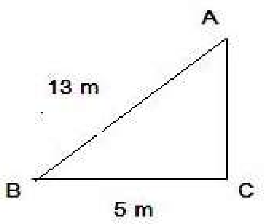
Chapter 13. Pythagoras Theorem [Proof and Simple Applications with Converse]

Exercise 13(A)

Solution 1:

The pictorial representation of the given problem is given below,



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

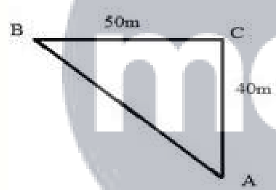
(i)Here, AB is the hypotenuse. Therefore applying the Pythagoras theorem we get,

$$AB^2 = BC^2 + CA^2$$
$$13^2 = 5^2 + CA^2$$
$$CA^2 = 13^2 - 5^2$$
$$CA^2 = 144$$
$$CA = 12 \text{ m}$$

Therefore, the distance of the other end of the ladder from the ground is 12m

Solution 2:

Here , we need to measure the distance AB as shown in the figure below,



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Therefore , in this case

$$AB^2 = BC^2 + CA^2$$
$$AB^2 = 50^2 + 40^2$$
$$AB^2 = 2500 + 1600$$
$$AB^2 = 4100$$
$$AB = 64.03$$

Therefore the required distance is 64.03 m.

Solution 3:

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the ΔPQS and applying Pythagoras theorem we get,

$$PQ^2 = PS^2 + QS^2$$
$$10^2 = PS^2 + 6^2$$
$$PS^2 = 100 - 36$$
$$PS = 8$$

Now, we consider the ΔPRS and applying Pythagoras theorem we get,

$$PR^2 = RS^2 + PS^2$$
$$PR^2 = 15^2 + 8^2$$
$$PR = 17$$

The length of PR 17 cm

Solution 4:

Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the ΔBDC and applying Pythagoras theorem we get,

$DB^2 = DC^2 + BC^2$
$DB^2 = 12^2 + 3^2$
$DB^2 = 144 + 9$
$DB^2 = 153$

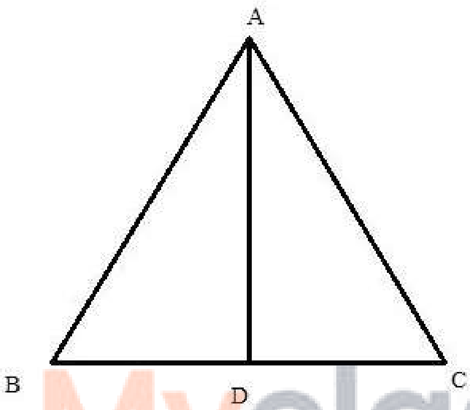
Now, we consider the ΔABD and applying Pythagoras theorem we get,

$DA^2 = DB^2 + BA^2$
$13^2 = 153 + BA^2$
$BA^2 = 169 - 153$
$BA = 4$

The length of AB is 4 cm.

Solution 5:

Since ABC is an equilateral triangle therefore, all the sides of the triangle are of same measure and the perpendicular AD will divide BC in two equal parts.



Pythagoras theorem states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Here, we consider the ΔABD and applying Pythagoras theorem we get,

$$AB^2 = AD^2 + BD^2$$

$$AD^2 = 100^2 - 5^2 \quad \begin{bmatrix} \text{Given, BC} = 10 \text{ cm} = AB, \\ BD = \frac{1}{2}BC \end{bmatrix}$$

Therefore, the length of AD is 8.7 cm

$$AD^2 = 100 - 25$$

$$AD^2 = 75$$

$$AD = 8.7$$

Solution 6:

We have Pythagoras theorem which states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

First, we consider the ΔABO, and applying Pythagoras theorem we get,

$$AB^2 = AO^2 + OB^2$$
$$AO^2 = AB^2 - OB^2$$
$$AO^2 = AB^2 - (BC + OC)^2$$
$$[\text{Let}, OC = x]$$
$$AO^2 = AB^2 - (BC + x)^2 \quad \ldots\ldots(i)$$

First, we consider the ΔACO, and applying Pythagoras theorem we get,

$$AC^2 = AO^2 + x^2$$
$$AO^2 = AC^2 - x^2 \quad \ldots\ldots(ii)$$

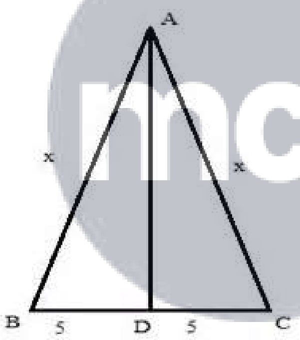
Now, from (i) and(ii),

$$AB^2 - (BC + x)^2 = AC^2 - x^2$$
$$8^2 - (6 + x)^2 = 3^2 - x^2 \quad [Given, AB = 8cm, BC = 8cm \text{ and } AC = 3cm]$$
$$x = 1\frac{7}{12}cm$$

Therefore, the length of OC will be $1\frac{7}{12}$ cm

Solution 7:

Here, the diagram will be,



We have Pythagoras theorem which states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Since, ABC is an isosceles triangle, therefore perpendicular from vertex will cut the base in two equal segments.

First, we consider the ΔABD, and applying Pythagoras theorem we get,

$$AB^2 = AD^2 + BD^2$$
$$AD^2 = x^2 - 5^2$$
$$AD^2 = x^2 - 25$$
$$AD = \sqrt{x^2 - 25} \quad \ldots\ldots(i)$$

Now,

$$\text{Area}=60$$
$$\frac{1}{2}\times 10\times \text{AD}=60$$
$$\frac{1}{2}\times 10\times \sqrt{x^2-25} = 60$$
$$x=13$$

Therefore, x is 13cm

Solution 8:

Let, the sides of the triangle be, x, $\sqrt{2}x$ and x

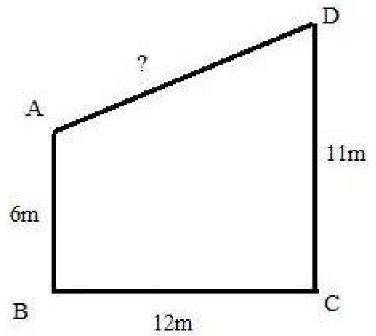
Now, $x^2 + x^2 = 2x^2 = \left(\sqrt{2}x\right)^2$ ......(i)

Here, in (i) it is shown that, square of one side of the given triangle is equal to the addition of square of other two sides. This is nothing but Pythagoras theorem which states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Therefore, the given triangle is a right angled triangle.

Solution 9:

The diagram of the given problem is given below,



We have Pythagoras theorem which states that in a right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the remaining two sides.

Here, 11 − 6 = 5m (Since DC is perpendicular to BC)

base = 12m

Applying Pythagoras theorem we get,

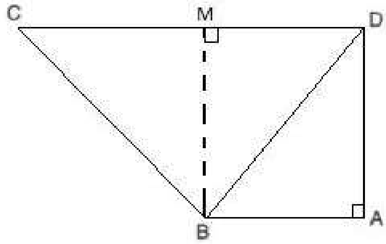
$$\text{hypotenuse}^2 = 5^2 + 12^2$$
$$h^2 = 25 + 144$$
$$h^2 = 169$$
$$h = 13$$

Therefore, the distance between the tips will be 13m

Solution 10:

Take M be the point on CD such that AB = DM.
So DM = 7cm and MC = 10 cm
Join points B and M to form the line segment BM.

So BM || AD also BM = AD.



In right-angled ΔBAD

$BD^2 = AD^2 + BA^2$

$(25)^2 = AD^2 + (7)^2$

$AD^2 = (25)^2 - (7)^2$

$AD^2 = 576$

$AD = 24$

In right-angled ΔCMB

$CB^2 = CM^2 + MB^2$

$CB^2 = (10)^2 + (24)^2 \quad [MB = AD]$

$CB^2 = 100 + 576$

$CB^2 = 676$

$CB = 26 cm$

Solution 11:

Given that AX:XB = 1:2.

Let n be the common multiple for which this proportion gets satisfied.

So, AX = 1(n) and XB = 2(n)

$AX + XB = 1(n) + 2(n)$

$\Rightarrow AB = n + 2n$

$\Rightarrow 12 = 3n$

$\Rightarrow n = 4$

AX = 1(n) = 4 and XB = 2(n) = 8

In ΔABC,

$XY \parallel BC$

$\frac{AB}{AX} = \frac{AC}{AY} = \frac{BC}{XY}$

$\Rightarrow \frac{AB}{AX} = \frac{AC}{AY}$

$\Rightarrow \frac{12}{4} = \frac{AC}{8}$

$\Rightarrow AC = 24 cm$

In right-angled ΔABC

$AC^2 = AB^2 + BC^2$

$\Rightarrow (24)^2 = (12)^2 + BC^2$

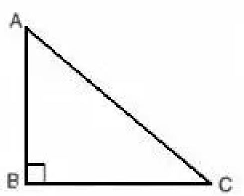
$\Rightarrow BC^2 = (24)^2 - (12)^2$

$\Rightarrow BC^2 = 576 - 144$

$\Rightarrow BC^2 = 432$

$\Rightarrow BC = 12\sqrt{3} cm$

Solution 12:



(i) In right-angled ΔABC

$AC^2 = AB^2 + BC^2$

$\Rightarrow (x+6)^2 = (x-3)^2 + (x+4)^2$

$\Rightarrow (x^2 + 12x + 36) = (x^2 - 6x + 9) + (x^2 + 8x + 16)$

$\Rightarrow x^2 - 10x - 11 = 0$

$\Rightarrow (x-11)(x+1) = 0$

$\Rightarrow x = 11$ or $x = -1$

But length of the side of a triangle can not be negative.

$\Rightarrow x = 11cm$

$\therefore AB = (x-3) = (11-3) = 8cm$

$BC = (x+4) = (11+4) = 15cm$

$AC = (x+6) = (11+6) = 17cm$

(ii) In right-angled ΔABC

$AC^2 = AB^2 + BC^2$

$\Rightarrow (4x+5)^2 = (x)^2 + (4x+4)^2$

$\Rightarrow (16x^2 + 40x + 25) = (x^2) + (16x^2 + 32x + 16)$

$\Rightarrow x^2 - 8x - 9 = 0$

$\Rightarrow (x-9)(x+1) = 0$

$\Rightarrow x = 9$ or $x = -1$

But length of the side of a triangle can not be negative.

$\Rightarrow x = 9cm$

$\therefore AB = x = 9cm$

$BC = (4x+4) = (36+4) = 40cm$

$AC = (4x+5) = (36+5) = 41cm$