

Exercise 20(C)

The surface area

of a sphere is 2464 cm^2 , find its volume. **Solution:**

Given,

$$\text{Surface area of the sphere} = 2464 \text{ cm}^2$$

Let the radius of the sphere be r .

$$\text{Then, surface area of the sphere} = 4\pi r^2$$

$$4\pi r^2 = 2464$$

$$4 \times \frac{22}{7} \times r^2 = 2464$$

$$r^2 = \frac{(2464 \times 7)}{(4 \times 22)} = 196$$

$$r = 14 \text{ cm}$$

$$\text{So, volume} = \frac{4}{3} \pi r^3$$

$$= \frac{4}{3} \times \frac{22}{7} \times 14 \times 14 \times 14$$

$$= 11498.67 \text{ cm}^3$$

1. The volume of a sphere is 38808 cm^3 ; find its diameter and the surface area.

Solution:

Given,

$$\text{Volume of the sphere} = 38808 \text{ cm}^3$$

Let the radius of the sphere = r

$$\frac{4}{3} \pi r^3 = 38808$$

$$\frac{4}{3} \times \frac{22}{7} \times r^3 = 38808$$

$$r^3 = \frac{(38808 \times 7 \times 3)}{(4 \times 22)} = 9261$$

$$r = 21 \text{ cm}$$

$$\text{So, the diameter} = 2r = 21 \times 2 = 42 \text{ cm}$$

And,

$$\text{Surface area} = 4\pi r^2 = 4 \times \frac{22}{7} \times 21 \times 21 \text{ cm}^2 = 5544 \text{ cm}^2$$

2. A spherical ball of lead has been melted and made into identical smaller balls with radius equal to half the radius of the original one. How many such balls can be made?

Solution:

Let the radius of the spherical ball = r

$$\text{So, the volume} = \frac{4}{3} \pi r^3$$

And, the radius of smaller ball = $r/2$

$$\text{Volume of smaller ball} = \frac{4}{3} \pi \left(\frac{r}{2}\right)^3 = \frac{4}{3} \pi r^3/8 = \pi r^3/6$$

$$\begin{aligned} \text{Thus, the number of balls made out of the given ball} &= \frac{(\frac{4}{3} \pi r^3)}{(\pi r^3/6)} \\ &= \frac{4}{3} \times 6 = 8 \end{aligned}$$

3. How many balls each of radius 1 cm can be made by melting a bigger ball whose diameter is 8 cm.

Solution:

Chapter 20: Cylinder, Cone and Sphere (Surface Area and Volume)

Given,

Diameter of bigger ball = 8 cm

So, radius of bigger ball = 4 cm

$$\text{Volume} = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi 4^3 = \frac{265\pi}{3} \text{ cm}^3$$

Radius of small ball = 1 cm

$$\text{Then, volume} = \frac{4}{3} \pi r^3 = \frac{4}{3} \pi 1^3 = \frac{4\pi}{3} \text{ cm}^3$$

Thus,

$$\begin{aligned} \text{The number of balls} &= \frac{(265\pi/3)}{(4\pi/3)} \\ &= \frac{265\pi/3 \times 3}{4\pi} \\ &= 64 \end{aligned}$$

4. 8 metallic sphere; each of radius 2 mm, are melted and cast into a single sphere. Calculate the radius of the new sphere.

Solution:

Radius of metallic sphere = 2mm = $\frac{1}{5}$ cm

$$\text{Volume} = \frac{4}{3} \pi r^3 = \frac{4}{3} \times \frac{22}{7} \times \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{88}{(21 \times 125)} \text{ cm}^3$$

$$\text{Volume of 8 spheres} = \frac{(88 \times 8)}{(21 \times 125)} = \frac{704}{(21 \times 125)} \text{ cm}^3 \dots\dots (i)$$

Let the radius of new sphere = R

$$\text{Volume} = \frac{4}{3} \pi R^3 = \frac{4}{3} \times \frac{22}{7} \times R^3 \dots\dots (ii)$$

According to the question, equating (i) and (ii) we have

$$\frac{704}{(21 \times 125)} = \frac{4}{3} \times \frac{22}{7} \times R^3$$

$$\frac{704}{(21 \times 125)} = \frac{88}{21} \times R^3$$

$$R^3 = \frac{(704 \times 21)}{(21 \times 125 \times 88)} = \frac{8}{125}$$

$$R = \frac{2}{5} = 0.4 \text{ cm} = 4 \text{ mm}$$

Therefore, the radius of the new sphere is 4 mm.

5. The volume of one sphere is 27 times that of another sphere. Calculate the ratio of their:

(i) radii

(ii) surface areas

Solution:

Given,

The volume of first sphere = 27 x volume of second sphere

Let the radius of the first sphere = r_1

And, radius of second sphere = r_2

(i) Then, according to the question we have

$$\frac{4}{3} \pi r_1^3 = 27 \left(\frac{4}{3} \pi r_2^3 \right)$$

$$r_1^3 / r_2^3 = 27$$

$$r_1 / r_2 = 3 / 1$$

$$\text{Thus, } r_1 : r_2 = 3 : 1$$

(ii) Surface area of the first sphere = $4 \pi r_1^2$

And the surface area of second sphere = $4 \pi r_2^2$

$$\text{Ratio of their surface areas} = \frac{4 \pi r_1^2}{4 \pi r_2^2} = \frac{r_1^2}{r_2^2} = \frac{3^2}{1^2} = 9$$

Hence, the ratio = 9: 1