EXERCISE 19.1

If the n^{th} term of a sequence is given by $a_n = n^2 - n+1$, write down its first five terms.

Solution:

Given:

$a_n = n^2 - n+1$

By using the values n = 1, 2, 3, 4, 5 we can find the first five terms.

When n = 1:

$a_1 = (1)^2 - 1 + 1$
$= 1 - 1 + 1$
$= 1$

When n = 2:

$a_2 = (2)^2 - 2 + 1$
$= 4 - 2 + 1$
$= 3$

When n = 3:

$a_3 = (3)^2 - 3 + 1$
$= 9 - 3 + 1$
$= 7$

When n = 4:

$a_4 = (4)^2 - 4 + 1$
$= 16 - 4 + 1$
$= 13$

When n = 5:

$a_5 = (5)^2 - 5 + 1$
$= 25 - 5 + 1$
$= 21$

∴ First five terms of the sequence are 1, 3, 7, 13, 21.

1. A sequence is defined by $a_n = n^3 - 6n^2 + 11n - 6$, $n \in N$. Show that the first three terms of the sequence are zero and all other terms are positive.

Solution:

Given:

$a_n = n^3 - 6n^2 + 11n - 6$, $n \in N$

By using the values n = 1, 2, 3 we can find the first three terms.
When n = 1:
$a_1 = (1)^3 – 6(1)^2 + 11(1) – 6$
$= 1 – 6 + 11 – 6$
$= 12 – 12$
$= 0$

When n = 2:
$a_2 = (2)^3 – 6(2)^2 + 11(2) – 6$
$= 8 – 6(4) + 22 – 6$
$= 8 – 24 + 22 – 6$
$= 30 – 30$
$= 0$

When n = 3:
$a_3 = (3)^3 – 6(3)^2 + 11(3) – 6$
$= 27 – 6(9) + 33 – 6$
$= 27 – 54 + 33 – 6$
$= 60 – 60$
$= 0$
This shows that the first three terms of the sequence is zero.
Now, let's check for when n = n:
$a_n = n^3 – 6n^2 + 11n – 6$
$= n^3 – 6n^2 + 11n – 6 – n + n – 2 + 2$
$= n^3 – 6n^2 + 12n – 8 – n + 2$
$= (n)^3 – 3×2n(n – 2) – (2)^3 – n + 2$
By using the formula, $\{(a – b)^3 = (a)^3 – (b)^3 – 3ab(a – b)\}$
$a_n = (n – 2)^3 – (n – 2)$
Here, n – 2 will always be positive for n > 3
∴ a_n is always positive for n > 3

2. Find the first four terms of the sequence defined by $a_1 = 3$ and $a_n = 3a_{n–1} + 2$, for all n > 1.
Solution:
Given:
$a_1 = 3$ and $a_n = 3a_{n–1} + 2$, for all n > 1
By using the values n = 1, 2, 3, 4 we can find the first four terms.
When n = 1:
$a_1 = 3$

When n = 2:
$a_2 = 3a_{2-1} + 2$
$= 3a_1 + 2$
$= 3(3) + 2$
$= 9 + 2$
$= 11$

When n = 3:
$a_3 = 3a_{3-1} + 2$
$= 3a_2 + 2$
$= 3(11) + 2$
$= 33 + 2$
$= 35$

When n = 4:
$a_4 = 3a_{4-1} + 2$
$= 3a_3 + 2$
$= 3(35) + 2$
$= 105 + 2$
$= 107$
∴ First four terms of sequence are 3, 11, 35, 107.

3. Write the first five terms in each of the following sequences:
(i) $a_1 = 1, a_n = a_{n-1} + 2, n > 1$
(ii) $a_1 = 1 = a_2, a_n = a_{n-1} + a_{n-2}, n > 2$
(iii) $a_1 = a_2 = 2, a_n = a_{n-1} - 1, n > 2$
Solution:
(i) $a_1 = 1, a_n = a_{n-1} + 2, n > 1$
By using the values n = 1, 2, 3, 4, 5 we can find the first five terms.
Given:
$a_1 = 1$

When n = 2:
$a_2 = a_{2-1} + 2$
$= a_1 + 2$
$= 1 + 2$
$= 3$

When n = 3:

$a_3 = a_{3-1} + 2$
$= a_2 + 2$
$= 3 + 2$
$= 5$

When n = 4:
$a_4 = a_{4-1} + 2$
$= a_3 + 2$
$= 5 + 2$
$= 7$

When n = 5:
$a_5 = a_{5-1} + 2$
$= a_4 + 2$
$= 7 + 2$
$= 9$
∴ First five terms of the sequence are 1, 3, 5, 7, 9.

(ii) $a_1 = 1 = a_2$, $a_n = a_{n-1} + a_{n-2}$, $n > 2$
By using the values n = 1, 2, 3, 4, 5 we can find the first five terms.
Given:
$a_1 = 1$
$a_2 = 1$

When n = 3:
$a_3 = a_{3-1} + a_{3-2}$
$= a_2 + a_1$
$= 1 + 1$
$= 2$

When n = 4:
$a_4 = a_{4-1} + a_{4-2}$
$= a_3 + a_2$
$= 2 + 1$
$= 3$

When n = 5:
$a_5 = a_{5-1} + a_{5-2}$
$= a_4 + a_3$

= 3 + 2
= 5
∴ First five terms of the sequence are 1, 1, 2, 3, 5.

(iii) $a_1 = a_2 = 2$, $a_n = a_{n-1} - 1$, $n > 2$
By using the values n = 1, 2, 3, 4, 5 we can find the first five terms.
Given:
$a_1 = 2$
$a_2 = 2$

When n = 3:
$a_3 = a_{3-1} - 1$
$= a_2 - 1$
$= 2 - 1$
$= 1$

When n = 4:
$a_4 = a_{4-1} - 1$
$= a_3 - 1$
$= 1 - 1$
$= 0$

When n = 5:
$a_5 = a_{5-1} - 1$
$= a_4 - 1$
$= 0 - 1$
$= -1$
∴ First five terms of the sequence are 2, 2, 1, 0, -1.

4. The Fibonacci sequence is defined by $a_1 = 1 = a_2$, $a_n = a_{n-1} + a_{n-2}$ for $n > 2$. Find $(a_{n+1})/a_n$ for n = 1, 2, 3, 4, 5.
Solution:
Given:
$a_1 = 1$
$a_2 = 1$
$a_n = a_{n-1} + a_{n-2}$
When n = 1:
$(a_{n+1})/a_n = (a_{1+1})/a_1$
$= a_2/a_1$

$= 1/1$
$= 1$
$a_3 = a_{3-1} + a_{3-2}$
$= a_2 + a_1$
$= 1 + 1$
$= 2$

When $n = 2$:
$(a_{n+1})/a_n = (a_{2+1})/a_2$
$= a_3/a_2$
$= 2/1$
$= 2$
$a_4 = a_{4-1} + a_{4-2}$
$= a_3 + a_2$
$= 2 + 1$
$= 3$

When $n = 3$:
$(a_{n+1})/a_n = (a_{3+1})/a_3$
$= a_4/a_3$
$= 3/2$
$a_5 = a_{5-1} + a_{5-2}$
$= a_4 + a_3$
$= 3 + 2$
$= 5$

When $n = 4$:
$(a_{n+1})/a_n = (a_{4+1})/a_4$
$= a_5/a_4$
$= 5/3$
$a_6 = a_{6-1} + a_{6-2}$
$= a_5 + a_4$
$= 5 + 3$
$= 8$

When $n = 5$:
$(a_{n+1})/a_n = (a_{5+1})/a_5$
$= a_6/a_5 = 8/5$
∴ Value of $(a_{n+1})/a_n$ when $n = 1, 2, 3, 4, 5$ are $1, 2, 3/2, 5/3, 8/5$