NCERT Solutions for Class-XI Maths

Chapter-16 Exercise-16.3

NCERT Math Class 11

1. Which of the following can not be valid assignment of probabilities for outcomes of sample Space $S = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5, \omega_6, \omega_7\}$

Assignment	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6	ω_7
(a)	0.1	0.01	0.05	0.03	0.01	0.2	0.6
(b)	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$	$\frac{1}{7}$
(c)	0.1	0.2	0.3	0.4	0.5	0.6	0.7
(d)	-0.1	0.2	0.3	0.4	-0.2	0.1	0.3
(e)	$\frac{1}{14}$	$\frac{7}{14}$	$\frac{3}{14}$	$\frac{4}{14}$	$\frac{5}{14}$	$\frac{6}{14}$	$\frac{7}{14}$

1.

a) Both the conditions of axiomatic approach holds true in the given assignment, that is

1) Each of the number $p(w_i)$ is less than zero and is positive
2) Sum of probabilities is

$0.01 + 0.05 + 0.03 + 0.01 + 0.2 + 0.6 = 1$

The given assignment is valid.

b) Both the conditions of axiomatic approach holds true in the given assignment, that is

1) Each of the number p (w_i) is less than zero and is positive
2) Sum of probabilities is

$$\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}+\frac{1}{7}=\frac{7}{7}=1$$

The given assignment is valid.

c) Both the conditions of axiomatic approach in the given assignment are

1) Each of the number $p(w_i)$ is less than zero and is positive
2) Sum of probabilities is

$0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 + 0.7 = 2.8 > 1$

So the 2nd condition is not satisfied

Which states that $p(w_i) \leq 1$

The given assignment is not valid.

d) The conditions of axiomatic approach don't hold true in the given assignment, that is

1) Each of the number $p(w_i)$ is less than zero but also negative

To be true each of the number $p(w_i)$ should be less than zero and positive

So the assignment is not valid

e) Both the conditions of axiomatic approach in the given assignment are

1) Each of the number $p(w_i)$ is less than zero and is positive

2) Sum of probabilities is

$$\frac{1}{14}+\frac{2}{14}+\frac{3}{14}+\frac{4}{14}+\frac{5}{14}+\frac{6}{14}+\frac{7}{14}=\frac{28}{14}\geq 1$$

The second condition doesn't hold true so the assignment is not valid.

2. A coin is tossed twice, what is the probability that at least one tail occurs?

2. When a coin is tossed twice, the sample space is given by

$S=\{HH, HT, TH, TT\}$

Let A be the event of the occurrence of at least one tail.

Accordingly, $A=\{HT, TH, TT\}$

$$\therefore P(A)=\frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}}$$

$$=\frac{n(A)}{n(S)}$$

$$=\frac{3}{4}$$

3. A die is thrown; find the probability of following events:

(i) A prime number will appear,

(ii) A number greater than or equal to 3 will appear,

(iii) A number less than or equal to one will appear,

(iv) A number more than 6 will appear,

(v) A number less than 6 will appear.

3. Here S = {1, 2, 3, 4, 5, 6}

∴ n(S) = 6

(i) Let A be the event of getting a prime number,

A = {2, 3, 5} and n(A) = 3

$$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(ii) Let A be the event of getting a number greater than or equal to 3,,

Then A = (3, 4, 5, 6) and n(A) = 3

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(iii) Let A be the event of getting a number less than or equal to 1,

Then A = (1) ∴ n (A) = 1

$$P(A) = \frac{n(A)}{n(S)} = \frac{1}{6}$$

(iv) Let A be the event of getting a number more than 6, then

Then A = (0), ∴ n (A) = 0

$$P(A) = \frac{n(A)}{n(S)} = \frac{0}{6} = 0$$

(v) Let A be the event of getting a number less than 6, then

Then A= (1, 2, 3, 4, 5), ∴ n (A) = 5

$$P(A) = \frac{n(A)}{n(S)} = \frac{5}{6}$$

4. A card is selected from a pack of 52 cards.

(a) How many points are there in the sample space?

(b) Calculate the probability that the card is an ace of spades.

(c) Calculate the probability that the card is (i) an ace (ii) black card.

4. (a) When a card is selected from a pack of 52 cards, the number of possible outcomes is 52 i.e., the sample space contains 52 elements.

Therefore, there are 52 points in the sample space.

(b) Let A be the event in which the card drawn is an ace of spades.

Accordingly, $n(\text{A}) = 1$

$$\therefore \text{P}(\text{A}) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}} = \frac{n(\text{A})}{n(\text{S})} = \frac{1}{52}$$

(c) (i)Let E be the event in which the card drawn is an ace.

Since there are 4 aces in a pack of 52 cards, $n(E) = 4$

$$\therefore P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total number of possible outcomes}} = \frac{n(\text{E})}{n(\text{S})} = \frac{4}{52} = \frac{1}{13}$$

(ii)Let F be the event in which the card drawn is black.

Since there are 26 black cards in a pack of 52 cards, $n(F) = 26$

$$\therefore \text{P}(\text{F}) = \frac{\text{Number of outcomes favourable to F}}{\text{Total number of possible outcomes}} = \frac{n(\text{F})}{n(\text{S})} = \frac{26}{52} = \frac{1}{2}$$

5. A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is (i) 3 (ii) 12

5. The coin and die are tossed together.

Let S be the sample space

= {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5),(6, 6)} n(S) = 12

(i) Let A be the event having sum of numbers as 3.

A = {(1, 2)} , ∴ n (A) = 1

$$\therefore \text{P}(\text{A}) = \frac{1}{12}$$

(ii) Let A be the event having sum of number as 12.

Then A = {(6, 6)}, n (A) = 1

$$\therefore \text{P}(\text{A}) = \frac{\text{n}(\text{A})}{\text{n}(\text{S})} = \frac{1}{12}$$

6. There are four men and six women on the city council. If one council member is selected for a committee at random, how likely is it that it is a woman?

6. There are four men and six women on the city council.

As one council member is to be selected for a committee at random, the sample space contains $10(4+6)$ elements.

Let A be the event in which the selected council member is a woman.

Accordingly, $n(\text{A}) = 6$

$$\therefore \text{P}(\text{A}) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}} = \frac{n(\text{A})}{n(\text{S})} = \frac{6}{10} = \frac{3}{5}$$

7. A fair coin is tossed four times, and a person win Re 1 for each head and lose Rs 1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses and the probability of having each of these amounts.

7. Here the sample space is,

S = (HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, THHT, HTTH, THTH, TTHH,

TTTH, TTHT, THTT, HTTT, TTTT)

According to the given condition , a person will win or lose money depending up on the face of the coin so,

(i) For 4 heads = 1 + 1 + 1 + 1 = Rs. 4 = he wins Rs. 4

(ii) For 3 heads and 1 tail = 1 + 1 + 1 – 1.50 = Rs. 1.50 = he will be winning Rs. 1.50

(iii) For 2 heads and 2 tails = 1 + 1 – 1.50 – 1.50 = - Rs. 1 = he will be losing Re. 1

(iv)For 1 head and 3 tails = 1 – 1.50 – 1.50 – 1.50 = - Rs. 3.50 = he will be losing Rs. 3.50

(v) For 4 tails = – 1.50 – 1.50 – 1.50 – 1.50 = - Rs. 6 = he will be losing Rs. 6

Now the sample space of amounts is

S= {4, 1.50, 1.50, 1.50, 1.50, – 1, – 1, – 1, – 1, – 1, – 1, – 3.50, – 3.50, – 3.50, – 3.50, – 6}

$\therefore$ n (S) = 16

$$\text{P}(\text{winning Rs.4}) = \frac{1}{16}$$

$P(\text{winning Rs.1.50}) = \frac{4}{16} = \frac{1}{4}$

$P(\text{losing Re.1}) = \frac{6}{16} = \frac{3}{8}$

$P(\text{losing Rs.3.50}) = \frac{4}{16} = \frac{1}{4}$

$P(\text{losing Rs.6}) = \frac{1}{16}$

8. Three coins are tossed once. Find the probability of getting

(i) 3 heads

(ii) 2 heads

(iii) atleast 2 heads

(iv) atmost 2 heads

(v) no head

(vi) 3 tails

(vi) exactly two tails

(vii) (viii) no tail

(viii) (ix) atmost two tails

8. When three coins are tossed once, the sample space is given by

$S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\}$

$\therefore$ Accordingly, $n(S) = 8$

It is known that the probability of an event A is given by

$$P(A) = \frac{\text{Number of outcomes favourable to A}}{\text{Total number of possible outcomes}} = \frac{n(A)}{n(S)}$$

(i) Let B be the event of the occurrence of 3 heads. Accordingly, $B = \{HHH\}$

$$\therefore P(B) = \frac{n(B)}{n(S)} = \frac{1}{8}$$

(ii) Let C be the event of the occurrence of 2 heads.

Accordingly, $C = \{HHT, HTH, THH\}$

$$\therefore P(C) = \frac{n(C)}{n(S)} = \frac{3}{8}$$

(iii) Let D be the event of the occurrence of at least 2 heads.

Accordingly, D $=\{\text{HHH}, \text{HHT}, \text{HTH}, \text{THH}\}$

$$\therefore P(D)=\frac{n(D)}{n(S)}=\frac{4}{8}=\frac{1}{2}$$

(iv) LetE be the event of the occurrence of at most 2 heads. Accordingly,

$E=\{\text{HHT}, \text{HTH}, \text{THH}, \text{HTT}, \text{THT}, \text{TTH}, \text{TTT}\}$

$$\therefore P(E)=\frac{n(E)}{n(S)}=\frac{7}{8}$$

(v) Let F be the event of the occurrence of no head.

Accordingly, $F=\{\text{TTT}\}$

$$\therefore P(F)=\frac{n(F)}{n(S)}=\frac{1}{8}$$

(vi) Let G be the event of the occurrence of 3 tails.

Accordingly, $G=\{\text{TTT}\}$

$$\therefore P(G)=\frac{n(G)}{n(S)}=\frac{1}{8}$$

(vii) Let H be the event of the occurrence of exactly 2 tails.

Accordingly, $H=\{\text{HTT}, \text{THT}, \text{TTH}\}$

$$\therefore P(H)=\frac{n(H)}{n(S)}=\frac{3}{8}$$

(viii) Let I be the event of the occurrence of no tail.

Accordingly, $I=\{\text{HHH}\}$

$$\therefore P(I)=\frac{n(I)}{n(S)}=\frac{1}{8}$$

(ix) Let J be the event of the occurrence of at most 2 tails.

Accordingly, $J=\{\text{HHH}, \text{HHT}, \text{HTH}, \text{THH}, \text{HTT}, \text{THT}, \text{TTH}\}$

$$\therefore P(J)=\frac{n(J)}{n(S)}=\frac{7}{8}$$

9. If $\frac{2}{11}$ is the probability of an event, what is the probability of the event 'not A'.

9. Given:

$$P(A) = \frac{2}{11}$$

$\therefore$ P(not A) = 1 – P(A)

$$\Rightarrow 1 - \frac{2}{11} = \frac{9}{11}$$

10. A letter is chosen at random from the word 'ASSASSINATION'. Find the probability that letter is

(i) a vowel

(ii) a consonant

10. There are 13 letters in the word ASSASSINATION.

$\therefore$ Hence, $n(S) = 13$

(i) There are 6 vowels in the given word.

$\therefore$ Probability $(\text{vowel}) = 6/13$

(ii) There are 7 consonants in the given word.

$\therefore$ Probability (consonant) $= 7/13$

11. In a lottery, a person choses six different natural numbers at random from 1 to 20, and if these six numbers match with the six numbers already fixed by the lottery committee, he wins the prize. What is the probability of winning the prize in the game? [Hint order of the numbers is not important.]

11. Total numbers of numbers in the draw = 20 and numbers to be selected = 6

$\therefore$ n (S) = 20_{c_6}

Let A be the event that six numbers match with the six numbers fixed by the lottery committee.

$\therefore$ $n(A) = 6_{c_6} = 1$

Probability of winning the prize

$$P(A) = \; = \frac{n(A)}{n(S)} = \frac{6_{c_6}}{20_{c_6}} = \frac{6!14!}{20!}$$

$$= \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 14!}{20 \times 19 \times 18 \times 17 \times 16 \times 15 \times 14!}$$

$= \frac{1}{38760}$

12. Check whether the following probabilities P(A) and P(B) are consistently defined
(i) $P(A) = 0.5, P(B) = 0.7, P(A \cap B) = 0.6$
(ii) $P(A) = 0.5, P(B) = 0.4, P(A \cup B) = 0.8$

12. (i) $P(A) = 0.5, P(B) = 0.7, P(A \cap B) = 0.6$

It is known that if E and F are two events such that $E \subset F$, then $P(E) \leq P(F)$.

However, here, $P(A \cap B) > P(A)$.

Hence, $P(A)$ and $P(B)$ are not consistently defined.

(ii) $P(A) = 0.5, P(B) = 0.4, P(A \cup B) = 0.8$

It is known that if E and F are two events such that $E \subset F$, then $P(E) \leq P(F)$.

Here, it is seen that $P(A \cup B) > P(A)$ and $P(A \cup B) >$ $P(B)$. Hence, $P(A)$ and $P(B)$ are consistently defined.

13. Fill in the blanks in following table:

	P(A)	P(B)	$P(A \cap B)$	$P(A \cup B)$
(i)	$\frac{1}{3}$	$\frac{1}{5}$	$\frac{1}{15}$	….
(ii)	0.35	……	0.25	0.6
(iii)	0.5	0.35	……	0.7

13. (i) here $P(A) = \frac{1}{3}, P(B) = \frac{1}{5}, P(A \cap B) = \frac{1}{15}$
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\frac{1}{3} + \frac{1}{5} - \frac{1}{15} = \frac{7}{15}$

(ii) here $P(A) = 0.35, P(B) = ?, P(A \cap B) = 0.25, P(A \cup B) = 0.6$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.6 = 0.35 + P(B) - 0.25$

$P(B) = 0.5$

(iii) Here $P(A) = 0.5$, $P(B) = 0.35$, $P(A \cap B) = ?$, $P(A \cup B) = 0.7$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$0.7 = 0.5 + 0.35 - P(A \cap B)$$

$$P(A \cap B) = 0.85 - 0.7 = 0.15$$

14. Given $P(A) = \frac{3}{5}$ and $P(B) = \frac{1}{5}$. Find P(A or B), if A and B are mutually exclusive events.

14. Here, $P(A) = 3/5, P(B) = 1/5$

For mutually exclusive events A and B,

$P(A \text{ or } B) = P(A) + P(B)$

$\therefore P(A \text{ or } B) = 3/5 + 1/5 = 4/5$

15. If E and F are events such that $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \text{ and } F) = \frac{1}{8}$, find

(i) P (E or F)

(ii) P (not E and not F).

15. $P(E) = \frac{1}{4}$, $P(F) = \frac{1}{2}$ and $P(E \cap F) = \frac{1}{8}$

(i) Now $P(E \cup F) = P(A) + P(B) - P(A \cap B)$

$$\frac{1}{4} + \frac{1}{2} - \frac{1}{8} = \frac{5}{8}$$

(ii) $P(\text{not E and not F}) = P(\bar{E} \cap \bar{F}) = P(\overline{E \cup F}) = 1 - P(E \cup F)$

$$1 - \frac{5}{8} = \frac{8-5}{8} = \frac{3}{8}$$

16. Events E and F are such that P (not E or not F) = 0.25, State whether E and F are mutually exclusive.

16. Given: P(not E and not F) = 0.25

$P(\bar{E} \cup \bar{F}) = 0.25$

$\Rightarrow P(\overline{E \cap F}) = 0.25$

$\Rightarrow 1 - P(E \cap F) = 0.25$

$\Rightarrow P(E \cap F) = 1 - 0.25 = 0.75 \neq 0$

∴ E and F are not mutually exclusive events.

17. A and B are events such that P(A) = 0.42, P(B) = 0.48 and P(A and B) = 0.16. Determine
(i) P(not A),
(ii) P(not B) and
(iii) P(A or B)

17. Given: P(A) = 0.42, P(B)= 0.48 and P(A and B) = 0.16
(i)P (not A)= $P(\bar{A})$=1-P(A)=1-0.42=0.58
(ii) P(not B)= $P(\bar{B})$=1-P(B)=1-0.48=0.52
(iii) PA not B)= P (A ∪B)= P(A) + P(B) –P(A ∩ B)
= 0.42 + 0.48 – 0.16 = 0.74

18. In Class XI of a school 40% of the students study Mathematics and 30% study Biology. 10% of the class study both Mathematics and Biology. If a student is selected at random from the class, find the probability that he will be studying Mathematics or Biology.

18. Let A be the event in which the selected student studies Mathematics and B be the event in which the selected student studies Biology.
Now,

$P(A) = 40\% = 40/100 = 2/5$

$P(B) = 30\% = 30/100 = 3/10$

$P(A \text{ and } B) = 10\% = 10/100 = 1/10$

We know that $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\therefore P(\text{A or B}) = \frac{2}{5} + \frac{3}{10} - \frac{1}{10} = \frac{6}{10} = 0.6$

Thus, the probability that the selected student will be studying Mathematics or Biology is 0.6

19. In an entrance test that is graded on the basis of two examinations, the probability of a randomly chosen student passing the first examination is 0.8 and the probability of passing the second examination is 0.7. The probability of passing atleast one of them is 0.95. What is the probability of passing both?

19. Let A be the event that the student passes the first examination and B be the event that the students passes the second examination.
P ($P(A \cup B)$ is probability of passing at least one of the examination

Then, P(A∪B)= 0.95 , P(A)=0.8, P(B)=0.7
∴ P(A∪B)=P(A) + P(B)-P(A∩B)

$0.95 = 0.8 + 0.7 - P(A \cap B)$
$P(A \cap B) = 1.5 - 0.95 = 0.55$
0.55 is the probability that student will pass both the examinations

20. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. If the probability of passing the English examination is 0.75, what is the probability of passing the Hindi examination?

20. Let A and B be the events of passing English and Hindi examinations respectively.

Accordingly, $P(A$ and $B) = 0.5, P($ not A and not $B) = 0.1$, i.e., $P(A' \cap B') = 0.1$

$P(A) = 0.75$

Now, $(A \cup B)' = (A' \cap B')$ [De Morgan's law]

$\therefore P(A \cup B)' = P(A' \cap B') = 0.1$

$P(A \cup B) = 1 - P(A \cup B)' = 1 - 0.1 = 0.9$

We know that $P(A$ or $B) = P(A) + P(B) - P(A$ and $B)$

$\therefore 0.9 = 0.75 + P(B) - 0.5$

$\Rightarrow P(B) = 0.9 - 0.75 + 0.5$

$\Rightarrow P(B) = 0.65$

Thus, the probability of passing the Hindi examination is 0.65.

21. In a class of 60 students, 30 opted for NCC, 32 opted for NSS and 24 opted for both NCC and NSS. If one of these students is selected at random, find the probability that
(i) The student opted for NCC or NSS.
(ii) The student has opted neither NCC nor NSS.
(iii) The student has opted NSS but not NCC.

21. Given: Total number of students = 60
So the sample space consist of n(S) = 60
Let A be the event that student opted for NCC and B be the event that the student opted for NSS.

Here $n(A) = 30$, $n(B) = 32$ and $n(A \cap B) = 24$ being number of students who have opted for both NCC and NSS

$P(A) = \frac{n(A)}{n(S)} = \frac{30}{60} = \frac{1}{2}$, $P(B) = \frac{n(B)}{n(S)} = \frac{32}{60} = \frac{8}{15}$, $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{24}{60} = \frac{2}{5}$

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(i) P(Student opted for NCC or NSS)

P (A or B) = P(A) + P(B) –P(A and B)

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$\frac{1}{2}+\frac{8}{15}-\frac{2}{5}=\frac{19}{30}$$

(ii) P(student opted neither NCC nor NSS)

P(not A and not B) = $P(\bar{A}\cap\bar{B})$

$$P(\bar{A}\cap\bar{B})=P(\overline{A\cap B})$$ (by De Morgan's law)

$$=1-P(A\cup B)=1-\frac{19}{30}=\frac{11}{30}$$

(iii) P(student opted NSS but not NCC)

n(B - A) = n(B) – n (A ∩ B)

⇒ 32 – 24 = 8

The probability that the selected student has opted for NSS and not NCC is

$$\Rightarrow \frac{8}{60}=\frac{2}{15}$$