EXERCISE 3.3

1. For which value(s) of λ , do the pair of linear equations

$\lambda x + y = \lambda^2$ and $x + \lambda y = 1$ have

(i) no solution?

(ii) infinitely many solutions?

(iii) a unique solution?

Solution:

The given pair of linear equations is

$\lambda x + y = \lambda^2$ and $x + \lambda y = 1$

$a_1 = \lambda$, $b_1 = 1$, $c_1 = -\lambda^2$

$a_2 = 1$, $b_2 = \lambda$, $c_2 = -1$

The given equations are;

$\lambda x + y - \lambda^2 = 0$

$x + \lambda y - 1 = 0$

Comparing the above equations with $ax + by + c = 0$;

We get,

$a_1 = \lambda$, $b_1 = 1$, $c_1 = -\lambda^2$;

$a_2 = 1$, $b_2 = \lambda$, $c_2 = -1$;

$a_1/a_2 = \lambda/1$

$b_1/b_2 = 1/\lambda$

$c_1/c_2 = \lambda^2$

(i) For no solution,

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

i.e. $\lambda = 1/\lambda \neq \lambda^2$

so, $\lambda^2 = 1$;

and $\lambda^2 \neq \lambda$

Here, we take only $\lambda = -1$,

Since the system of linear equations has infinitely many solutions at $\lambda = 1$,

(ii) For infinitely many solutions,

$a_1/a_2 = b_1/b_2 = c_1/c_2$

i.e. $\lambda = 1/\lambda = \lambda^2$

so $\lambda = 1/\lambda$ gives $\lambda = \pm 1$;

$\lambda = \lambda^2$ gives $\lambda = 1,0$;

Hence satisfying both the equations

$\lambda = 1$ is the answer.

(iii) For a unique solution,

$a_1/a_2 \neq b_1/b_2$

so $\lambda \neq 1/\lambda$

hence, $\lambda^2 \neq 1$;

$\lambda \neq \pm 1$;

So, all real values of λ except ± 1

2. For which value(s) of *k* will the pair of equations

$$kx + 3y = k - 3$$
$$12x + ky = k$$

have no solution?

Solution:

The given pair of linear equations is

kx + 3y = k - 3 …(i)

12x + ky = k …(ii)

On comparing the equations (i) and (ii) with ax + by = c = 0,

We get,

$a_1 = k, b_1 = 3, c_1 = -(k - 3)$

$a_2 = 12, b_2 = k, c_2 = - k$

Then,

$a_1 /a_2 = k/12$

$b_1 /b_2 = 3/k$

$c_1 /c_2 = (k-3)/k$

For no solution of the pair of linear equations,

$a_1/a_2 = b_1/b_2 \neq c_1/c_2$

$k/12 = 3/k \neq (k-3)/k$

Taking first two parts, we get

$k/12 = 3/k$

$k^2 = 36$

$k = \pm 6$

Taking last two parts, we get

$3/k \neq (k-3)/k$

$3k \neq k(k - 3)$

$k^2 - 6k \neq 0$

so, $k \neq 0,6$

Therefore, value of k for which the given pair of linear equations has no solution is k = - 6.

3. For which values of *a* and *b*, will the following pair of linear equations have infinitely many solutions?

$$x + 2y = 1$$
$$(a - b)x + (a + b)y = a + b - 2$$

Solution:

The given pair of linear equations are:

x + 2y = 1 …(i)

(a-b)x + (a + b)y = a + b - 2 …(ii)

On comparing with ax + by = c = 0 we get

$a_1 = 1, b_1 = 2, c_1 = - 1$

$a_2 = (a - b), b_2 = (a + b), c_2 = - (a + b - 2)$

$a_1 /a_2 = 1/(a-b)$

$b_1 /b_2 = 2/(a+b)$

$c_1 /c_2 = 1/(a+b-2)$

For infinitely many solutions of the, pair of linear equations,

$a_1/a_2 = b_1/b_2 = c_1/c_2$(coincident lines)

so, $1/(a-b) = 2/(a+b) = 1/(a+b-2)$
Taking first two parts,
$1/(a-b) = 2/(a+b)$
$a + b = 2(a - b)$
$a = 3b$ …(iii)

Taking last two parts,
$2/(a+b) = 1/(a+b-2)$
$2(a + b - 2) = (a + b)$
$a + b = 4$ …(iv)
Now, put the value of a from Eq. (iii) in Eq. (iv), we get
$3b + b = 4$
$4b = 4$
$b = 1$
Put the value of b in Eq. (iii), we get
$a = 3$
So, the values (a,b) = (3,1) satisfies all the parts. Hence, required values of a and b are 3 and 1 respectively for which the given pair of linear equations has infinitely many solutions.

4. Find the value(s) of *p* in (i) to (iv) and *p* and *q* in (v) for the following pair of equations:
(i) $3x - y - 5 = 0$ and $6x - 2y - p = 0$, if the lines represented by these equations are parallel.
Solution:
Given pair of linear equations is
$3x - y - 5 = 0$ …(i)
$6x - 2y - p = 0$ …(ii)
On comparing with $ax + by + c = 0$ we get
We get,
$a_1 = 3, b_1 = -1, c_1 = -5$;
$a_2 = 6, b_2 = -2, c_2 = -p$;
$a_1/a_2 = 3/6 = 1/2$
$b_1/b_2 = 1/2$
$c_1/c_2 = 5/p$
Since, the lines represented by these equations are parallel, then
$a_1/a_2 = b_1/b_2 \neq c_1/c_2$
Taking last two parts, we get $1/2 \neq 5/p$
So, $p \neq 10$
Hence, the given pair of linear equations are parallel for all real values of p except 10.

(ii) $-x + py = 1$ and $px - y = 1$, if the pair of equations has no solution.
Solution:
Given pair of linear equations is
$-x + py = 1$ …(i)
$px - y - 1 = 0$ …(ii)
On comparing with $ax + by + c = 0$,
We get,
$a_1 = -1, b_1 = p, c_1 = -1$;

$a_2 = p$, $b_2 = -1$, $c_2 = -1$;
$a_1/a_2 = -1/p$
$b_1/b_2 = -p$
$c_1/c_2 = 1$
Since, the lines equations has no solution i.e., both lines are parallel to each other.
$a_1/a_2 = b_1/b_2 \neq c_1/c_2$
$-1/p = -p \neq 1$
Taking last two parts, we get
$p \neq -1$
Taking first two parts, we get
$p^2 = 1$
$p = \pm 1$
Hence, the given pair of linear equations has no solution for p = 1.

(iii) $-3x + 5y = 7$ and $2px - 3y = 1$, if the lines represented by these equations are intersecting at a unique point.

Solution:

Given, pair of linear equations is
$-3x + 5y = 7$
$2px - 3y = 1$
On comparing with $ax + by + c = 0$, we get
Here, $a_1 = -3$, $b_1 = 5$, $c_1 = -7$;
And $a_2 = 2p$, $b_2 = -3$, $c_2 = -1$;
$a_1/a_2 = -3/2p$
$b_1/b_2 = -5/3$
$c_1/c_2 = 7$
Since, the lines are intersecting at a unique point i.e., it has a unique solution
$a_1/a_2 \neq b_1/b_2$
$-3/2p \neq -5/3$
$p \neq 9/10$
Hence, the lines represented by these equations are intersecting at a unique point for all real values of p except 9/10

(iv) $2x + 3y - 5 = 0$ and $px - 6y - 8 = 0$, if the pair of equations has a unique solution.

Solution:

Given, pair of linear equations is
$2x + 3y - 5 = 0$
$px - 6y - 8 = 0$
On comparing with $ax + by + c = 0$ we get
Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -5$;
And $a_2 = p$, $b_2 = -6$, $c_2 = -8$;
$a_1/a_2 = 2/p$
$b_1/b_2 = -3/6 = -½$
$c_1/c_2 = 5/8$
Since, the pair of linear equations has a unique solution.
$a_1/a_2 \neq b_1/b_2$

so $2/p \neq -\frac{1}{2}$
$p \neq -4$
Hence, the pair of linear equations has a unique solution for all values of p except – 4.

(v) $2x + 3y = 7$ and $2px + py = 28 – qy$, if the pair of equations have infinitely many solutions.
Solution:

Given pair of linear equations is
$2x + 3y = 7$
$2px + py = 28 – qy$
or $2px + (p + q)y - 28 = 0$
On comparing with $ax + by + c = 0$,
We get,
Here, $a_1 = 2$, $b_1 = 3$, $c_1 = -7$;
And $a_2 = 2p$, $b_2 = (p + q)$, $c_2 = -28$;
$a_1/a_2 = 2/2p$
$b_1/b_2 = 3/(p+q)$
$c_1/c_2 = \frac{1}{4}$
Since, the pair of equations has infinitely many solutions i.e., both lines are coincident.
$a_1/a_2 = b_1/b_2 = c_1/c_2$
$1/p = 3/(p+q) = \frac{1}{4}$
Taking first and third parts, we get
$p = 4$
Again, taking last two parts, we get
$3/(p+q) = \frac{1}{4}$
$p + q = 12$
Since $p = 4$
So, $q = 8$
Here, we see that the values of $p = 4$ and $q = 8$ satisfies all three parts.
Hence, the pair of equations has infinitely many solutions for all values of $p = 4$ and $q = 8$.

5. Two straight paths are represented by the equations $x – 3y = 2$ and $–2x + 6y = 5$.Check whether the paths cross each other or not.
Solution:

Given linear equations are
$x - 3y - 2 = 0$ …(i)
$-2x + 6y - 5 = 0$ …(ii)
On comparing with ax + by c=0,
We get
$a_1 =1$, $b_1 =-3$, $c_1 =-2$;
$a_2 = -2$, $b_2 =6$, $c_2 =-5$;
$a_1/a_2 = -\frac{1}{2}$
$b_1/b_2 = -3/6 = -\frac{1}{2}$
$c_1/c_2 = 2/5$
i.e., $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ [parallel lines]
Hence, two straight paths represented by the given equations never cross each other, because they are parallel to each other.

6. Write a pair of linear equations which has the unique solution *x* = – 1, *y* =3. How many such pairs can you write?

Solution:

Condition for the pair of system to have unique solution

$a_1/a_2 \neq b_1/b_2$

Let the equations be,

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Since, x = - 1 and y = 3 is the unique solution of these two equations, then

It must satisfy the equations –

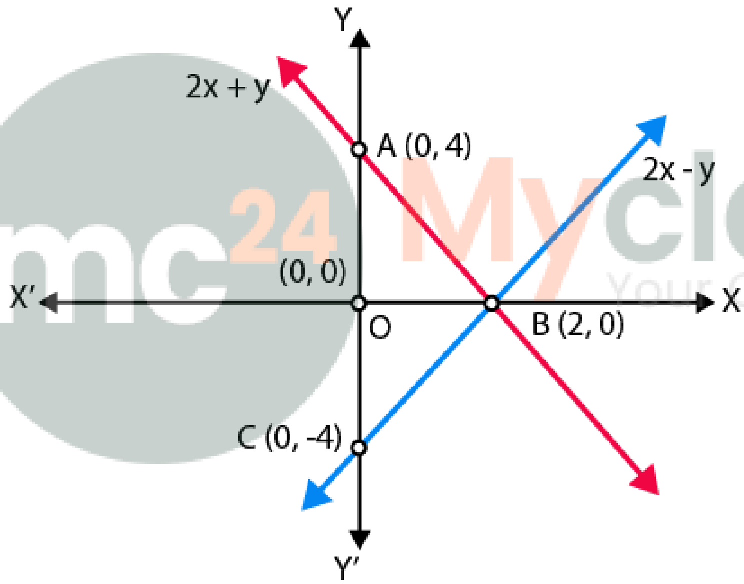
$a_1(-1) + b_1(3) + c_1 = 0$

$- a_1 + 3b_1 + c_1 = 0$ …(i)

and $a_2(- 1) + b_2(3) + c_2 = 0$

$- a_2 + 3b_2 + c_2 = 0$ …(ii)

Since for the different values of a_1, b_1, c_1 and a_2, b_2, c_2 satisfy the Eqs. (i) and (ii).



Hence, infinitely many pairs of linear equations are possible.

7. If 2*x* + *y* = 23 and 4*x* – *y* = 19, find the values of 5*y* – 2*x* and y/x - 2.

Solution:

Given equations are

2x + y = 23 …(i)

4x - y = 19 …(ii)

On adding both equations, we get

6x = 42

So, x = 7

Put the value of x in Eq. (i), we get

2(7) + y = 23

y = 23 – 14

so, y = 9
Hence 5y - 2x = 5(9) - 2(7) = 45 - 14 = 31
y/x – 2 = 9/7 -2 = -5/7
Hence, the values of (5y - 2x) and y/x – 2 are 31 and -5/7 respectively.

8. Find the values of *x* and *y* in the following rectangle [see Fig. 3.2].

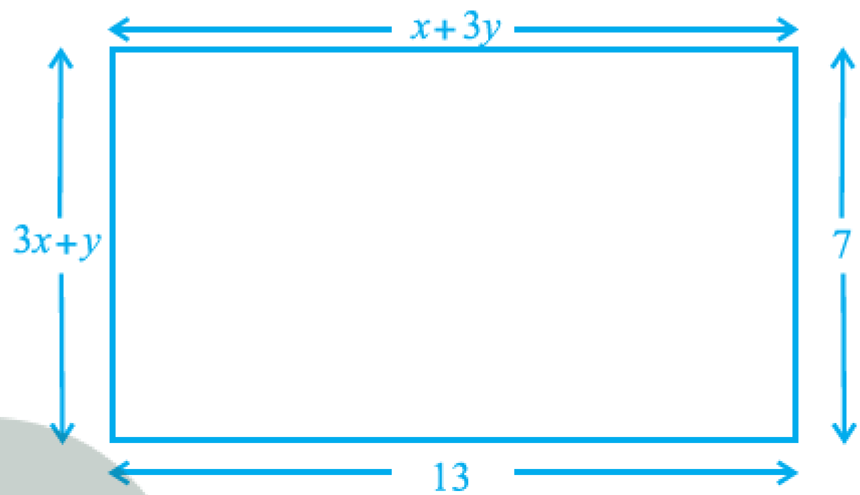


Fig. 3.2

Solution:

Using property of rectangle,
We know that,
Lengths are equal,
i.e., CD = AB
Hence, x + 3y = 13 …(i)
Breadth are equal,
i.e., AD = BC
Hence, 3x + y = 7 …(ii)
On multiplying Eq. (ii) by 3 and then subtracting Eq. (i),
We get,
8x = 8
So, x = 1
On substituting x = 1 in Eq. (i),
We get,
y = 4
Therefore, the required values of x and y are 1 and 4, respectively.