NCERT Solutions for Class-XII Maths

Chapter-1
Exercise- 2.1

NCERT Math Class 12

1. Find the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$

1. Let $\sin^{-1}\left(-\frac{1}{2}\right) = y$, then $\sin y = \left(\frac{\pi}{6}\right) = -\sin = \sin\left(-\frac{\pi}{6}\right)$ We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}$ Therefore, the principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is $-\frac{\pi}{6}$

2. Find the principal value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

2. Let us take $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = x$

Then, $\cos x = \frac{\sqrt{3}}{2} = \cos\left(\frac{\pi}{6}\right)$

We know that principle value range of $\cos^{-1}$ is $[0, \pi]$

And $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$

Therefore, principle value of $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ is $\frac{\pi}{6}$

3. Find the principal value of $\text{cosec}^{-1}(2)$.

3. Let $\text{cosec}^{-1}(2) = y$. then, $\text{cosec}\, y = 2 = \text{cosec}\left(\frac{\pi}{6}\right)$

We know that the range of the principal value branch of cosec^{-1} is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]-\{0\}$ and

$$\text{cosec}\left(\frac{\pi}{6}\right)=2$$

Therefore, the principal value of $\text{cosec}^{-1}1$ (2) is $\frac{\pi}{6}$

4. Find the principal value of $\tan^{-1}\left(-\sqrt{3}\right)$

4. Let us take $\tan^{-1}\left(-\sqrt{3}\right)=x$

Then we get,

$$\tan x=-\sqrt{3}=-\tan\frac{\pi}{3}=\tan\left(-\frac{\pi}{3}\right)$$

And $\tan\left(-\frac{\pi}{3}\right)=-\sqrt{3}$

We know that principle value range of $\tan^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Therefore, principle value of $\tan^{-1}\left(-\sqrt{3}\right)$ is $\left(-\frac{\pi}{3}\right)$.

5. Find the principal value of $\cos^{-1}-\left(\frac{1}{2}\right)$.

5. Let $\cos^{-1}-\left(\frac{1}{2}\right)=y$, then $\cos y=-\frac{1}{2}=-\cos\frac{\pi}{3}=\cos\left(\pi-\frac{\pi}{3}\right)=\cos\left(\frac{2\pi}{3}\right)$

We know that the range of the principal value branch of cos–1 is [O, π:] and cos $\left(\frac{2\pi}{3}\right)=-\frac{1}{2}$

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{2}\right)$ is $\frac{2\pi}{3}$.

6. Find the principal value of $\tan^{-1}$ (–1).

6. Let us take $\tan^{-1}(-1)=x$ then we get,

$$n\tan x=-1=-\tan\left(\frac{\pi}{4}\right)$$

$$=\tan\left(-\frac{\pi}{4}\right)$$

And $\tan\left(-\frac{\pi}{4}\right)=-1$

We know that principle value range of $\tan^{-1}$ is $\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

Therefore, principle value of $\tan^{-1}(-1)$ is $-\frac{\pi}{4}$

7. Find the principal value of $\sec^{-1}$

7. Let $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)=$ y, then sec y = $\frac{2}{\sqrt{3}}=\sec\left(\frac{\pi}{6}\right)$

We know that the range of the principal value branch of sec–1 is [O, rr] $-\left\{\frac{\pi}{2}\right\}$

and $\sec\left(\frac{\pi}{6}\right)=\frac{2}{\sqrt{3}}$.

Therefore, the principal value of $\sec^{-1}\left(\frac{2}{\sqrt{3}}\right)$ is `, $\frac{\pi}{6}$.

8. Find the principal value of $\cot^{-1}\sqrt{3}$

8. Let us consider $\cot^{-1}\left(-\sqrt{3}\right)=x$

Then we get,

$\cot x=-\sqrt{3}=-\cot\left(\frac{\pi}{6}\right)=\cot\left(\pi-\frac{\pi}{6}\right)=\cot\frac{5\pi}{6}$

We know that range of the principle value branch of $\cot^{-1}$ is [0, π]

And $\cot\left(\frac{5\pi}{6}\right)=-\sqrt{3}$

Therefore, principle value of $\cot^{-1}\left(-\sqrt{3}\right)$ is $\frac{5\pi}{6}$

9. Find the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$

9. Let $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)=$ y, then cos y = $-\frac{1}{\sqrt{2}}=\cos\left(\frac{\pi}{4}\right)=\cos\left(\pi-\frac{\pi}{4}\right)=\cos\left(\frac{3\pi}{4}\right)$.

We know that the range of the principal value branch of $\cos^{-1}$ is [0, π] and cos $\left(\frac{3\pi}{4}\right)=-\frac{1}{\sqrt{2}}$.

Therefore, the principal value of $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$ is $\frac{3\pi}{4}$.

10. Find the principal value of $\text{cosec}^{-1}\left(-\sqrt{2}\right)$.

10. Let us take the values of $\text{cosec}^{-1}\left(-\sqrt{2}\right) = x$

Then,

$\text{cosec}\, x = -\sqrt{2} = \text{cosec}\left(-\frac{\pi}{4}\right)$

We know that range of the principle value branch of cosec^{-1} is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$

And $\text{cosec}\left(-\frac{\pi}{4}\right) = -\sqrt{2}$

Therefore principle value of $\text{cosec}^{-1}\left(-\sqrt{2}\right)$ is $-\frac{\pi}{4}$

11. Find the value of $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$.

11. Let $\tan^{-1}(1) = x$, then $\tan x = 1 = \tan\frac{\pi}{4}$.

We know that the range of the principal value branch of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

$\therefore \tan^{-1}(1) = \frac{\pi}{4}$

Let $\cos^{-1}\left(-\frac{1}{2}\right) = y$, then

$\cos y = -\frac{1}{2} = -\cos\frac{\pi}{3} = \cos\left(\pi - \frac{\pi}{3}\right) = \cos\left(\frac{2\pi}{3}\right)$

We know that the range of the principal value branch of $\cos^{-1}$ is $[O, \pi]$.

$\therefore \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

Let $\sin^{-1}\left(-\frac{1}{2}\right) = z$, then

$\sin z = -\frac{1}{2} = -\sin\frac{\pi}{6} = \sin\left(-\frac{\pi}{6}\right)$

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$\therefore \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

Now,

$\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(-\frac{1}{2}\right)$

$= \frac{\pi}{4} + \frac{2\pi}{3} - \frac{\pi}{6} = \frac{3\pi + 8\pi - 2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$

12. Find the value of $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

12. Let $\cos^{-1}\left(\frac{1}{2}\right) = x$

Then, we get,

$\cos x = \frac{1}{2} = \cos\frac{\pi}{3}$

We know that range of the principle value branch of $\cos^{-1}$ is $[0, \pi]$

Therefore $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

Let $\sin^{-1}\left(\frac{1}{2}\right) = y$ then $\sin y = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

We know that range of the principle value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

Now,

$\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3} + 2 \times \frac{\pi}{6} = \frac{2\pi}{3}.$

13. If $\sin^{-1} x = y$, then

(A) $0 < y < \pi$

(B) $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

(C) $0 < y < \pi$

(D) $-\frac{\pi}{2} < y < \frac{\pi}{2}$

13. It is given that $\sin^{-1} x = y$.

We know that the range of the principal value branch of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Therefore, $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

Hence, the option (B) is correct.

14. $\tan^{-1}\sqrt{3} = \sec^{-1}(-2)$ is equal to

(A) π (B) $-\frac{\pi}{3}$

(C) $\frac{\pi}{3}\frac{2\pi}{3}$ (D)

15. The correct option is (B).

Explanation:

Let us take

$\tan^{-1}\left(\sqrt{3}\right) = x$ Then we get,

$\tan x = \sqrt{3} = \tan\frac{\pi}{3}$

We know that range of the principle value branch of $\tan^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Therefore, $\tan^{-1}\left(\sqrt{3}\right) = \frac{\pi}{3}$

Let $\sec^{-1}(-2) = y$ Then we get,

$\sec y = -2 = -\sec\frac{\pi}{3} = \sec\left(\pi - \frac{\pi}{3}\right) = \sec\left(\frac{2\pi}{3}\right)$

We know that range of the principle value branch of $\sec^{-1}$ is $[0, \pi] - \left\{\frac{\pi}{2}\right\}$

Therefore, $\sec^{-1}(-2) = \frac{2\pi}{3}$

Now,

$\tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$

Hence, the option (B) is correct.