin x.\frac{d}{dx}(x)\right)$$

.... $\left(\because \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} \;\&\; \frac{d}{dx}(u.v) = u.\frac{d}{dx}(v) + v.\frac{d}{dx}(u)\right)$

$$= \frac{1}{2\sqrt{x.\sin x}}.(x.(\cos x) + \sin x.(1))$$ .... $\left(\because \frac{d}{dx}(kx) = k \;\&\; \frac{d}{dx}(\sin x) = \cos x\right)$

$$= \frac{(x.\cos x + \sin x)}{2\sqrt{x.\sin x}}$$

24. Question

Differentiate each of the following w.r.t. x:

$\sqrt{\cot\sqrt{x}}$

Answer

__**Formulae**__ **:**

• $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$

• $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

Let,

$y = \sqrt{\cot\sqrt{x}}$

And $u = \sqrt{x}$

therefore, $y = \sqrt{\cot u}$

let, v = cot u

therefore, $y = \sqrt{v}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(\sqrt{v}).\frac{d}{du}(\cot u).\frac{d}{dx}(\sqrt{x})$$

$$= \frac{1}{2\sqrt{v}}.(-\operatorname{cosec}^2 u).\frac{1}{2\sqrt{x}}$$

.... $\left(\because \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} \ \& \ \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2\right)$

$$= \frac{1}{2\sqrt{\cot u}}.(-\operatorname{cosec}^2 u).\frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2\sqrt{\cot\sqrt{x}}}.(-\operatorname{cosec}^2\sqrt{x}).\frac{1}{2\sqrt{x}}$$

$$= \frac{-\operatorname{cosec}^2\sqrt{x}}{4\sqrt{x}\sqrt{\cot\sqrt{x}}}$$

25. Question

Differentiate each of the following w.r.t. x:

$\cot^3 x^2$

Answer

Formulae :

- $\frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x$
- $\frac{d}{dx}(x^n) = n.x^{n-1}$

Let,

$y = \cot^3 x^2$

and $u = x^2$

therefore, $y = \cot^3 u$

let, $v = \cot u$

therefore, $y = v^3$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{du} \cdot \frac{du}{dx}$ ………… By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}(v^3) \cdot \frac{d}{du}(\cot u) \cdot \frac{d}{dx}(x^2)$$

$$= 3v^2 \cdot (-\operatorname{cosec}^2 u) \cdot 2x \ \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}(x^n) = n.x^{n-1} \ \&\ \frac{d}{dx}(\cot x) = -\operatorname{cosec}^2 x\right)$$

$$= 3\cot^2 u \cdot (-\operatorname{cosec}^2 u) \cdot 2x$$

$$= -6x.\cot^2 u \cdot \operatorname{cosec}^2 u$$

$$= -6x.\cot^2(x^2) \cdot \operatorname{cosec}^2(x^2)$$

26. Question

Differentiate each of the following w.r.t. x:

$$\cos\left(\sin\sqrt{ax+b}\right)$$

Answer

<u>Formulae</u> :

- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$$y = \cos\left(\sin\sqrt{ax+b}\right)$$

and $u = ax + b$

therefore, $y = \cos\left(\sin\sqrt{u}\right)$

let, $v = \sqrt{u}$

therefore, $y = \cos(\sin v)$

let, $w = \sin v$

therefore, $y = \cos w$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dw}.\frac{dw}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dw}(\cos w).\frac{d}{dv}(\sin v).\frac{d}{du}(\sqrt{u}).\frac{d}{dx}(ax + b)$$

$$= (-\sin w).(\cos v).\left(\frac{1}{2\sqrt{u}}\right).\left(\frac{d}{dx}(ax) + \frac{d}{dx}(b)\right)$$

.... $\left(\because \frac{d}{dx}(\cos x) = -\sin x, \frac{d}{dx}(\sin x) = \cos x, \frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}} \text{ \& } \frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$$= (-\sin(\sin v)).(\cos\sqrt{u}).\left(\frac{1}{2\sqrt{ax + b}}\right).(a + 0)$$

$$= (-\sin(\sin\sqrt{u})).(\cos\sqrt{ax + b}).\left(\frac{1}{2\sqrt{ax + b}}\right).(a)$$

$$= \left(\frac{-a.\cos\sqrt{ax + b}}{2\sqrt{ax + b}}\right).\left(\sin\left(\sin\sqrt{ax + b}\right)\right)$$

27. Question

Differentiate each of the following w.r.t. x:

$$\sqrt{\operatorname{cosec}(x^3 + 1)}$$

Answer

<u>Formulae</u> :

- $\frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x . \cot x$
- $\frac{d}{dx}(x^n) = n.x^{n-1}$
- $\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$
- $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

$y = \sqrt{\operatorname{cosec}(x^3+1)}$

and $u = x^3 + 1$

therefore, $y = \sqrt{\operatorname{cosec} u}$

let, $v = \operatorname{cosec} u$

therefore, $y = \sqrt{v}$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{dy}{dv}.\frac{dv}{du}.\frac{du}{dx}$ .... By chain rule

$$\therefore \frac{dy}{dx} = \frac{d}{dv}\left(\sqrt{v}\right).\frac{d}{du}(\operatorname{cosec} u).\frac{d}{dx}(x^3+1)$$

$$= \frac{1}{2\sqrt{v}}.(-\operatorname{cosec} u.\cot u).\left(\frac{d}{dx}(x^3) + \frac{d}{dx}(1)\right)$$

.... $\left(\because \frac{d}{dx}(\operatorname{cosec} x) = -\operatorname{cosec} x.\cot x, \frac{d}{dx}\left(\sqrt{x}\right) = \frac{1}{2\sqrt{x}} \;\&\; \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$

$$= \frac{1}{2\sqrt{\operatorname{cosec} u}}.(-\operatorname{cosec}(x^3+1).\cot(x^3+1)).(3x^2+0)$$

.... $\left(\because \frac{d}{dx}(x^n) = n.x^{n-1}\right)$

$$= \frac{1}{2\sqrt{\operatorname{cosec}(x^3+1)}}.(-\operatorname{cosec}(x^3+1).\cot(x^3+1)).(3x^2)$$

$$= \frac{-3x^2}{2}.\sqrt{\operatorname{cosec}(x^3+1)}.\cot(x^3+1)$$

28. Question

Differentiate each of the following w.r.t. x:

sin $5x$ cos $3x$

Answer

<u>**Formulae**</u> **:**

- $(2\sin a.\cos b) = \sin(a+b) + \sin(a-b)$
- $\frac{d}{dx}(\sin x) = \cos x$

• $\frac{d}{dx}(kx) = k$

• $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

y = sin 5x. cos 3x

$$y = \frac{1}{2}(2\sin 5x.\cos 3x)$$

$$y = \frac{1}{2}(\sin(5x + 3x) + \sin(5x - 3x)) \text{ .... } (\because (2\sin a.\cos b) = \sin(a + b) + \sin(a - b))$$

$$y = \frac{1}{2}(\sin(8x) + \sin(2x))$$

Differentiating above equation w.r.t. x,

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}(\sin(8x) + \sin(2x))\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\left(\frac{d}{dx}\sin 8x + \frac{d}{dx}\sin 2x\right) \text{ .... } \left(\because \frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}\right)$$

$$= \frac{1}{2}(8\cos 8x + 2\cos 2x) \text{ .... } \left(\because \frac{d}{dx}(\sin x) = \cos x \ \& \ \frac{d}{dx}(kx) = k\right)$$

$$= 4\cos 8x + \cos 2x$$

29. Question

Differentiate each of the following w.r.t. x:

sin $2x$ sin x

Answer

<u>Formulae</u> :

• $(2\sin a.\sin b) = \cos(a - b) - \cos(a + b)$

• $\frac{d}{dx}(\cos x) = -\sin x$

• $\frac{d}{dx}(kx) = k$

• $\frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}$

Let,

y = sin 2x. sin x

$y = \frac{1}{2}(2\sin 2x.\sin x)$

$y = \frac{1}{2}(\cos(2x - x) - \cos(2x + x))$ .... $(\because (2\sin a.\sin b) = \cos(a - b) - \cos(a + b))$

$y = \frac{1}{2}(\cos x - \cos 3x)$

Differentiating above equation w.r.t. x,

$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}(\cos x - \cos 3x)\right)$

$\therefore \frac{dy}{dx} = \frac{1}{2}\left(\frac{d}{dx}\cos x - \frac{d}{dx}\cos 3x\right)$ .... $\left(\because \frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}\right)$

$= \frac{1}{2}(-\sin x + 3\sin 3x)$ .... $\left(\because \frac{d}{dx}(\cos x) = -\sin x \ \& \ \frac{d}{dx}(kx) = k\right)$

$= \frac{3}{2}\sin 3x - \frac{1}{2}\sin x$

30. Question

Differentiate each of the following w.r.t. x:

cos $4x$ cos $2x$

Answer

Formulae :

- $(2\cos a.\cos b) = \cos(a + b) + \cos(a - b)$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(kx) = k$
- $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

Let,

y = cos 4x. cos 2x

$y = \frac{1}{2}(2\cos 4x.\cos 2x)$

$y = \frac{1}{2}(\cos(4x + 2x) + \cos(4x - 2x))$ .... $(\because (2\cos a.\cos b) = \cos(a + b) + \cos(a - b))$

$y = \frac{1}{2}(\cos 6x + \cos 2x)$

Differentiating above equation w.r.t. x,

$$\therefore \frac{dy}{dx} = \frac{d}{dx}\left(\frac{1}{2}(\cos 6x + \cos 2x)\right)$$

$$\therefore \frac{dy}{dx} = \frac{1}{2}\left(\frac{d}{dx}\cos 6x + \frac{d}{dx}\cos 2x\right) \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}\right)$$

$$= \frac{1}{2}(-6\sin 6x - 2\sin 2x) \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}(\cos x) = -\sin x \ \&\ \frac{d}{dx}(kx) = k\right)$$

= -3 sin 6x – sin 2x

= - (3 sin 6x + sin 2x)

31. Question

Find $\frac{dy}{dx}$, when:

$$y = \sin\left(\frac{1+x^2}{1-x^2}\right)$$

Answer

<u>**Formulae**</u> **:**

- $\frac{2\tan x}{1+\tan^2 x} = \sin 2x$
- $\frac{1+\tan^2 x}{1-\tan^2 x} = \cos 2x$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $1 + \tan^2 x = \sec^2 x$

Given,

$$y = \sin\left(\frac{1+x^2}{1-x^2}\right)$$

Put x = tan a

Therefore, $\frac{dx}{da} = \sec^2 a$ ………….. eq (1)

$$y = \sin\left(\frac{1+\tan^2 a}{1-\tan^2 a}\right)$$

y = sin (cos 2a) ………… $\left(\because \frac{1+\tan^2 x}{1-\tan^2 x} = \cos 2x\right)$

Differentiating above equation w.r.t. a ,

$$\frac{dy}{da} = \frac{d}{da}(\sin(\cos 2a))$$

$$= (\cos(\cos 2a))\frac{d}{da}(\cos 2a) \text{ ………… } \left(\because \frac{d}{dx}(\sin x) = \cos x\right)$$

$$= (\cos(\cos 2a)).(-\sin 2a).\frac{d}{da}(2a) \text{ ………… } \left(\because \frac{d}{dx}(\cos x) = -\sin x\right)$$

$$= (-2\sin 2a).(\cos(\cos 2a))$$

$$= -2\left(\frac{2\tan a}{1+\tan^2 a}\right).\left(\cos\left(\frac{1+\tan^2 a}{1-\tan^2 a}\right)\right) \text{ ………… } \left(\because \frac{1+\tan^2 x}{1-\tan^2 x} = \cos 2x \;\&\; \frac{2\tan x}{1+\tan^2 x} = \sin 2x\right)$$

But, x = tan a

$$\frac{dy}{da} = -2\left(\frac{2x}{1+x^2}\right).\left(\cos\left(\frac{1+x^2}{1-x^2}\right)\right)$$

$$\frac{dy}{da} = \left(\frac{-4x}{1+x^2}\right).\left(\cos\left(\frac{1+x^2}{1-x^2}\right)\right) \text{ …………………… eq (2)}$$

Now,

$\frac{dy}{dx} = \frac{dy}{da}.\frac{da}{dx}$ ………… By chain rule

$$\therefore \frac{dy}{dx} = \left(\frac{-4x}{1+x^2}\right).\left(\cos\left(\frac{1+x^2}{1-x^2}\right)\right).\frac{1}{\sec^2 a} \text{ ……………. from eq (1) \& eq (2)}$$

$$= \left(\frac{-4x}{1+x^2}\right).\left(\cos\left(\frac{1+x^2}{1-x^2}\right)\right).\frac{1}{1+\tan^2 a} \text{ ………. } (\because 1+\tan^2 x = \sec^2 x)$$

$$= \left(\frac{-4x}{1+x^2}\right).\left(\cos\left(\frac{1+x^2}{1-x^2}\right)\right).\frac{1}{1+x^2} \text{ ………. } (\because x = \tan a)$$

$$\therefore \frac{dy}{dx} = \frac{-4x}{(1+x^2)^2}.\left(\cos\left(\frac{1+x^2}{1-x^2}\right)\right)$$

32. Question

Find $\frac{dy}{dx}$, when:

$$y = \frac{(\sin x + x^2)}{\cot 2x}$$

Answer

Formulae :

- $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u.\frac{d}{dx}(v)}{(v)^2}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cot x) = -\mathrm{cosec}^2 x$
- $\frac{d}{dx}(x^n) = n.x^{n-1}$

Given,

$$y = \frac{\sin x + x^2}{\cot 2x}$$

Differentiating above equation w.r.t. x ,

$$\frac{dy}{dx} = \frac{d}{dx}\left(\frac{\sin x + x^2}{\cot 2x}\right)$$

$$= \frac{\cot 2x.\frac{d}{dx}(\sin x + x^2) - (\sin x + x^2).\frac{d}{dx}(\cot 2x)}{(\cot 2x)^2} \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v.\frac{d}{dx}(u) - u.\frac{d}{dx}(v)}{(v)^2}\right)$$

$$= \frac{\cot 2x.(\cos 2x + 2x) - (\sin x + x^2).(-2\mathrm{cosec}^2 2x)}{(\cot 2x)^2}$$

$\ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}(\sin x) = \cos x, \frac{d}{dx}(x^n) = n.x^{n-1} \;\&\; \frac{d}{dx}(\cot x) = -\mathrm{cosec}^2\right)$

$$= \frac{(\cos 2x + 2x)}{\cot 2x} - \frac{(\sin x + x^2).(-2\,\mathrm{cosec}^2 2x)}{(\cot 2x)^2}$$

$$= \tan 2x.(\cos 2x + 2x) + \frac{(\sin x + x^2).\left(\frac{2}{\sin^2 x}\right)}{\frac{\cos^2 x}{\sin^2 x}}$$

$$= \tan 2x.(\cos 2x + 2x) + \frac{2(\sin x + x^2)}{\cos^2 x}$$

$$= \tan 2x.(\cos 2x + 2x) + 2\sec^2 2x.(\sin x + x^2)$$

$$\therefore \frac{dy}{dx} = \tan 2x.(\cos 2x + 2x) + 2\sec^2 2x.(\sin x + x^2)$$

33. Question

If $y = \frac{(\cos x - \sin x)}{(\cos x + \sin x)}$, prove that $\frac{dy}{dx} + y^2 + 1 = 0$.

Answer

Formulae :

- $\frac{\sin x}{\cos x} = \tan x$
- $\frac{1 - \tan x}{1 + \tan x} = \tan\left(\frac{\pi}{4} - x\right)$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}$
- $\frac{d}{dx}(kx) = k$
- $\frac{d}{dx}(k) = 0$
- $\tan^2 x + 1 = \sec^2 x$

Given,

$$y = \frac{(\cos x - \sin x)}{(\cos x + \sin x)}$$

Dividing numerator and denominator by cosx,

$$y = \frac{\left(1 - \frac{\sin x}{\cos x}\right)}{\left(1 + \frac{\sin x}{\cos x}\right)}$$

$$y = \frac{1 - \tan x}{1 + \tan x} \;\ldots\ldots\ldots\ldots \left(\because \frac{\sin x}{\cos x} = \tan x\right)$$

$$y = \tan\left(\frac{\pi}{4} - x\right) \;\ldots\ldots\ldots\ldots \left(\because \frac{1 - \tan x}{1 + \tan x} = \tan\left(\frac{\pi}{4} - x\right)\right)$$

Differentiating above equation w.r.t. x ,

$$\frac{dy}{dx} = \frac{d}{dx}\tan\left(\frac{\pi}{4} - x\right)$$

$$= \sec^2\left(\frac{\pi}{4} - x\right).\frac{d}{dx}\left(\frac{\pi}{4} - x\right) \;\ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}(\tan x) = \sec^2 x\right)$$

$$= \sec^2\left(\frac{\pi}{4} - x\right).\left(\frac{d}{dx}\left(\frac{\pi}{4}\right) - \frac{d}{dx}(x)\right) \;\ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}\right)$$

$= \sec^2\left(x+\frac{\pi}{4}\right).(0-1)$ .... $\left(\because \frac{d}{dx}(kx) = k \;\&\; \frac{d}{dx}(k) = 0\right)$

$= -\sec^2\left(x+\frac{\pi}{4}\right)$

$\therefore \frac{dy}{dx} = -\sec^2\left(x+\frac{\pi}{4}\right)$

Now,

$\frac{dy}{dx} + y^2 + 1 = -\sec^2\left(x+\frac{\pi}{4}\right) + \left(\tan^2\left(x+\frac{\pi}{4}\right) + 1\right)$

$= -\sec^2\left(x+\frac{\pi}{4}\right) + \left(\sec^2\left(x+\frac{\pi}{4}\right)\right)$ .... $(\because \tan^2 x + 1 = \sec^2 x)$

= 0

$\therefore \frac{dy}{dx} + y^2 + 1 = 0$

Hence Proved.

34. Question

If $y = \frac{(\cos x + \sin x)}{(\cos x - \sin x)}$, prove that $\frac{dy}{dx} = \sec^2\left(x+\frac{\pi}{4}\right)$.

Answer

Formulae :

- $\frac{\sin x}{\cos x} = \tan x$
- $\frac{1+\tan x}{1-\tan x} = \tan\left(x+\frac{\pi}{4}\right)$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(u+v) = \frac{du}{dx} + \frac{dv}{dx}$
- $\frac{d}{dx}(kx) = k$
- $\frac{d}{dx}(k) = 0$

Given,

$y = \frac{(\cos x + \sin x)}{(\cos x - \sin x)}$

Dividing numerator and denominator by cosx,

$$y = \frac{\left(1+\frac{\sin x}{\cos x}\right)}{\left(1-\frac{\sin x}{\cos x}\right)}$$

$$y = \frac{1+\tan x}{1-\tan x} \ldots\ldots\ldots\ldots \left(\because \frac{\sin x}{\cos x} = \tan x\right)$$

$$y = \tan\left(x+\frac{\pi}{4}\right) \ldots\ldots\ldots\ldots \left(\because \frac{1+\tan x}{1-\tan x} = \tan\left(x+\frac{\pi}{4}\right)\right)$$

Differentiating above equation w.r.t. x ,

$$\frac{dy}{dx} = \frac{d}{dx}\tan\left(x+\frac{\pi}{4}\right)$$

$$= \sec^2\left(x+\frac{\pi}{4}\right).\frac{d}{dx}\left(x+\frac{\pi}{4}\right) \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}(\tan x) = \sec^2 x\right)$$

$$= \sec^2\left(x+\frac{\pi}{4}\right).\left(\frac{d}{dx}(x)+\frac{d}{dx}\left(\frac{\pi}{4}\right)\right) \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}(u+v) = \frac{du}{dx}+\frac{dv}{dx}\right)$$

$$= \sec^2\left(x+\frac{\pi}{4}\right).(1+0) \ldots\ldots\ldots\ldots \left(\because \frac{d}{dx}(kx) = k \;\&\; \frac{d}{dx}(k) = 0\right)$$

$$= \sec^2\left(x+\frac{\pi}{4}\right)$$

$$\therefore \frac{dy}{dx} = \sec^2\left(x+\frac{\pi}{4}\right)$$

Hence Proved.

Exercise 10B

1. Question

Differentiate each of the following w.r.t. x:

(i) e^{4x}

(ii) e^{-5x}

(iii) $(e)^{x^3}$

Answer

(i) Let $y = e^{4x}$ $z = 4x$

Formula : $\frac{d(e^x)}{dx} = e^x$

According to chain rule of differentiation

$^{dy}/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= (e^{4x}) \times 4$

$= 4e^{4x}$

(ii) Let y = e^{-5x} z = -5x

Formula : $\frac{d(e^x)}{dx} = e^x$

According to chain rule of differentiation

$^{dy}/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= (e^{-5x}) \times (-5)$

$= -5e^{-5x}$

(iii) Let y =$(e)^{x^3}$ z = x^3

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(x^n)}{dx} = n \times x^{n-1}$

According to chain rule of differentiation

$^{dy}/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$

$= \left((e)^{x^3}\right) \times 3x^2$

$= 3x^2 (e)^{x^3}$

2. Question

Differentiate each of the following w.r.t. x:

(i) $e^{2/x}$

(ii) $e^{\sqrt{x}}$

(iii) $e^{-2\sqrt{x}}$

Answer

(i) Let y =$e^{2/x}$ z = 2/x

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(x^n)}{dx} = n \times x^{n-1}$

According to chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left(e^{2/x}\right) \times \left(\frac{-2}{x^2}\right)$$

$$= \frac{-2}{x^2} \times e^{\frac{2}{x}}$$

(ii) Let y $= e^{\sqrt{x}}$ z = $\sqrt{x}$

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(x^n)}{dx} = n \times x^{n-1}$

According to chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left(e^{\sqrt{x}}\right) \times \left(\frac{1}{2} \times x^{-0.5}\right) = \left(e^{\sqrt{x}}\right) \times \left(\frac{1}{2 \times \sqrt{x}}\right)$$

$$= \frac{e^{\sqrt{x}}}{2\sqrt{x}}$$

(iii) Let y $= e^{-2\sqrt{x}}$ z = -2$\sqrt{x}$

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(x^n)}{dx} = n \times x^{n-1}$

According to chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left(e^{-2\sqrt{x}}\right) \times \left(-2 \times \frac{1}{2} \times x^{-0.5}\right) = \left(e^{-2\sqrt{x}}\right) \times \left(\frac{-1}{\sqrt{x}}\right)$$

$$= \frac{-e^{-2\sqrt{x}}}{\sqrt{x}}$$

3. Question

Differentiate each of the following w.r.t. x:

(i) $e^{\cot x}$

(ii) $e^{-\sin 2x}$

(iii) $e^{\sqrt{\sin x}}$

Answer

(i) Let y $= e^{\cot x}$ z = $\cot x$

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(\cot x)}{dx} = -\operatorname{cosec}^2 x$

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= (e^{\cot x}) \times (-\operatorname{cosec}^2 x)$$

$$= -\operatorname{cosec}^2 x \; e^{\cot x}$$

(ii) Let y $= e^{-\sin 2x}$ z = $-\sin 2x$

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(\sin x)}{dx} = \cos x$

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left(e^{-\sin 2x}\right) \times (-\cos 2x \times 2)$$

$$= (-2\cos 2x)\; e^{-\sin 2x}$$

(iii) Let y $= e^{\sqrt{\sin x}}$ z = $\sqrt{\sin x}$

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(\sin x)}{dx} = \cos x$

According to chain rule of differentiation

$$dy/dx = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left(e^{\sqrt{\sin x}}\right) \times \left(\frac{1}{2} \times (\sin x)^{-0.5} \times \cos x\right) = \left(e^{\sqrt{\sin x}}\right) \times \left(\frac{1 \times \cos x}{2\sqrt{\sin x}}\right)$$

$$= \frac{\cos x}{2\sqrt{\sin x}} e^{\sqrt{\sin x}}$$

4. Question

Differentiate each of the following w.r.t. x:

(i) tan (log x)

(ii) log (sec x)

(iii) log (sin (x/2))

Answer

(i) Let y =tan(log x) z = log x

Formula : $\frac{d(\tan x)}{dx} = \sec^2 x, \frac{d(\log x)}{dx} = {}^1/_x$

According to chain rule of differentiation

$$ {}^{dy}/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx} $$

$$ = (\sec^2 \log x) \times \left(\frac{1}{x}\right) $$

$$ = \frac{\sec^2 (\log x)}{x} $$

(ii) Let y = log (sec x) z = sec x

Formula : $\frac{d(\sec x)}{dx} = \sec x \times \tan x, \frac{d(\log x)}{dx} = {}^1/_x$

According to chain rule of differentiation

$$ {}^{dy}/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx} $$

$$ = \left(\frac{1}{\sec x}\right)(\sec x \times \tan x) $$

$$ = \tan x $$

(iii) Let y = log (sin (x/2)) z = sin (x/2)

Formula : $\frac{d(\sin x)}{dx} = \cos x, \frac{d(\log x)}{dx} = {}^1/_x$

According to chain rule of differentiation

$$ {}^{dy}/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx} $$

$$ = \left(\frac{1}{\sin (x/2)}\right)\left(\cos (x/2) \times \frac{1}{2}\right) $$

$$ = \frac{1}{2} \times \cot (x/2) $$

5. Question

Differentiate each of the following w.r.t. x:

(i) $\log_3 x$

(ii) 2^{-x}

(iii) 3^{x+2}

Answer

(i) Let y = $\log_3 x$

Formula : $\log_a b = \frac{\log b}{\log a}, \frac{d(\log x)}{dx} = 1/x$

Therefore y = $\frac{\log x}{\log 3}$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dx}$$

$$= \left(\frac{1}{\log 3}\right)\left(\frac{1}{x}\right)$$

$$= \frac{1}{x(\log 3)}$$

(ii) Let y = 2^{-x} z = -x

Formula : $\frac{d(a^x)}{dx} = a^x (\log a)$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= (2^{-x}) \times (\log 2)(-1)$$

$$= -2^{-x}(\log 2)$$

(iii) Let y = 3^{x+2} z = x

Therefore Y = $3^2 \times 3^x$

Formula : $\frac{d(a^x)}{dx} = a^x (\log a)$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= 9(3^x) \times (\log 3)$$

6. Question

Differentiate each of the following w.r.t. x:

(i) $\log\left(x+\frac{1}{x}\right)$

(ii) log (sin (3x))

(iii) $\log\left(x+\sqrt{1+x^2}\right)$

Answer

(i) Let y = $\log(x+\frac{1}{x})$ z = $x+\frac{1}{x}$

Formula : $\frac{d(\log x)}{dx} = \frac{1}{x}, \frac{d(x^n)}{dx} = n \times x^{n-1}$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left(\frac{1}{x+\frac{1}{x}}\right) \times (1-\frac{1}{x^2})$$

$$= \left(\frac{x}{x^2+1}\right) \times (\frac{x^2-1}{x^2})$$

$$= \left(\frac{x^2-1}{x(x^2+1)}\right)$$

(ii) Let y = log (sin (3x)) z = sin (3x)

Formula : $\frac{d(\sin x)}{dx} = \cos x, \frac{d(\log x)}{dx} = 1/_x$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left(\frac{1}{\sin(3x)}\right)(\cos(3x) \times 3)$$

$$= 3 \times \cot(3x)$$

(iii) Let y = $\log(x+\sqrt{1+x^2})$ z = $x+\sqrt{1+x^2}$

Formula : $\frac{d(\log x)}{dx} = \frac{1}{x}, \frac{d(x^n)}{dx} = n \times x^{n-1}$

According to chain rule of differentiation

$$\frac{dy}{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= \left(\frac{1}{x + \sqrt{1 + x^2}}\right) \times \left(1 + \frac{1}{2}(1 + x^2)^{-0.5}\, 2x\right)$$

$$= \left(\frac{1}{x + \sqrt{1 + x^2}}\right) \times \left(1 + \frac{x}{1}(1 + x^2)^{-0.5}\right)$$

$$= \left(\frac{1}{x + \sqrt{1 + x^2}}\right) \times \left(1 + \frac{x}{\sqrt{1 + x^2}}\right)$$

$$= \left(\frac{1}{x + \sqrt{1 + x^2}}\right) \times \left(\frac{\sqrt{1 + x^2} + x}{\sqrt{1 + x^2}}\right)$$

$$= \left(\frac{1}{\sqrt{1 + x^2}}\right)$$

7. Question

Differentiate each of the following w.r.t. x:

$$e^{\sqrt{x}} \log x$$

Answer

Let y = $e^{\sqrt{x}} \log x$, z = $e^{\sqrt{x}}$ and w = log (x)

Formula : $\frac{d(e^x)}{dx} = e^x, \frac{d(\log x)}{dx} = \frac{1}{x}$

According to product rule of differentiation

$$\frac{dy}{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\log (x) \times \left(e^{\sqrt{x}}\right) \times \frac{1}{2\sqrt{x}}] + [e^{\sqrt{x}} \times \frac{1}{x}]$$

$$= e^{\sqrt{x}} \times [\frac{\log (x)}{2\sqrt{x}} + \frac{1}{x}]$$

$$= e^{\sqrt{x}} \times [\frac{\sqrt{x} \log (x)}{2x} + \frac{2}{2x}]$$

$$= e^{\sqrt{x}} \times [\frac{2 + \sqrt{x} \log (x)}{2x}]$$

8. Question

Differentiate each of the following w.r.t. x:

$\log \sin \sqrt{x^2 + 1}$

Answer

Let y = $\log \sin \sqrt{1 + x^2}$, z = $\sin \sqrt{1 + x^2}$

Formula : $\frac{d(\sin x)}{dx} = \cos x$, $\frac{d(\log x)}{dx} = \frac{1}{x}$

According to chain rule of differentiation

$$dy/_{dx} = \frac{dy}{dz} \times \frac{dz}{dx}$$

$$= [\frac{1}{\sin \sqrt{1 + x^2}}] \times [\cos \sqrt{1 + x^2}] \times [\frac{1}{2} \times \frac{1}{\sqrt{1 + x^2}} \times 2\, x]$$

$$= [\cot \sqrt{1 + x^2}] \times [\frac{1}{1} \times \frac{1}{\sqrt{1 + x^2}} \times x]$$

$$= \frac{x}{\sqrt{x^2 + 1}} \cot \sqrt{x^2 + 1}$$

9. Question

Differentiate each of the following w.r.t. x:

e^{2x} sin 3x

Answer

Let y = e^{2x} sin 3x, z = e^{2x} and w = sin 3x

Formula : $\frac{d(e^x)}{dx} = e^x$ and $\frac{d(\sin x)}{dx} = \cos x$

According to product rule of differentiation

$$dy/_{dx} = w \times \frac{dz}{dx} + z \times \frac{dw}{dx}$$

$$= [\sin 3x \times (2 \times e^{2x})] + [e^{2x} \times 3 \cos 3x]$$

$$= e^{2x} \times [2 \sin 3x + 3 \cos 3x]$$

10. Question

Differentiate each of the following w.r.t. x:

e^{3x} cos 2x

Answer

Let y = e^{3x} cos 2x, z = e^{3x} and w = cos 2x