Exercise 5(D)

Factorize:

1. a^3 - 27

Solution:

We have, $a^3 - 27$
$= a^3 - 3^3$
$= (a - 3) [a^2 + (a \times 3) + 3^2]$ [As, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]
$= (a - 3)(a^2 + 3a + 9)$

2. 1 - $8a^3$

Solution:

We have, $1 - 8a^3$
$= 1^3 - (2a)^3$
$= (1 - 2a) [1^2 + (1 \times 2a) + (2a)^2]$
$= (1 - 2a)(1 + 2a + 4a^2)$ [As, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

3. 64 - a^3b^3

Solution:

We have, $64 - a^3b^3$
$= 4^3 - (ab)^3$
$= (4 - ab) [4^2 + (4 \times ab) + (ab)^2]$
$= (4 - ab)(16 + 4ab + a^2b^2)$ [As, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

4. $a^6 + 27b^3$

Solution:

We have, $a^6 + 27b^3$
$= (a^2)^3 + (3b)^3$
$= (a^2 + 3b) [(a^2)^2 - (a^2 \times 3b) + (3b)^2]$
$= (a^2 + 3b)(a^4 - 3a^2b + 9b^2)$ [As, $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$]

5. $3x^7y - 81x^4y^4$

Solution:

We have, $3x^7y - 81x^4y^4$
$= 3xy (x^6 - 27x^3y^3)$
$= 3xy [(x^2)^3 - (3xy)^3]$
$= 3xy (x^2 - 3xy) [(x^2)^2 + (x^2 \times 3xy) + (3xy)^2]$ [As, $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]
$= 3xy (x^2 - 3xy)(x^4 + 3x^3y + 9x^2y^2)$
$= 3xy. x(x - 3y). x^2(x^2 + 3xy + 9y^2)$ [Taking common from terms]
$= 3x^4y (x - 3y)(x^2 + 3xy + 9y^2)$

6. $a^3 – 27/a^3$
Solution:

We have, $a^3 – 27/a^3$
$= a^3 – (3/a)^3$
$= (a – 3/a) [a^2 + a \times 3/a + (3/a)^2]$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]
$= (a – 3/a) (a^2 + 3 + 9/a^2)$

7. $a^3 + 0.064$
Solution:

We have, $a^3 + 0.064$
$= a^3 + (0.4)^3$
$= (a + 0.4) [a^2 – (a \times 0.4) + 0.4^2]$
$= (a + 0.4) (a^2 – 0.4a + 0.16)$ [As, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$]

8. $a^4 - 343a$
Solution:

We have, $a^4 - 343a$
$= a (a^3 - 343)$
$= a (a^3 - 7^3)$
$= a (a - 7) [a^2 + (a \times 7) + 7^2]$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]
$= a (a - 7) (a^2 + 7a + 49)$

9. $(x - y)^3 - 8x^3$
Solution:

We have, $(x - y)^3 - 8x^3$
$= (x - y)^3 - (2x)^3$
$= (x - y - 2x) [(x - y)^2 + 2x(x - y) + (2x)^2]$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]
$= (-x – y) [x^2 + y^2 - 2xy + 2x^2 - 2xy + 4x^2]$
$= -(x + y) [7x^2 - 4xy + y^2]$

10. $8a^3/27 – b^3/8$
Solution:

We have, $8a^3/27 – b^3/8$
$= (2a/3)^3 – (b/2)^3$
$= (2a/3 – b/2) [(2a/3)^2 + (2a/3 \times b/2) + (b/2)^2]$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]
$= (2a/3 – b/2) (4a^2/9 + ab/3 + b^2/4)$

11. $a^6 - b^6$
Solution:

We have, $a^6 - b^6$

$= (a^3)^2 – (b^3)^2$
$= (a^3 + b^3) (a^3 - b^3)$ [As $x^2 – y^2 = (x + y) (x - y)$]
Now,
$= [(a + b) (a^2 – ab + b^2)] [(a – b) (a^2 + ab + b^2)]$ [Using identities]
$= (a + b) (a - b) (a^2 – ab + b^2) (a^2 + ab + b^2)$

12. $a^6 - 7a^3 - 8$
Solution:

We have, $a^6 - 7a^3 – 8$
By splitting the middle term,
$= a^6 - 8a^3 + a^3 – 8$
$= a^3(a^3 - 8) + 1(a^3 - 8)$
$= (a^3 + 1) (a^3 - 8)$
We know that,
$a^3 – b^3 = (a - b) (a^2 + ab + b^2)$ … (1)
$a^3 + b^3 = (a + b) (a^2 - ab + b^2)$ … (2)
Now,
$(a^3 + 1) (a^3 - 8)$
$= [(a + 1) (a^2 – a + 1)] [(a - 2)(a^2 + 2a + 4)]$ … [Using (1) and (2)]
$= (a + 1) (a - 2) (a^2 + 2a + 4) (a^2 – a + 1)$

13. $a^3 - 27b^3 + 2a^2b - 6ab^2$
Solution:

We have, $a^3 - 27b^3 + 2a^2b - 6ab^2$
$= [a^3 – (3b)^3] + 2ab(a – 3b)$
$= (a – 3b) (a^2 + 3ab + 9b^2) + 2ab(a – 3b)$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]
Now, taking (a – 3b) as common
$= (a – 3b) [(a^2 + 3ab + 9b^2) + 2ab]$
$= (a – 3b) (a^2 + 5ab + 9b^2)$

14. $8a^3 - b^3 - 4ax + 2bx$
Solution:

We have, $8a^3 - b^3 - 4ax + 2bx$
$= (2a)^3 - b^3 – 2x(2a - b)$
$= (2a - b) [(2a)^2 – 2ab + b^2] – 2x(2a - b)$
Taking (2a - b) as common,
$= (2a - b) [(4a^2 + 2ab + b^2) – 2x]$
$= (2a – b) (4a^2 + 2ab + b^2 – 2x)$

15. $a - b - a^3 + b^3$
Solution:

We have, $a - b - a^3 + b^3$
$= (a - b) – (a^3 – b^3)$
$= (a - b) – [(a - b) (a^2 + ab + b^2)]$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]
Now, taking (a - b) as common
$= (a - b) [1 - (a^2 + ab + b^2)]$
$= (a - b) (1 - a^2 - ab - b^2)$

16. $2x^3 + 54y^3 - 4x - 12y$
Solution:

We have, $2x^3 + 54y^3 - 4x - 12y$
$= 2(x^3 + 27y^3 - 2x - 6y)$
Now,
$= 2 \{[(x)^3 + (3y)^3] - 2(x + 3y)\}$
$= 2 \{[(x + 3y) (x^2 - 3xy + 9y^2)] - 2(x + 3y)\}$ [As, $a^3 + b^3 = (a + b) (a^2 - ab + b^2)$]
$= 2 (x + 3y) (x^2 - 3xy + 9y^2 - 2)$

17. $1029 - 3x^3$
Solution:

We have, $1029 - 3x^3$
$= 3(343 - x^3)$
$= 3(7^3 - x^3)$
$= 3(7 - x) (7^2 + 7x + x^2)$ [As, $a^3 – b^3 = (a - b) (a^2 + ab + b^2)$]
$= 3(7 - x) (49 + 7x + x^2)$

18. Show that:
(i) $13^3 - 5^3$ is divisible by 8
(ii) $35^3 + 27^3$ is divisible by 62
Solution:

(i) We have, $(13^3 - 5^3)$
Now, using identity $(a^3 - b^3) = (a - b) (a^2 + ab + b^2)$
$= (13 - 5) (13^2 + 13 \times 5 + 5^2)$
$= 8 \times (169 + 65 + 25)$

Hence, the number is divisible by 8.

(ii) $(35^3 + 27^3)$
Now, using identity $(a^3 + b^3) = (a + b) (a^2 - ab + b^2)$
$= (35 + 27) (35^2 + 35 \times 27 + 27^2)$
$= 62 \times (35^2 + 35 \times 27 + 27^2)$

Hence, the number is divisible by 62.

19. Evaluate:

$$\frac{5.67 \times 5.67 \times 5.67 + 4.33 \times 4.33 \times 4.33}{5.67 \times 5.67 - 5.67 \times 4.33 + 4.33 \times 4.33}$$

Solution:

Let a = 5.67 and b = 4.33

Then,

$$\frac{5.67 \times 5.67 \times 5.67 + 4.33 \times 4.33 \times 4.33}{5.67 \times 5.67 - 5.67 \times 4.33 + 4.33 \times 4.33}$$

$$= \frac{a \times a \times a + b \times b \times b}{a \times a - a \times b + b \times b}$$

$$= \frac{a^3 + b^3}{a^2 - ab + b^2}$$

$$= \frac{(a + b)(a^2 - ab + b^2)}{a^2 - ab + b^2}$$

= a + b

= 5.67 + 4.33

= 10