

# NCERT Solutions for Class-XII Maths

## Chapter-1 Exercise- 1.3

- Let  $f: \{1, 3, 4\} \rightarrow \{1, 2, 5\}$  and  $g: \{1, 2, 5\} \rightarrow \{1, 3\}$  be given by  $f = \{(1, 2), (3, 5), (4, 1)\}$  and  $g = \{(1, 3), (2, 3), (5, 1)\}$ . Write down  $gof$ .
- The functions  $f: \{1, 3, 4\} \rightarrow \{1, 2, 5\}$  and  $g: \{1, 2, 5\} \rightarrow \{1, 3\}$  are defined as  $f = \{(1, 2), (3, 5), (4, 1)\}$  and  $g = \{(1, 3), (2, 3), (5, 1)\}$ .  
 $gof(1) = g[f(1)] = g(2) = 3$  [as  $f(1) = 2$  and  $g(2) = 3$ ]  
 $gof(3) = g[f(3)] = g(5) = 1$  [as  $f(3) = 5$  and  $g(5) = 1$ ]  
 $gof(4) = g[f(4)] = g(1) = 3$  [as  $f(4) = 1$  and  $g(1) = 3$ ]  
 $\therefore gof = \{(1, 3), (3, 1), (4, 3)\}$

- Let  $f, g$  and  $h$  be functions from  $R$  to  $R$ . Show that

$$(f + g)oh = foh + goh$$

$$(f \cdot g)oh = (f \cdot goh)$$

- (i)  $(f + g)oh = foh + goh$

$$\text{Let us consider } ((f + g)oh)(x) = (f + g)(h(x))$$

$$= f(h(x)) + g(h(x))$$

$$= (foh)(x) + (goh)(x)$$

$$= \{(f \cdot g)oh\}(x)$$

$$\text{Then, } ((f + g)oh)(x) = \{(foh) + (goh)\}(x)$$

$$\forall x \in R$$

**Therefore,  $(f + g)oh = foh + goh$ .**

$$\text{(ii) } (f \cdot g)oh = (f \cdot goh)$$

$$\text{Let us consider } ((f \cdot g)oh)(x) = (f \cdot g)(h(x))$$

$$= f(h(x)) \cdot g(h(x))$$

$$= f(h(x)) \cdot g(h(x))$$

$$= (f \cdot g)(h(x))$$

$$= \{(f \cdot g)oh\}(x)$$

$$\text{Then, } ((f \cdot g)oh)(x) = \{(f \cdot g)oh\}(x) \quad \forall x \in R$$

**Therefore,  $(f \cdot g)oh = (f \cdot goh)$ .**

- Find  $gof$  and  $fog$ , if

$$\text{(i) } f(x) = |x| \text{ and } g(x) = |5x - 2|$$

$$\text{(ii) } f(x) = 8x^3 \text{ and } g(x) = x$$

- (i)  $f(x) = |x|$  and  $g(x) = |5x - 2|$

$$\therefore \text{gof}(x) = g(f(x)) = g(|x|) = |5|x|-2|$$

$$\text{fog}(x) = f(g(x)) = f(|5x-2|) = ||5x-2|| = |5x-2|$$

(ii).  $f(x) = 8x^3$  and  $g(x) = x^{\frac{1}{3}}$

$$\therefore \text{gof}(x) = g(f(x)) = g(8x^3) = (8x^3)^{\frac{1}{3}} = 2x$$

$$\text{fog}(x) = f(g(x)) = f(x^{\frac{1}{3}}) = 8(x^{\frac{1}{3}})^3 = 8x$$

4. If  $f(x) = \frac{(4x+3)}{(6x-4)}, x \neq \frac{2}{3}$ , show that  $\text{fof}(x) = x$ , for all  $x \neq \frac{2}{3}$ . What is the inverse of  $f$ ?

4. It is given that  $f(x) = \frac{(4x+3)}{(6x-4)}, x \neq \frac{2}{3}$

$$(\text{fof})(x) = f(f(x)) = f\left(\frac{4x+3}{6x-4}\right) = \frac{4\left(\frac{4x+3}{6x-4}\right)+3}{6\left(\frac{4x+3}{6x-4}\right)-4} = \frac{16x+12+18x-12}{24x+18-24x+16} = \frac{34x}{34} = x$$

Therefore,  $\text{fof}(x) = x$ , for all  $x \neq \frac{2}{3}$

$\Rightarrow \text{fof} = 1$

Therefore, the given function  $f$  is invertible and the inverse of  $f$  is itself.

5. State with reason whether following functions have inverse

(i)  $f: \{1, 2, 3, 4\} \rightarrow \{10\}$  with

$$f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$$

(ii)  $g: \{5, 6, 7, 8\} \rightarrow \{1, 2, 3, 4\}$  with

$$g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$$

(iii)  $h: \{2, 3, 4, 5\} \rightarrow \{7, 9, 11, 13\}$  with  $h = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$

5. (i)  $f: \{1, 2, 3, 4\} \rightarrow \{10\}$  defined as  $f = \{(1, 10), (2, 10), (3, 10), (4, 10)\}$  From the given definition of  $f$ , we can see that  $f$  is a many one function as  $f(1) = f(2) = f(3) = f(4) = 10$

$\therefore f$  is not one – one.

Hence, function  $f$  does not have an inverse.

(ii)  $g: \{5, 6, 7, 8\} \rightarrow \{1, 2, 3, 4\}$  defined as

$$g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$$

From the given definition of  $g$ , it is seen that  $g$  is a many one function as  $g(5) = g(7) = 4$ .

$\therefore g$  is not one – one.

Hence, function  $g$  does not have an inverse.

(iii)  $h: \{2, 3, 4, 5\} \rightarrow \{7, 9, 11, 13\}$  defined as

$$h = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$$

It is seen that all distinct elements of the set  $\{2, 3, 4, 5\}$  have distinct images under  $h$ .

$\therefore$  Function  $h$  is one – one.

Also,  $h$  is onto since for every element  $y$  of the set  $\{7, 9, 11, 13\}$ , there exists an element  $x$  in the set  $\{2, 3, 4, 5\}$ , such that  $h(x) = y$ .

Thus,  $h$  is a one – one and onto function.

Hence,  $h$  has an inverse.

6. Show that  $f: [-1, 1] \rightarrow \mathbb{R}$ , given by  $f(x) = \frac{x}{x+2}$  is one – one. Find the inverse of the function  
 $f: [-1, 1] \rightarrow \text{Range } f$ .

(Hint: For  $y \in \text{Range } f$ ,  $y = f(x) = \frac{x}{x+2}$ , for some  $x$  in  $[-1, 1]$ , i.e.,  $x = \frac{2y}{1-y}$ )

6. It is given that  $f: [-1, 1] \rightarrow \mathbb{R}$ , given by  $f(x) = \frac{x}{x+2}$

Now, Let  $f(x) = f(y)$

$$\Rightarrow \frac{x}{x+2} = \frac{y}{y+2}$$

$$\Rightarrow xy + 2x = xy + 2y$$

$$\Rightarrow 2x = 2y$$

$$\Rightarrow x = y$$

$\Rightarrow f$  is a one- one function.

Now, Let  $y = \frac{x}{x+2}$ ,  $xy = x + 2y$  so  $x = \frac{y}{1-y}$

So, for every  $y$  in the range there exists  $x$  in the domain such that  $f(x) = y$

$\Rightarrow f$  is onto function.

$\Rightarrow f: [-1,1] \rightarrow \text{Range } f$  is one-one and onto

$\Rightarrow$  the inverse of the function :  $f: [-1,1] \rightarrow \text{Range } f$  exists.

Let  $g: \text{Range } f \rightarrow [-1,1]$  be the inverse of range  $f$ .

Let  $y$  be an arbitrary element of range  $f$ .

Since,  $f: [-1,1] \rightarrow \text{Range } f$  is onto, we get:

$y = f(x)$  for some  $x \in [-1,1]$

$$\Rightarrow y = \frac{x}{x+2}$$

$$\Rightarrow xy + 2y = x$$

$$\Rightarrow x(1 - y) = 2y$$

$$\Rightarrow x = \frac{2y}{1-y}, y \neq 1$$

Now, Let us define  $g: \text{Range } f \rightarrow [-1,1]$

$$g(y) = \frac{2y}{1-y}, y \neq 1$$

$$\text{Now, } (g \circ f)(x) = g(f(x)) = g\left(\frac{x}{x+2}\right) = \frac{2\left(\frac{x}{x+2}\right)}{1 - \frac{x}{x+2}} = \frac{2x}{x+2-x} = \frac{2x}{2} = x$$

$$(f \circ g)(y) = f(g(y)) = f\left(\frac{2y}{1-y}\right) = \frac{\frac{2y}{1-y}}{\frac{2y}{1-y} + 2} = \frac{2y}{2y+2-2y} = \frac{2y}{2} = y$$

Thus,  $g \circ f = I_{[-1,1]}$  and  $f \circ g = I_{\text{Range } f}$

$$\Rightarrow f^{-1} = g$$

Therefore,  $f^{-1}(y) = \frac{2y}{1-y}, y \neq 1$

7. Consider  $f: \mathbb{R} \rightarrow \mathbb{R}$  given by  $f(x) = 4x + 3$ . Show that  $f$  is invertible. Find the inverse of  $f$ .

7.  $f: \mathbb{R} \rightarrow \mathbb{R}$  is given by,  $f(x) = 4x + 3$

For one – one

$$\text{Let } f(x) = f(y)$$

$$\Rightarrow 4x + 3 = 4y + 3$$

$$\Rightarrow 4x = 4y$$

$$\Rightarrow x = y$$

$\therefore f$  is a one – one function.

For onto

For  $y \in \mathbb{R}$ , let  $y = 4x + 3$ .

$$\Rightarrow x = \frac{y-3}{4} \in \mathbb{R}$$

Therefore, for any  $y \in \mathbb{R}$ , there exists  $x = \frac{y-3}{4} \in \mathbb{R}$ , such that

$$f(x) = f\left(\frac{y-3}{4}\right) = 4\left(\frac{y-3}{4}\right) + 3 = y.$$

$\therefore f$  is onto.

Thus,  $f$  is one – one and onto and therefore,  $f^{-1}$  exists.

Let us define  $g: \mathbb{R} \rightarrow \mathbb{R}$  by  $g(x) = \frac{y-3}{4}$

$$\text{Now, } (g \circ f)(x) = g(f(x)) = g(4x + 3) = \frac{(4x+3)-3}{4} = \frac{4x}{4} = x \text{ and}$$

$$(f \circ g)(y) = f(g(y)) = f\left(\frac{y-3}{4}\right) = 4\left(\frac{y-3}{4}\right) + 3 = y-3 + 3 = y$$

$$\therefore \text{gof} = \text{fog} = \text{I}_R$$

Hence,  $f$  is invertible and the inverse of  $f$  is given by  $f^{-1}(y) = g(y) = \frac{y-3}{4}$

8. Consider  $f: R_+ \rightarrow [4, \infty)$  given by  $f(x) = x^2 + 4$ . Show that  $f$  is invertible with the inverse  $f^{-1}$  of given  $f$  by  $f^{-1}(y) = \sqrt{y-4}$ , where  $R_+$  is the set of all non-negative real numbers.

8. It is given that  $f: R_+ \rightarrow [4, \infty)$  given by  $f(x) = x^2 + 4$ .

Now, Let  $f(x) = f(y)$

$$\Rightarrow x^2 + 4 = y^2 + 4$$

$$\Rightarrow x^2 = y^2$$

$$\Rightarrow x = y$$

$\Rightarrow f$  is one-one function.

Now, for  $y \in [4, \infty)$ , let  $y = x^2 + 4$ .

$$\Rightarrow x^2 = y - 4 \geq 0.$$

$$\Rightarrow x = \sqrt{y-4} \geq 0$$

$\Rightarrow$  for any  $y \in R$ , there exists  $x = \sqrt{y-4} \in R$  such that

$$\Rightarrow f(x) = f(\sqrt{y-4}) = (\sqrt{y-4})^2 + 4 = y - 4 + 4 = y.$$

$\Rightarrow f$  is onto function.

Therefore,  $f$  is one-one and onto function, so  $f^{-1}$  exists.

Now, let us define  $g: [4, \infty) \rightarrow R_+$  by,

$$g(y) = \sqrt{y-4}$$

$$\text{Now, } \text{gof}(x) = g(f(x)) = g(x^2 + 4) = \sqrt{(x^2 + 4) - 4} = \sqrt{x^2} = x$$

$$\text{And, } \text{fog}(y) = f(g(y)) = f(\sqrt{y-4}) = (\sqrt{y-4})^2 + 4 = (y-4) + 4 = y$$

Therefore,  $\text{gof} = \text{fog} = \text{I}_R$ .

**Therefore,  $f$  is invertible and the inverse of  $f$  is given by**

$$\mathbf{f^{-1}(y) = g(y) = \sqrt{y-4}.$$

9. Consider  $f: R_+ \rightarrow [-5, \infty)$  given by  $f(x) = 9x^2 + 6x - 5$ . Show that  $f$  is invertible with  $f^{-1}$

$$^{-1}(y) = \left( \frac{(\sqrt{y+6})-1}{3} \right)$$

9.  $f: R_+ \rightarrow [-5, \infty)$  is given as  $f(x) = 9x^2 + 6x - 5$ .

Let  $y$  be an arbitrary element of  $[-5, \infty)$ .

$$\text{Let } y = 9x^2 + 6x - 5$$

$$\Rightarrow y = (3x + 1)^2 - 1 - 5 = (3x + 1)^2 - 6$$

$$\Rightarrow y + 6 = (3x + 1)^2$$

$$\Rightarrow 3x + 1 = \sqrt{y + 6} \quad [\text{as } y \geq -5 \Rightarrow y + 6 > 0]$$

$$\Rightarrow x = \frac{(y + 6) - 1}{3}$$

$\therefore f$  is onto, thereby range  $f = [-5, \infty)$ .

$$\text{Let us define } g: [-5, \infty) \rightarrow \mathbb{R}_+ \text{ as } g(y) = \frac{(\sqrt{y + 6}) - 1}{3}$$

$$\text{Now, } (g \circ f)(x) = g(f(x)) = g(9x^2 + 6x - 5) = g((3x + 1)^2 - 6)$$

$$= \sqrt{(3x + 1)^2 - 6 + 6} - 1$$

$$= \frac{3x + 1 - 1}{3} = \frac{3x}{3} = x$$

$$\text{and } (f \circ g)(y) = f(g(y)) = f\left(\frac{\sqrt{y + 6} - 1}{3}\right) = \left[3\left(\frac{\sqrt{y + 6} - 1}{3}\right) + 1\right]^2 - 6$$

$$= (\sqrt{y + 6})^2 - 6 = y + 6 - 6 = y$$

$$\therefore g \circ f = x = I_{\mathbb{R}} \text{ and } f \circ g = y = I_{\text{Range } f}$$

Hence,  $f$  is invertible and the inverse of  $f$  is given by

$$f^{-1}(y) = g(y) = \left(\frac{(\sqrt{y + 6}) - 1}{3}\right)$$

**10.** Let  $f: X \rightarrow Y$  be an invertible function. Show that  $f$  has unique inverse. (Hint: suppose  $g_1$  and  $g_2$  are two inverses of  $f$ . Then for all  $y \in Y$ ,  $f \circ g_1(y) = I_Y(y) = f \circ g_2(y)$ . Use one – one ness of  $f$ ).

**10.** It is given that  $f: X \rightarrow Y$  be an invertible function.

Also, suppose  $f$  has two inverse

Then, for all  $y \in Y$ , we get:

$$f \circ g_1(y) = I_1(y) = f \circ g_2(y)$$

$$\Rightarrow f(g_1(y)) = f(g_2(y))$$

$$\Rightarrow g_1(y) = g_2(y)$$

$$\Rightarrow g_1 = g_2$$

**Therefore,  $f$  has a unique inverse.**

**11.** Consider  $f: \{1, 2, 3\} \rightarrow \{a, b, c\}$  given by  $f(1) = a, f(2) = b$  and  $f(3) = c$ . Find  $f^{-1}$  and show that  $(f^{-1})^{-1} = f$ .

11. Function  $f: \{1, 2, 3\} \rightarrow \{a, b, c\}$  is given by  $f(1) = a$ ,  $f(2) = b$ , and  $f(3) = c$

If we define  $g: \{a, b, c\} \rightarrow \{1, 2, 3\}$  as  $g(a) = 1$ ,  $g(b) = 2$ ,  $g(c) = 3$ .

We have

$$(f \circ g)(a) = f(g(a)) = f(1) = a$$

$$(f \circ g)(b) = f(g(b)) = f(2) = b$$

$$(f \circ g)(c) = f(g(c)) = f(3) = c$$

and

$$(g \circ f)(1) = g(f(1)) = g(a) = 1$$

$$(g \circ f)(2) = g(f(2)) = g(b) = 2$$

$$(g \circ f)(3) = g(f(3)) = g(c) = 3$$

$\therefore g \circ f = I_X$  and  $f \circ g = I_Y$ , where  $X = \{1, 2, 3\}$  and  $Y = \{a, b, c\}$ .

Thus, the inverse of  $f$  exists and  $f^{-1} = g$ .

$\therefore f^{-1}: \{a, b, c\} \rightarrow \{1, 2, 3\}$  is given by  $f^{-1}(a) = 1, f^{-1}(b) = 2, f^{-1}(c) = 3$

Let us now find the inverse of  $f^{-1}$  i.e., find the inverse of  $g$ .

If we define  $h: \{1, 2, 3\} \rightarrow \{a, b, c\}$  as  $h(1) = a, h(2) = b, h(3) = c$

We have

$$(g \circ h)(1) = g(h(1)) = g(a) = 1$$

$$(g \circ h)(2) = g(h(2)) = g(b) = 2$$

$$(g \circ h)(3) = g(h(3)) = g(c) = 3$$

and

$$(h \circ g)(a) = h(g(a)) = h(1) = a$$

$$(h \circ g)(b) = h(g(b)) = h(2) = b$$

$$(h \circ g)(c) = h(g(c)) = h(3) = c$$

$\therefore g \circ h = I_X$  and  $h \circ g = I_Y$ , where  $X = \{1, 2, 3\}$  and  $Y = \{a, b, c\}$ .

Thus, the inverse of  $g$  exists and  $g^{-1} = h \Rightarrow (f^{-1})^{-1} = h$ .

It can be noted that  $h = f$ .

Hence,  $(f^{-1})^{-1} = f$ .

12. Let  $f: X \rightarrow Y$  be an invertible function. Show that the inverse of  $f^{-1}$  is  $f$ , i.e.,  $(f^{-1})^{-1} = f$ .

12. It is given that  $f: X \rightarrow Y$  be an invertible function.

Then, there exists a function  $g: Y \rightarrow X$  such that  $g \circ f = I_X$  and  $f \circ g = I_Y$ .

Then,  $f^{-1} = g$ .

Now,  $g \circ f = I_x$  and  $f \circ g = I_y$

$\Rightarrow f^{-1} \circ f = I_x$  and  $f \circ f^{-1} = I_y$

Thus,  $f^{-1}: Y \rightarrow X$  is invertible and  $f$  is the inverse of  $f^{-1}$ .

**Therefore,  $(f^{-1})^{-1} = f$ .**

13. If  $f: \mathbb{R} \rightarrow \mathbb{R}$  be given by  $f(x) = (3 - x^3)^{\frac{1}{3}}$ , then  $f \circ f(x)$  is

(A)  $\frac{1}{x^3}$

(B)  $x^3$

(C)  $x$

(D)  $(3 - x^3)$

13.  $f: \mathbb{R} \rightarrow \mathbb{R}$  be given as  $f(x) = (3 - x^3)^{\frac{1}{3}}$

$$\therefore f \circ f(x) = f(f(x)) = f\left((3 - x^3)^{\frac{1}{3}}\right) = \left[3 - \left((3 - x^3)^{\frac{1}{3}}\right)^3\right]^{\frac{1}{3}}$$

$$= [3 - (3 - x^3)]^{\frac{1}{3}} = (x^3)^{\frac{1}{3}} = x$$

$$\therefore f \circ f(x) = x$$

The correct answer is C.

14. Let  $f: \mathbb{R} - \left\{-\frac{4}{3}\right\} \rightarrow \mathbb{R}$  be a function as  $f(x) = \frac{4x}{3x+4}$ . The inverse of  $f$  is map  $g: \text{Range } f \rightarrow \mathbb{R} - \left\{-\frac{4}{3}\right\}$  given by

$f \rightarrow \mathbb{R} - \left\{-\frac{4}{3}\right\}$  given by

(A)  $g(y) = \frac{3y}{3-4y}$

(B)  $g(y) = \frac{4y}{4-3y}$

(C)  $g(y) = \frac{4y}{3-4y}$

(D)  $g(y) = \frac{3y}{4-3y}$

14. The correct option is (B).

**Explanation:** It is given that  $f: \mathbb{R} - \left\{-\frac{4}{3}\right\} \rightarrow \mathbb{R}$  be a function defined as  $f(x) = \frac{4x}{3x+4}$ .

Let  $y$  be any element of Range  $f$ .

Then, there exists  $x \in \mathbb{R} - \left\{-\frac{4}{3}\right\}$  such that  $y = f(x)$

$$\Rightarrow y = \frac{4x}{3x+4}$$

$$\Rightarrow 3xy + 4y = 4x$$

$$\Rightarrow x(4 - 3y) = 4y$$

$$\Rightarrow x = \frac{4y}{4-3y}$$

Let us define  $g: \text{Range } f \rightarrow \mathbb{R} - \left\{-\frac{4}{3}\right\}$  as  $g(y) = \frac{4y}{4-3y}$

$$\text{Now, } (g \circ f)(x) = g(f(x)) = g\left(\frac{4x}{3x+4}\right) = \frac{4\left(\frac{4x}{3x+4}\right)}{4-3\left(\frac{4x}{3x+4}\right)} = \frac{16x}{12x+16-12x} = \frac{16x}{16} = x$$

$$\text{And, } (f \circ g)(y) = f(g(y)) = f\left(\frac{4y}{4-3y}\right) = \frac{4\left(\frac{4y}{4-3y}\right)}{3\left(\frac{4y}{4-3y}\right)+4} = \frac{16y}{12y+16-12y} = \frac{16y}{16} = y$$

Therefore,  $g \circ f = I_{\mathbb{R} - \left\{-\frac{4}{3}\right\}}$  and  $f \circ g = I_{\text{Range } f}$

Thus,  $g$  is the inverse of  $f$

Therefore, The inverse of  $f$  is the map

$g: \text{Range } f \rightarrow \mathbb{R} - \left\{-\frac{4}{3}\right\}$ , which is given by  $g(y) = \frac{4y}{4-3y}$ .