$= (b^2)^2 - (bc)^2 - (bc)^2 + (c^2)^2$

$= b^4 - 2b^2c^2 + c^4$

$(a^2 - b^2)(c^2 - d^2) = (b^2 - c^2)^2$

From the above equation we can say that $(a^2 - b^2), (b^2 - c^2), (c^2 - d^2)$ are in GP

Q. 17. If a, b, c, d are in GP, then prove that

$\frac{1}{(a^2+b^2)}, \frac{1}{(b^2+c^2)}, \frac{1}{(c^2+d^2)}$ **are in GP**

Answer : To prove: $\frac{1}{(a^2+b^2)}, \frac{1}{(b^2+c^2)}, \frac{1}{(c^2+d^2)}$ are in GP.

Given: a, b, c, d are in GP

Proof: When a,b,c,d are in GP then

$\Rightarrow \frac{b}{a} = \frac{c}{b} = \frac{d}{c}$

From the above, we can have the following conclusion

$\Rightarrow bc = ad$ … (i)

$\Rightarrow b^2 = ac$ … (ii)

$\Rightarrow c^2 = bd$ … (iii)

Considering $\frac{1}{(a^2+b^2)}, \frac{1}{(b^2+c^2)}, \frac{1}{(c^2+d^2)}$

$\frac{1}{(a^2+b^2)} \times \frac{1}{(c^2+d^2)} = \frac{1}{a^2c^2+a^2d^2+b^2c^2+b^2d^2}$

$= \frac{1}{(ac)^2+(ad)^2+(bc)^2+(bd)^2}$

From eqn. (i) , (ii) and (iii)

$= \frac{1}{(b^2)^2 + (bc)^2 + (bc)^2 + (c^2)^2}$

$= \frac{1}{b^4 + 2b^2c^2 + c^4}$

$\frac{1}{(a^2+b^2)} \times \frac{1}{(c^2+d^2)} = \frac{1}{(b^2+c^2)^2}$

From the above equation, we can say that $\frac{1}{(a^2+b^2)}, \frac{1}{(b^2+c^2)}, \frac{1}{(c^2+d^2)}$ are in GP.

Q. 18. If $(p^2 + q^2)$, $(pq + qr)$, $(q^2 + r^2)$ are in GP then prove that p, q, r are in GP

Answer : To prove: p, q, r are in GP

Given: $(p^2 + q^2)$, $(pq + qr)$, $(q^2 + r^2)$ are in GP

Formula used: When a,b,c are in GP, $b^2 = ac$

Proof: When $(p^2 + q^2)$, $(pq + qr)$, $(q^2 + r^2)$ are in GP,

$(pq + qr)^2 = (p^2 + q^2)(q^2 + r^2)$

$p^2q^2 + 2pq^2r + q^2r^2 = p^2q^2 + p^2r^2 + q^4 + q^2r^2$

$2pq^2r = p^2r^2 + q^4$

$pq^2r + pq^2r = p^2r^2 + q^4$

$pq^2r - q^4 = p^2r^2 - pq^2r$

$q^2(pr - q^2) = pr(pr - q^2)$

$q^2 = pr$

From the above equation we can say that p, q and r are in G.P.

Q. 19. If a, b, c are in AP, and a, b, d are in GP, show that a, (a – b) and (d – c) are in GP.

Answer : To prove: a, (a – b) and (d – c) are in GP.

Given: a, b, c are in AP, and a, b, d are in GP

Proof: As a,b,d are in GP then

$b^2 = ad$ … (i)

As a, b, c are in AP

$2b = (a + c)$ … (ii)

Considering a, (a – b) and (d – c)

$(a - b)^2 = a^2 - 2ab + b^2$

$= a^2 - (2b)a + b^2$

From eqn. (i) and (ii)

$= a^2 - (a+c)a + ad$

$= a^2 - a^2 - ac + ad$

$= ad - ac$

$(a - b)^2 = a (d - c)$

From the above equation we can say that a, (a – b) and (d – c) are in GP.

Q. 20. If a, b, c are in AP, and a, x, b and b, y, c are in GP then show that x^2, b^2, $y^2$are in AP.

Answer : To prove: x^2, b^2, y^2 are in AP.

Given: a, b, c are in AP, and a, x, b and b, y, c are in GP

Proof: As, a,b,c are in AP

$\Rightarrow 2b = a + c$ … (i)

As, a,x,b are in GP

$\Rightarrow x^2 = ab$ … (ii)

As, b,y,c are in GP

$\Rightarrow y^2 = bc$ … (iii)

Considering x^2, b^2, y^2

$x^2 + y^2 = ab + bc$ [From eqn. (ii) and (iii)]

$= b (a + c)$

$= b(2b)$ [From eqn. (i)]

$x^2 + y^2 = 2b^2$

From the above equation we can say that x^2, b^2, y^2 are in AP.

Exercise 12F

Q. 1. Find two positive numbers a and b, whose
(i) AM = 25 and GM = 20
(ii) AM = 10 and GM = 8

Answer : (i) AM = 25 and GM = 20

To find: Two positive numbers a and b

Given: AM = 25 and GM = 20

Formula used: (i) Arithmetic mean between a and b $= \frac{a+b}{2}$

(ii) Geometric mean between a and b $= \sqrt{ab}$

Arithmetic mean of two numbers $= \frac{a+b}{2}$

$\frac{a+b}{2} = 25$

$\Rightarrow a + b = 50$

$\Rightarrow b = 50 - a \ldots$ (i)

Geometric mean of two numbers $= \sqrt{ab}$

$\Rightarrow \sqrt{ab} = 20$

$\Rightarrow ab = 400$

Substituting value of b from eqn. (i)

$a(50 - a) = 400$

$\Rightarrow 50a - a^2 = 400$

On rearranging

$\Rightarrow a^2 - 50a + 400 = 0$

$\Rightarrow a^2 - 40a - 10a + 400$

$\Rightarrow a(a - 40) - 10(a - 40) = 0$

$\Rightarrow (a - 10)(a - 40) = 0$

$\Rightarrow a = 10, 40$

Substituting, a = 10 Or a = 40 in eqn. (i)

b = 40 Or b = 10

Therefore two numbers are 10 and 40

(ii) AM = 10 and GM = 8

To find: Two positive numbers a and b

Given: AM = 10 and GM = 8

Formula used: (i) Arithmetic mean between a and b = $\frac{a+b}{2}$

(ii) Geometric mean between a and b = $\sqrt{ab}$

Arithmetic mean of two numbers = $\frac{a+b}{2}$

$$\frac{a+b}{2} = 10$$

$\Rightarrow a + b = 20$

$\Rightarrow a = 20 - b \ldots (i)$

Geometric mean of two numbers $=\sqrt{ab}$

$\Rightarrow \sqrt{ab}=8$

$\Rightarrow ab=64$

Substituting value of a from eqn. (i)

b(20 – b) = 64

$\Rightarrow 20b - b^2 = 64$

On rearranging

$\Rightarrow b^2 - 20b + 64 = 0$

$\Rightarrow b^2 - 16b - 4b + 64$

$\Rightarrow b(b - 16) - 4(b - 16) = 0$

$\Rightarrow (b - 16)(b - 4) = 0$

$\Rightarrow b = 16, 4$

Substituting, b = 16 Or b = 4 in eqn. (i)

a = 4 Or b = 16

Therefore two numbers are 16 and 4

Q. 2. Find the GM between the numbers

(i) 5 and 125
(ii) 1 and $\frac{9}{16}$
(iii) 0.15 and 0.0015
(iv) -8 and -2
(v) -6.3 and -2.8
(vi) ad ab^3

Answer : (i) 5 and 125

To find: Geometric Mean

Given: The numbers are 5 and 125

Formula used: (i) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Geometric mean of two numbers $= \sqrt{ab}$

$= \sqrt{5 \times 25}$

$= \sqrt{625}$

= 25

The geometric mean between 5 and 125 is 25

(ii) 1 and $\frac{9}{16}$

To find: Geometric Mean

Given: The numbers are 1 and $\frac{9}{16}$

Formula used: (i) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Geometric mean of two numbers $= \sqrt{ab}$

$= \sqrt{1 \times \frac{9}{16}}$

$= \sqrt{\frac{9}{16}}$

$= \frac{3}{4}$

The geometric mean between 1 and $\frac{9}{16}$ is $\frac{3}{4}$.

(iii) 0.15 and 0.0015

To find: Geometric Mean

Given: The numbers are 0.15 and 0.0015

Formula used: (i) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Geometric mean of two numbers $= \sqrt{ab}$

$= \sqrt{0.15 \times 0.0015}$

$= \sqrt{0.000225}$

= 0.015

The geometric mean between 0.15 and 0.0015 is 0.015.

(iv) -8 and -2

To find: Geometric Mean

Given: The numbers are -8 and -2

Formula used: (i) Geometric mean between $a \text{ and } b = \sqrt{ab}$

Geometric mean of two numbers $= \sqrt{ab}$

$= \sqrt{-8 \times -2}$

$= \sqrt{16}$

= ± 4

Mean is a number which has to fall between two numbers.

Therefore we will take -4 as our answer as +4 doesn’t lie between -8 and -2

The geometric mean between -8 and -2 is -4.

(v) -6.3 and -2.8

To find: Geometric Mean

Given: The numbers are -6.3 and -2.8

Formula used: (i) Geometric mean between a and $b=\sqrt{ab}$

Geometric mean of two numbers $=\sqrt{ab}$

$=\sqrt{-6.3 \times -2.8}$

$=\sqrt{17.64}$

$= \pm 4.2$

Mean is a number which has to fall between two numbers.

Therefore we will take -4.2 as our answer as +4.2 doesn't lie between -6.3 and -2.8

The geometric mean between -6.3 and -2.8 is -4.2.

(vi) a^3b and ab^3

To find: Geometric Mean

Given: The numbers are a^3b and ab^3

Formula used: (i) Geometric mean between a and $b=\sqrt{ab}$

Geometric mean of two numbers $=\sqrt{ab}$

$=\sqrt{a^3b \times ab^3}$

$=\sqrt{a^4b^4}$

$= a^2b^2$

The geometric mean between a^3b and ab^3 is a^2b^2.

Q. 13. Insert two geometric means between 9 and 243.

Answer : To find: Two geometric Mean

Given: The numbers are 9 and 243

Formula used: (i) r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$, where n is the number of

geometric mean

Let G_1 and G_2 be the three geometric mean

Then r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$\Rightarrow$ r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

$\Rightarrow$ r $= \left(\frac{243}{9}\right)^{\frac{1}{2+1}}$

$\Rightarrow$ r $= 27^{\frac{1}{3}}$

$\Rightarrow$ r = 3

$G_1 = ar = 9 \times 3 = 27$

$G_2 = ar^2 = 9 \times 3^2 = 9 \times 9 = 81$

Two geometric mean between 9 and 243 are 27 and 81.

Q. 4. Insert three geometric means between $\frac{1}{3}$ and 432.

Answer : To find: Three geometric Mean

Given: The numbers $\frac{1}{3}$ and 432

Formula used: (i) r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$, where n is the number of

geometric mean

Let G_1, G_2 and G_3 be the three geometric mean

Then r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

⇒ r $= \left(\frac{b}{a}\right)^{\frac{1}{3+1}}$

⇒ r $= \left(\frac{432}{\left(\frac{1}{3}\right)}\right)^{\frac{1}{2+1}}$

⇒ r $= \left(\frac{432\times3}{1}\right)^{\frac{1}{3+1}}$

⇒ r $=(1296)^{\frac{1}{4}}$

⇒ r = 6

$G_1 = ar = \left(\frac{1}{3}\right)\times6 = 2$

$G_2 = ar^2 = \left(\frac{1}{3}\right)\times6^2 = \left(\frac{1}{3}\right)\times36 = 12$

$G_3 = ar^3 = \left(\frac{1}{3}\right)\times6^3 = \left(\frac{1}{3}\right)\times216 = 72$

Three geometric mean between $\frac{1}{3}$ and 432 are 2, 12 and 72.

Q. 5. Insert four geometric means between 6 and 192.

Answer : To find: Four geometric Mean

Given: The numbers 6 and 192

Formula used: (i) r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$, where n is the number of geometric mean

Let G_1, G_2, G_3 and G_4 be the three geometric mean

Then r $= \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$

⇒ r $= \left(\frac{b}{a}\right)^{\frac{1}{4+1}}$

⇒ r $= \left(\frac{192}{6}\right)^{\frac{1}{4+1}}$

⇒ r $= (32)^{\frac{1}{5}}$

⇒ r = 2

$G_1 = ar = 6 \times 2 = 12$

$G_2 = ar^2 = 6 \times 2^2 = 24$

$G_3 = ar^3 = 6 \times 2^3 = 48$

$G_4 = ar^4 = 6 \times 2^4 = 96$

Four geometric mean between 6 and 192 are 12, 24, 48 and 96.

Q. 6. The AM between two positive numbers a and b(a>b) is twice their GM. Prove that a:b $=\left(2+\sqrt{3}\right):\left(2-\sqrt{3}\right)$**.**

Answer : To prove: Prove that a:b $=(2+\sqrt{3}):(2-\sqrt{3})$

Given: Arithmetic mean is twice of Geometric mean.

Formula used: (i) Arithmetic mean between a and b $= \frac{a+b}{2}$

(ii) Geometric mean between a and b $= \sqrt{ab}$

AM = 2(GM)

$$\frac{a+b}{2} = 2\left(\sqrt{ab}\right)$$

$\Rightarrow a + b = 4(\sqrt{ab})$

Squaring both side

$\Rightarrow (a + b)^2 = 16ab$ … (i)

We know that $(a - b)^2 = (a + b)^2 - 4ab$

From eqn. (i)

$\Rightarrow (a - b)^2 = 16ab - 4ab$

$\Rightarrow (a - b)^2 = 12ab$ … (ii)

Dividing eqn. (i) and (ii)

$$\frac{(a+b)^2}{(a-b)^2} = \frac{16ab}{12ab}$$

$$\Rightarrow \left(\frac{a+b}{a-b}\right)^2 = \frac{16}{12}$$

Taking square root both side

$$\Rightarrow \frac{a+b}{a-b} = \frac{4}{2\sqrt{3}}$$

$$\Rightarrow \frac{a+b}{a-b} = \frac{2}{\sqrt{3}}$$

Applying componendo and dividend

$$\Rightarrow \frac{a+b+a-b}{a+b-a+b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

$$\Rightarrow \frac{2a}{2b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

$$\Rightarrow \frac{a}{b} = \frac{2+\sqrt{3}}{2-\sqrt{3}}$$

Hence Proved

Q. 7. If a, b, c are in AP, x is the GM between a and b; y is the GM between b and c; then show that b^2 is the AM between x^2 and y^2.

Answer : To prove: b^2 is the AM between x^2 and y^2.

Given: (i) a, b, c are in AP

(ii) x is the GM between a and b

(iii) y is the GM between b and c

Formula used: (i) Arithmetic mean between a and b = $\frac{a+b}{2}$

(ii) Geometric mean between a and b$=\sqrt{ab}$

As a, b, c are in A.P.

⇒ 2b = a + c … (i)

As x is the GM between a and b

⇒ x = $(\sqrt{ab})$

⇒ x^2 = ab … (ii)

As y is the GM between b and c

⇒ y = $(\sqrt{bc})$

⇒ y^2 = bc … (iii)

Arithmetic mean of x^2 and y^2 is $\left(\frac{x^2+y^2}{2}\right)$

Substituting the value from (ii) and (iii)

$$\left(\frac{x^2+y^2}{2}\right)=\left(\frac{ab+bc}{2}\right)$$

$$= \frac{b(a+c)}{2}$$

Substituting the value from eqn. (i)

$$= \frac{b(2b)}{2}$$

$= b^2$

Hence Proved

Q. 8. Show that the product of n geometric means between a and b is equal to the nth power of the single GM between a and b.

Answer : To prove: Product of n geometric means between a and b is equal to the nth power of the single GM between a and b.

Formula used:(i) Geometric mean between $\text{a and b} = \sqrt{ab}$

(ii) Sum of n terms of A.P. $= \frac{(n)(n+1)}{2}$

Let the n geometric means between and b be $G_1, G_2, G_3, \ldots G_n$

Hence a, $G_1, G_2, G_3, \ldots G_n$, b are in GP

$\Rightarrow G_1 = ar, G_2 = ar^2$ and so on …

Now, we have n+2 term

$\Rightarrow b = ar^{n+2-1}$

$\Rightarrow b = ar^{n+1}$

$\Rightarrow r = \left(\frac{b}{a}\right)^{\frac{1}{n+1}}$ … (i)

The product of n geometric means is $G_1 \times G_2 \times G_3 \times \ldots G_n$

$= ar \times ar^2 \times ar^3 \times \ldots ar^n$

$= a^n \times r^{(1+2+3\ldots+n)}$

$= a^n \times r^{(n)\left(\frac{n+1}{2}\right)}$ $\left[\text{Sum of n terms of A.P.} = \frac{(n)(n+1)}{2}\right]$

Substituting the value of r from eqn. (i)

$= a^n \times \left(\frac{b}{a}\right)^{\left(\frac{1}{n+1}\right)(n)\left(\frac{n+1}{2}\right)}$

$= a^n \times \left(\frac{b}{a}\right)^{\left(\frac{n}{2}\right)}$

$= a^n \times \frac{b^{\frac{n}{2}}}{a^{\frac{n}{2}}}$

$= a^{\frac{n}{2}} \times b^{\frac{n}{2}}$

$= (ab)^{\frac{n}{2}}$

$= \left(\sqrt{ab}\right)^n$ ... (ii)

Single geometric mean between a and b $= \sqrt{ab}$

n^{th} power of single geometric mean between a and b $= \left(\sqrt{ab}\right)^n$

Hence Proved

Q. 9. If AM and GM of the roots of a quadratic equation are 10 and 8 respectively then obtain the quadratic equation.

Answer : To find: The quadratic equation.

Given: (i) AM of roots of quadratic equation is 10

(ii) GM of roots of quadratic equation is 8

Formula used: (i) Arithmetic mean between a and b $= \frac{a+b}{2}$

(ii) Geometric mean between a and $b = \sqrt{ab}$

Let the roots be p and q

Arithmetic mean of roots p and q $= \frac{p+q}{2}$ = 10

⇒ $\frac{p+q}{2}$ = 10

⇒ p + q = 20 = sum of roots … (i)

Geometric mean of roots p and q = $\sqrt{pq}$ = 8

⇒ pq = 64 = product of roots … (ii)

Quadratic equation = x^2 – (sum of roots)x + (product of roots)

From equation (i) and (ii)

Quadratic equation = x^2 – (20)x + (64)

= x^2 –20x + 64

x^2 –20x + 64

Exercise 12G

Q. 1. Find the sum of each of the following infinite series :

$8 + 4\sqrt{2} + 4 + 2\sqrt{2} + \ldots\ldots \infty$

Answer : It is Infinite Geometric Series.

Here, a=8,

$$r = \frac{4\sqrt{2}}{8} = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$$

The formula used: Sum of an infinite Geometric series $= \frac{a}{1-r}$

$\therefore$Sum$= \frac{8}{1-\frac{1}{\sqrt{2}}} = \frac{8\sqrt{2}}{\sqrt{2}-1}$

Sum$= \frac{8\sqrt{2}}{\sqrt{2}-1}$

Q. 2. Find the sum of each of the following infinite series :

6 + 1.2 + 0.24 + ….. ∞

Answer : It is Infinite Geometric Series.

Here, a=6,

$$r = \frac{1.2}{6} = \frac{2}{10} = 0.2$$

The formula used: Sum of an infinite Geometric series $= \frac{a}{1-r}$

$\therefore$Sum$=\frac{6}{1-0.2} = \frac{6}{0.8} = \frac{15}{2}$

Sum$= \frac{15}{2}$

Q. 3. Find the sum of each of the following infinite series :

$$\sqrt{2} - \frac{1}{\sqrt{2}} + \frac{1}{2\sqrt{2}} - \frac{1}{4\sqrt{2}} + \ldots.. \infty$$

Answer : It is Infinite Geometric Series

Here, a=√2

$$r = \frac{\frac{-1}{\sqrt{2}}}{\sqrt{2}} = \frac{-1}{2}$$

$\therefore$Sum$= \frac{a}{1-r} = \frac{\sqrt{2}}{1-\frac{-1}{2}} = \frac{\sqrt{2}}{1+\frac{1}{2}} = \frac{2\sqrt{2}}{3}$

Sum$= \frac{2\sqrt{2}}{3}$

Q. 4. Find the sum of each of the following infinite series :

10 – 9 + 8.1 - ……∞

Answer : It is Infinite Geometric Series

Here, a=10

r$= \frac{-9}{10} = -0.9$

$\therefore$Sum $= \frac{a}{1-r} = \frac{10}{1-(-0.9)} = \frac{10}{1+0.9} = \frac{10}{1.9} = \frac{100}{19}$

Sum$= \frac{100}{19}$

Q. 5. Find the sum of each of the following infinite series :

$$\frac{2}{5} + \frac{3}{5^2} + \frac{2}{5^3} + \frac{3}{5^4} + \ldots\ldots_{\infty}$$

Answer : This geometric series is the sum of two geometric series:

$$\frac{2}{5} + \frac{2}{5^3} + \frac{2}{5^5} + \cdots \infty \ \& \ \frac{3}{5^2} + \frac{3}{5^4} + \frac{4}{5^6} + \cdots \infty$$

Sum of geometric series: $\frac{2}{5} + \frac{2}{5^3} + \frac{2}{5^5} + \cdots \infty$

Here, a$= \frac{2}{5}$

$$r = \frac{\frac{2}{5^3}}{\frac{2}{5}} = \frac{1}{5^2} = \frac{1}{25}$$

$$\therefore \text{Sum} = \frac{a}{1-r} = \frac{\frac{2}{5}}{1-\frac{1}{25}} = \frac{\frac{2}{5}}{\frac{25-1}{25}} = \frac{2 \times 25}{24 \times 5} = \frac{5}{12}$$

Sum of geometric series: $\frac{3}{5^2} + \frac{3}{5^4} + \frac{4}{5^6} + \cdots \infty$

Here, a= $\frac{3}{5^2}$

$$r = \frac{\frac{3}{5^4}}{\frac{3}{5^2}} = \frac{1}{5^2} = \frac{1}{25}$$

$$\therefore \text{Sum} = \frac{a}{1-r} = \frac{\frac{3}{5^2}}{1-\frac{1}{25}} = \frac{\frac{3}{5^2}}{\frac{25-1}{25}} = \frac{3 \times 25}{25 \times 24} = \frac{1}{8}$$

∴Sum of the given infinite series=sum of both the series $= \frac{5}{12} + \frac{1}{8} = \frac{(5\times2)+(1\times3)}{24}$

$$= \frac{10+3}{24} = \frac{13}{24}$$

Sum $= \frac{13}{24}$

Q. 6. Prove that $9^{1/3} \times 9^{1/9} \times 9^{1/27} \times \ldots..\infty = 3$

Answer : L.H.S=$9^{1/3} \times 9^{1/9} \times 9^{1/27} \times \ldots..\infty$

$=9^{(1/3)+(1/9)+(1/27)+\ldots\infty}$

The series in the exponent is an infinite geometric series

Whose, a = $\frac{1}{3}$

$$r = \frac{\frac{1}{9}}{\frac{1}{3}} = \frac{1 \times 3}{1 \times 9} = \frac{1}{3}$$

∴Sum of the series in the exponent $= \frac{a}{1-r} = \frac{\frac{1}{3}}{1-\frac{1}{3}} = \frac{1\times3}{3\times2} = \frac{1}{2}$

∴L.H.S=$9^{1/2}$

=3=R.H.S

Hence, Proved that $9^{1/3} \times 9^{1/9} \times 9^{1/27} \times \ldots..\infty = 3$

Q. 7. Find the rational number whose decimal expansion is given below :

(i) $0.\overline{3}$ **(ii)** $0.\overline{231}$

(iii) $3.\overline{52}$

Answer : (i) Let, x=0.3333…

⇒ x=0.3+0.03+0.003+…

⇒ x=3(0.1+0.01+0.001+0.0001+…∞)

⇒ $x=3\left(\frac{1}{10}+\frac{1}{100}+\frac{1}{1000}+\frac{1}{10000}+\cdots\infty\right)$

This is an infinite geometric series.

Here, a=1/10 and r=1/10

$$\therefore \text{Sum} = \frac{a}{1-r} = \frac{\frac{1}{10}}{1-\frac{1}{10}} = \frac{1 \times 10}{9 \times 10} = \frac{1}{9}$$

∴x $= 3 \times \frac{1}{9} = \frac{1}{3}$

$0.\overline{3} = \frac{1}{3}$

(ii) Let, x=0.231231231….

⇒ x=0.231+0.000231+0.000000231+…∞

⇒ x=231(0.001+0.000001+0.000000001+…∞)

⇒ x=231($\frac{1}{10^3}$+$\frac{1}{10^6}+\frac{1}{10^9}+\frac{1}{10^{12}}$+…∞)

This is an infinite geometric series.

Here, a $=\frac{1}{10^3}$ and r=$\frac{1}{10^3}$

$$\therefore \text{Sum} = \frac{a}{1-r} = \frac{\frac{1}{10^3}}{1-\frac{1}{10^3}} = \frac{1\times1000}{999\times1000} = \frac{1}{999}$$

⇒ $x = 231 \times \frac{1}{999} = \frac{231}{999}$

$0.\overline{231} = \frac{231}{999}$

(iii) Let, x=3.525252552…

⇒ x=3+0.52+0.0052+0.000052+…∞

⇒ x=3+52(0.01+0.0001+…∞)

⇒ x=3+52($\frac{1}{10^2}$+$\frac{1}{10^4}+\frac{1}{10^6}+\frac{1}{10^8}$+…∞)

Here, a $=\frac{1}{10^2}$ and r=$\frac{1}{10^2}$

$$\therefore \text{Sum} = \frac{a}{1-r} = \frac{\frac{1}{10^2}}{1-\frac{1}{10^2}} = \frac{1\times100}{99\times100} = \frac{1}{99}$$

⇒ $x = 3 + \left(52 \times \frac{1}{99}\right) = \frac{297+52}{999} = \frac{349}{999}$

$3.\overline{52} = \frac{349}{999}$

Q. 8. Express the recurring decimal 0.125125125 …. = $0.\overline{125}$ as a rational number.

Answer : Let, x=0.125125125… …(i)

Multiplying this equation by 1000 on both the sides so that repetitive terms cancel out and we get:

1000x=125.125125125… …(ii)

Equation (ii)-(i),

⇒ 1000x-x=125.125125125-0.125125125=125

⇒ 999x=125

⇒ $x = \frac{125}{999}$

$0.\overline{125} = \frac{125}{999}$

Q. 9. Write the value of $0.\overline{423}$ in the form of a simple fraction.

Answer : Let, x=0.423423423… …(i)

Multiplying this equation by 1000 on both the sides so that repetitive terms cancel out and we get:

1000x=423.423423423… …(ii)

Equation (ii)-(i),

⇒ 1000x-x=423.423423423-0.423423423=423

⇒ 999x=423

⇒ $x = \frac{423}{999} = \frac{47}{111}$

$0.\overline{423} = \frac{47}{111}$

Q. 10. Write the value of $2.\overline{134}$ in the form of a simple fraction.

Answer : Let, x=2.134134134… …(i)

Multiplying this equation by 1000 on both the sides so that repetitive terms cancel out and we get:

1000x=2134.134134134… …(ii)

Equation (ii)-(i),

⇒ 1000x-x=2134.134134134-2.134134134=2132

⇒ 999x=2132

⇒ $x = \frac{2132}{999}$

$2.\overline{134} = \frac{2132}{999}$

Q. 11. The sum of an infinite geometric series is 6. If its first term is 2, find its common ratio.

Answer :

Given: $\frac{a}{1-r} = 6$, a=2

To find:r=?

$\therefore \frac{2}{1-r} = 6$

$\Rightarrow 1 - r = \frac{2}{6} = \frac{1}{3}$

⇒ 3(1-r)=1

⇒ 3-3r=1

⇒ 3r=3-1

⇒ r= $\frac{2}{3}$

Common ratio r= $\frac{2}{3}$

Q. 12. The sum of an infinite geometric series is 20, and the sum of the squares of these terms is 100. Find the series.

Answer :

Given: $\frac{a}{1-r} = 20$ & $\frac{a^2}{1-r^2} = 100$

(Because on squaring both first term a and common ratio r will be squared.)

To find: the series

a=20(1-r)…(i)

$\Rightarrow \frac{a^2}{1-r^2} = 100 = \frac{(20\times(1-r))^2}{(1-r)(1+r)}$ ...(from (i))

$\Rightarrow 100 = 400 \times \frac{1-r}{1+r}$

⇒ 100(1+r)=400(1-r)

⇒ 100+100r=400-400r

⇒ 100r+400r=400-100

⇒ 500r=300

⇒ 5r=3

$\Rightarrow r = \frac{3}{5}$

Put this value of r in equation (i) we get

$a = 20\left(1-\frac{3}{5}\right) = \frac{20\times 2}{5} = 8$

∴The infinite geometric series is:8, $\frac{24}{5}, \frac{72}{25}, \frac{216}{125}, \frac{648}{625}, \ldots \infty$

Q. 13. The sum of an infinite GP is 57, and the sum of their cubes is 9747. Find the GP.

Answer : Let the first term Of G.P. be a, and common ratio be r.

$\therefore \frac{a}{1-r} = 57 \ldots (1)$

On cubing each term will become,

a^3, a^3r^3, ...

∴This sum $= \frac{a^3}{1-r^3} = 9747 \ldots (2)$

a=57(1-r) put this in equation 2 we get

$$\frac{(57 \times (1-r))^3}{1-r^3} = 9747$$

$\Rightarrow \frac{57^3 \times (1-r)^3}{1-r^3} = 9747$

$\Rightarrow \frac{(1-r)\times(1-r)^2}{(1-r)(1+r+r^2)} = \frac{9747}{57\times57\times57} = \frac{1}{19}$

$\Rightarrow 19(1-2r+r^2)=1+r+r^2$

$\Rightarrow 19r^2-r^2-38r-r+19-1=0$

$\Rightarrow 18r^2-39r+18=0$

$\Rightarrow 6r^2-13r+6=0$

$\Rightarrow (2r-3)(3r-2)=0$

$\Rightarrow r= 2/3, 3/2$

But $-1<r<1$

$\Rightarrow r=2/3$

Substitute this value of r in equation 1 we get

$$a = 57 \times \left(1-\frac{2}{3}\right) = 19$$

Thus the first term of G.P. is 19, and the common ratio is 2/3

$\therefore G.P=19, \frac{38}{3}, \frac{76}{9}, ...$

$19, \frac{38}{3}, \frac{76}{9}, ...$

Exercise 12H

Q. 1. If the 5th term of a GP is 2, find the product of its first nine terms.

Answer : Given: 5^{th} term of a GP is 2.

To find: the product of its first nine terms.

First term is denoted by a, the common ratio is denote by r.

$\therefore ar^4 = 2$

We have to find the value of: $a \times ar^1 \times ar^2 \times ar^3 \times \ldots \times ar^8$

$= a^9 r^{1 + 2 + 3 + 4 + \ldots + 8}$

$= a^9 r^{36}$

$= (ar^4)^9$

$= (2)^9$

$= 512$

Ans: 512.

Q. 2. If the (p + q)th and (p – q)th terms of a GP are m and n respectively, find its pth term.

Answer : Let,

$t_{p+q} = m = Ar^{p+q-1} = Ar^{p-1}r^q$

And

$t_{p-q} = n = Ar^{p-q-1} = Ar^{p-1}r^{-q}$

We know that p^{th} term $= Ar^{p-1}$

$\therefore m \times n = A^2 r^{2p-2}$

$\Rightarrow Ar^{p-1} = (mn)^{1/2}$

$\Rightarrow p^{th}$ term $= (mn)^{1/2}$

Ans: p^{th} term $= (mn)^{1/2}$

Q. 3. If 2^{nd}, 3^{rd} and 6^{th} terms of an AP are the three consecutive terms of a GP then find the common ratio of the GP.

Answer : We have been given that 2nd, 3rd and 6th terms of an AP are the three consecutive terms of a GP.

Let the three consecutive terms of the G.P. be a, ar, ar^2.

Where a is the first consecutive term and r is the common ratio.

2nd, 3rd terms of the A.P. are a and ar respectively as per the question.

∴ The common difference of the A.P. = ar - a

And the sixth term of the A.P. = ar^2

Since the second term is a and the sixth term is ar^2(In A.P.)

We use the formula:t = a + (n - 1)d

∴ $ar^2 = a + 4(ar - a)$…(the difference between 2nd and 6th term is 4(ar - a))

⇒ $ar^2 = a + 4ar - 4a$

⇒ $ar^2 + 3a - 4ar = 0$

⇒ $a(r^2 - 4r + 3) = 0$

⇒ $a(r - 1)(r - 3) = 0$

Here, we have 3 possible options:

1)a = 0 which is not expected because all the terms of A.P. and G.P. will be 0.

2)r = 1,which is also not expected because all th terms would be equal to first term.

3)r = 3,which is the required answer.

Ans: Common ratio = 3

Q. 4. Write the quadratic equation, the arithmetic and geometric means of whose roots are A and G respectively.

Answer : Let the roots of the required quadratic equation be a and b.

The arithmetic and geometric means of roots are A and G respectively.

⇒ $A = (a + b)/2$…(i)

And G = $\sqrt{ab}$ ...(ii)

We know that the equation whose roots are given is =

$x^2 - (a + b)x + ab = 0$

From (i) and (ii) we get:

$x^2 - 2A + G^2 = 0$

Thus, $x^2 - 2A + G^2 = 0$ is the required quadratic equation.

Ans: $x^2 - 2A + G^2 = 0$ is the required quadratic equation.

Q. 5. If a, b, c are in GP and $a^{1/x} = b^{1/y} = c^{1/z}$ then prove that x, y, z are in AP.

Answer : It is given that:

$a^{1/x} = b^{1/y} = c^{1/z}$

Let $a^{1/x} = b^{1/y} = c^{1/z} = k$

⇒ $a^{1/x} = k$

⇒ $(a^{1/x})^x = k^x$...(Taking power of x on both sides.)

⇒ $a^{1/x \times x} = k^x$

⇒ $a = k^x$

Similarly $b = k^y$

And $c = k^z$

It is given that a,b,c are in G.P.

⇒ $b^2 = ac$

Substituting values of a,b,c calculated above, we get:

⇒ $(k^y)^2 = k^x k^z$

⇒ $k^{2y} = k^{x + z}$

Comparing the powers we get,

2y = x + z

Which is the required condition for x,y,z to be in A.P.

Hence, proved that x,y,z, are in A.P.

Q. 6. If a, b, c are in AP and x, y, z are in GP then prove that the value of x^{b-c}. y^{c-a}. z^{a-b} is 1.

Answer : To prove: $x^{b-c}.\ y^{c-a}.\ z^{a-b} = 1$....(i)

It is given that a,b,c are in A.P.

⇒ 2b = a + c...(ii)

And x,y,z, are in G.P.

⇒ $y^2 = xz$

⇒ $x = y^2/z$

Substitute this value of x in equation (i),we get

L.H.S =

⇒ $\left(\frac{y^2}{z}\right)^{b-c} \times y^{c-a} \times z^{a-b}$

⇒ $y^{2(b-c)+c-a}.z^{a-b-(b-c)}$

⇒ $y^{2b-2c+c-a}.z^{a+c-b-b}$

⇒ $y^{2b-c-a}.z^{a+c-2b}$

⇒ $y^0.z^0$...(Using equation (i))

= 1 = R.H.S

Hence, proved that . If a, b, c are in AP and x, y, z are in GP then $x^{b-c}.\ y^{c-a}.\ z^{a-b} = 1$

Q. 7. Prove that $\left(1-\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+\frac{1}{3^4}....\infty\right)=\frac{3}{4}$

Answer : It is Infinite Geometric Series.

Here, a = 1,

$$r = \frac{\frac{-1}{3}}{1} = \frac{-1}{3}$$

Formula used: Sum of an infinite Geometric series $= \frac{a}{1-r}$

$$\therefore \text{Sum} = \frac{1}{1-\frac{-1}{3}} = \frac{1 \times 3}{3+1} = \frac{3}{4} = \text{R.H.S.}$$

Hence, Proved that $\left(1-\frac{1}{3}+\frac{1}{3^2}-\frac{1}{3^3}+\frac{1}{3^4}....\infty\right)=\frac{3}{4}$

Q. 8. Express $0.\overline{123}$ as a rational number.

Answer : Let, x = 0.123123123....

⇒ x = 0.123 + 0.000123 + 0.000000123 + …∞

⇒ x = 123(0.001 + 0.000001 + 0.000000001 + …∞)

⇒ x = 123($\frac{1}{10^3} + \frac{1}{10^6} + \frac{1}{10^9} + \frac{1}{10^{12}}$ + …∞)

This is an infinite geometric series.

Here, a $= \frac{1}{10^3}$ and r = $\frac{1}{10^3}$

$$\therefore \text{Sum} = \frac{a}{1-r} = \frac{\frac{1}{10^3}}{1-\frac{1}{10^3}} = \frac{1 \times 1000}{999 \times 1000} = \frac{1}{999}$$

⇒ $x = 123 \times \frac{1}{999} = \frac{123}{999}$

Ans : $0.\overline{123} = \frac{123}{999}$

Q. 9. Express $0.\overline{6}$ as a rational number.

Answer : Let ,x = 0.6666…

⇒ x = 0.6 + 0.06 + 0.006 + …

⇒ x = 6(0.1 + 0.01 + 0.001 + 0.0001 + …∞)

⇒ x = 6($\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + \frac{1}{10000} + \ldots\infty$)

This is an infinite geometric series.

Here,a = 1/10 and r = 1/10

$$\therefore \text{ Sum } = \frac{a}{1-r} = \frac{\frac{1}{10}}{1-\frac{1}{10}} = \frac{1 \times 10}{9 \times 10} = \frac{1}{9}$$

$$\therefore x = 6 \times \frac{1}{9} = \frac{6}{9} = \frac{2}{3}$$

Ans: $0.\overline{6} = \frac{2}{3}$

Q. 10. Express $0.\overline{68}$ as a rational number.

Answer : Let, x = 0.68686868…

⇒ x = 0.68 + 0.0068 + 0.000068 + …∞

⇒ x = 68(0.01 + 0.0001 + …∞)

⇒ x = 68($\frac{1}{10^2} + \frac{1}{10^4} + \frac{1}{10^6} + \frac{1}{10^8} + \ldots\infty$)

Here, a $= \frac{1}{10^2}$ and r = $\frac{1}{10^2}$

$$\therefore \text{ Sum } = \frac{a}{1-r} = \frac{\frac{1}{10^2}}{1-\frac{1}{10^2}} = \frac{1 \times 100}{99 \times 100} = \frac{1}{99}$$

$\Rightarrow x = \left(68 \times \frac{1}{99}\right) = \frac{68}{999} = \frac{68}{999}$

Ans: $0.\overline{68} = \frac{68}{999}$

Q. 11. The second term of a GP is 24 and its fifth term is 81. Find the sum of its first five terms.

Answer : Given: second term of a GP is 24 and its fifth term is 81.

To find: sum of first five terms of the G.P.

$ar = 24$ & $ar^4 = 81$

dividing these two terms we get:

$\Rightarrow \frac{ar^4}{ar} = \frac{81}{24}$

$\Rightarrow r^3 = \frac{27}{8}$

Taking cube root on both the sides we get,

$\Rightarrow r = \frac{3}{2}$

Substituting this value of r in ar = 24 we get

a = 24/(3/2) = (24 × 2)/3 = 16

∴ Sum of first Five terms of a G.P. = $a(r^n - 1)/(r - 1)$

$= 16 \times \frac{(\frac{3}{2})^5 - 1}{\frac{3}{2} - 1} = 16 \times \frac{\frac{243}{32} - 1}{\frac{3}{2} - 1}$

$= 16 \times \frac{242 \times 2}{32 \times 1} = 242$

Ans: 242

Q. 12. The ratio of the sum of first three terms is to that of first six terms of a GP is 125 : 152. Find the common ratio.

Answer : The first three terms of a G.P. are:a,ar,ar^2

The first six terms of a G.P. are:a,ar,ar^2,ar^3,ar^4,ar^5

It is given that the ratio of the sum of first three terms is to that of first six terms of a GP is 125 : 152.

⇒ $a + ar + ar^2 = 125x$ & $a + ar + ar^2 + ar^3 + ar^4 + ar^5 = 152x$

⇒ $a + ar + ar^2 + r^3(a + ar + ar^2) = 152x$

⇒ $125x + r^3(125x) = 152x$

⇒ $r^3(125x) = 152x - 125x = 27x$

⇒ $r^3 = \frac{27}{125} = \left(\frac{3}{5}\right)^3$

⇒ r = 3/5

Ans: common ratio = $\frac{3}{5}$

Q. 13. The sum of first three terms of a GP is $\frac{39}{10}$ and their product is 1. Find the common ratio and these three terms.

Answer : Let the first three terms of G.P. be $\frac{a}{r}, a, ar$

It is given that $\frac{a}{r} \times a \times ar = 1$

⇒ $a^3 = 1$

⇒ a = 1

And

$\frac{a}{r} + a + ar = \frac{39}{10}$

⇒ $a\left(\frac{1}{r} + 1 + r\right) = \frac{39}{10}$

$\Rightarrow \left(\frac{1}{r} + 1 + r\right) = \frac{39}{10}$ ...(a = 1)

$\Rightarrow \left(\frac{1}{r} + r\right) = \frac{39}{10} - 1 = \frac{29}{10}$

$\Rightarrow 10(1 + r^2) = 29r$

$\Rightarrow 10r^2 - 29r + 10 = 0$

$\Rightarrow 10r^2 - 25r - 4r + 10 = 0$

$\Rightarrow 5r(2r - 5) - 2(2r - 5) = 0$

$\Rightarrow (2r - 5)(5r - 2) = 0$

$\Rightarrow r = \frac{5}{2}, \frac{2}{5}$

Therefore the first three terms are:

i) If $r = \frac{5}{2}$ then

$\frac{2}{5}, 1, \frac{5}{2}$

ii) If $r = \frac{2}{5}$ then

$\frac{5}{2}, 1, \frac{2}{5}$

Ans: Common ratio $r = \frac{5}{2}, \frac{2}{5}$ and the first three terms are:

i) if $r = \frac{5}{2}$ then

$\frac{2}{5}, 1, \frac{5}{2}$

ii) If $r = \frac{2}{5}$ then

$\frac{5}{2}, 1, \frac{2}{5}$