

Exercise 5.3

1. Evaluate the following:

(i) $\sin 20^\circ / \cos 70^\circ$

(ii) $\cos 19^\circ / \sin 71^\circ$

(iii) $\sin 21^\circ / \cos 69^\circ$

(iv) $\tan 10^\circ / \cot 80^\circ$

(v) $\sec 11^\circ / \operatorname{cosec} 79^\circ$

Solution:

(i) We have,
 $\sin 20^\circ / \cos 70^\circ = \sin (90^\circ - 70^\circ) / \cos 70^\circ = \cos 70^\circ / \cos 70^\circ = 1$ [$\because \sin (90 - \theta) = \cos \theta$]

(ii) We have,
 $\cos 19^\circ / \sin 71^\circ = \cos (90^\circ - 71^\circ) / \sin 71^\circ = \sin 71^\circ / \sin 71^\circ = 1$ [$\because \cos (90 - \theta) = \sin \theta$]

(iii) We have,
 $\sin 21^\circ / \cos 69^\circ = \sin (90^\circ - 69^\circ) / \cos 69^\circ = \cos 69^\circ / \cos 69^\circ = 1$ [$\because \sin (90 - \theta) = \cos \theta$]

(iv) We have,
 $\tan 10^\circ / \cot 80^\circ = \tan (90^\circ - 10^\circ) / \cot 80^\circ = \cot 80^\circ / \cot 80^\circ = 1$ [$\because \tan (90 - \theta) = \cot \theta$]

(v) We have,
 $\sec 11^\circ / \operatorname{cosec} 79^\circ = \sec (90^\circ - 79^\circ) / \operatorname{cosec} 79^\circ = \operatorname{cosec} 79^\circ / \operatorname{cosec} 79^\circ = 1$ [$\because \sec (90 - \theta) = \operatorname{cosec} \theta$]

2. Evaluate the following:

(i) $\left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2$

Solution:

We have, [$\because \sin (90 - \theta) = \cos \theta$ and $\cos (90 - \theta) = \sin \theta$]

$$\begin{aligned} & \left(\frac{\sin 49^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\cos 41^\circ}{\sin 49^\circ}\right)^2 \\ &= \left(\frac{\sin(90^\circ - 41^\circ)}{\cos 41^\circ}\right)^2 + \left(\frac{\cos(90^\circ - 49^\circ)}{\sin 49^\circ}\right)^2 \\ &= \left(\frac{\cos 41^\circ}{\cos 41^\circ}\right)^2 + \left(\frac{\sin 49^\circ}{\sin 49^\circ}\right)^2 \end{aligned}$$

$$= 1^2 + 1^2 = 1 + 1$$

$$= 2$$

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(ii) $\cos 48^\circ - \sin 42^\circ$

Solution:

We know that, $\cos(90^\circ - \theta) = \sin \theta$.

So,

$$\cos 48^\circ - \sin 42^\circ = \cos(90^\circ - 42^\circ) - \sin 42^\circ = \sin 42^\circ - \sin 42^\circ = 0$$

Thus the value of $\cos 48^\circ - \sin 42^\circ$ is 0.

(iii) $\frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2} \left(\frac{\cos 35^\circ}{\sin 55^\circ} \right)$

Solution:

We have,

$$[\because \cot(90 - \theta) = \tan \theta \text{ and } \cos(90 - \theta) = \sin \theta]$$

$$\frac{\cot 40^\circ}{\tan 50^\circ} - \frac{1}{2} \left(\frac{\cos 35^\circ}{\sin 55^\circ} \right)$$

$$= \frac{\cot(90^\circ - 50^\circ)}{\tan 50^\circ} - \frac{1}{2} \left(\frac{\cos(90^\circ - 55^\circ)}{\sin 55^\circ} \right)$$

$$= \frac{\tan 50^\circ}{\tan 50^\circ} - \frac{1}{2} \left(\frac{\sin 55^\circ}{\sin 55^\circ} \right)$$

$$= 1 - \frac{1}{2}(1)$$

$$= \frac{1}{2}$$

(iv) $\left(\frac{\sin 27^\circ}{\cos 63^\circ} \right)^2 - \left(\frac{\cos 63^\circ}{\sin 27^\circ} \right)^2$

Solution:

We have,

$$[\because \sin(90 - \theta) = \cos \theta \text{ and } \cos(90 - \theta) = \sin \theta]$$

$$\left(\frac{\sin 27^\circ}{\cos 63^\circ} \right)^2 - \left(\frac{\cos 63^\circ}{\sin 27^\circ} \right)^2$$

$$= \left(\frac{\sin(90^\circ - 63^\circ)}{\cos 63^\circ} \right)^2 - \left(\frac{\cos(90^\circ - 27^\circ)}{\sin 27^\circ} \right)^2$$

$$= \left(\frac{\cos 63^\circ}{\cos 63^\circ} \right)^2 - \left(\frac{\sin 27^\circ}{\sin 27^\circ} \right)^2$$

$$= 1 - 1$$

$$= 0$$

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$$(v) \frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ} - 1$$

Solution:

We have,

$$[\because \cot (90 - \theta) = \tan \theta \text{ and } \tan (90 - \theta) = \cot \theta]$$

$$\frac{\tan 35^\circ}{\cot 55^\circ} + \frac{\cot 78^\circ}{\tan 12^\circ} - 1$$

$$= \tan (90^\circ - 55^\circ) / \cot 55^\circ + \cot (90^\circ - 12^\circ) / \tan 12^\circ - 1$$

$$= \cot 55^\circ / \cot 55^\circ + \tan 12^\circ / \tan 12^\circ - 1$$

$$= 1 + 1 - 1$$

$$= 1$$

$$(vi) \frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ} + \frac{\sin 59^\circ}{\cos 31^\circ}$$

Solution:

We have,

$$[\because \sin (90 - \theta) = \cos \theta \text{ and } \sec (90 - \theta) = \operatorname{cosec} \theta]$$

$$\frac{\sec 70^\circ}{\operatorname{cosec} 20^\circ} + \frac{\sin 59^\circ}{\cos 31^\circ}$$

$$= \sec (90^\circ - 20^\circ) / \operatorname{cosec} 20^\circ + \sin (90^\circ - 31^\circ) / \cos 31^\circ$$

$$= \operatorname{cosec} 20^\circ / \operatorname{cosec} 20^\circ + \cos 12^\circ / \cos 12^\circ$$

$$= 1 + 1$$

$$= 2$$

$$(vii) \operatorname{cosec} 31^\circ - \sec 59^\circ$$

Solution:

We have,

$$\operatorname{cosec} 31^\circ - \sec 59^\circ$$

$$\text{Since, } \operatorname{cosec} (90 - \theta) = \cos \theta$$

So,

$$\operatorname{cosec} 31^\circ - \sec 59^\circ = \operatorname{cosec} (90^\circ - 59^\circ) - \sec 59^\circ = \sec 59^\circ - \sec 59^\circ = 0$$

Thus,

$$\operatorname{cosec} 31^\circ - \sec 59^\circ = 0$$

$$(viii) (\sin 72^\circ + \cos 18^\circ) (\sin 72^\circ - \cos 18^\circ)$$

Solution:

We know that,

$$\sin (90 - \theta) = \cos \theta$$

So, the given can be expressed as

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$$\begin{aligned} & (\sin 72^\circ + \cos 18^\circ) (\sin (90 - 18)^\circ - \cos 18^\circ) \\ &= (\sin 72^\circ + \cos 18^\circ) (\cos 18^\circ - \cos 18^\circ) \\ &= (\sin 72^\circ + \cos 18^\circ) \times 0 \\ &= 0 \end{aligned}$$

(ix) $\sin 35^\circ \sin 55^\circ - \cos 35^\circ \cos 55^\circ$

Solution:

We know that,
 $\sin (90 - \theta) = \cos \theta$

So, the given can be expressed as
 $\sin (90 - 55)^\circ \sin (90 - 35)^\circ - \cos 35^\circ \cos 55^\circ$
 $= \cos 55^\circ \cos 35^\circ - \cos 35^\circ \cos 55^\circ$
 $= 0$

(x) $\tan 48^\circ \tan 23^\circ \tan 42^\circ \tan 67^\circ$

Solution:

We know that,
 $\tan (90 - \theta) = \cot \theta$

So, the given can be expressed as
 $\tan (90 - 42)^\circ \tan (90 - 67)^\circ \tan 42^\circ \tan 67^\circ$
 $= \cot 42^\circ \cot 67^\circ \tan 42^\circ \tan 67^\circ$
 $= (\cot 42^\circ \tan 42^\circ)(\cot 67^\circ \tan 67^\circ)$
 $= 1 \times 1$
 $= 1$

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$$[\because \tan \theta \times \cot \theta = 1]$$

(xi) $\sec 50^\circ \sin 40^\circ + \cos 40^\circ \operatorname{cosec} 50^\circ$

Solution:

We know that,
 $\sin (90 - \theta) = \cos \theta$ and $\cos (90 - \theta) = \sin \theta$

So, the given can be expressed as
 $\sec 50^\circ \sin (90 - 50)^\circ + \cos (90 - 50)^\circ \operatorname{cosec} 50^\circ$
 $= \sec 50^\circ \cos 50^\circ + \sin 50^\circ \operatorname{cosec} 50^\circ$
 $= 1 + 1$
 $= 2$

$$[\because \sin \theta \times \operatorname{cosec} \theta = 1 \text{ and } \cos \theta \times \sec \theta = 1]$$

3. Express each one of the following in terms of trigonometric ratios of angles lying between 0° and 45°

(i) $\sin 59^\circ + \cos 56^\circ$

(ii) $\tan 65^\circ + \cot 49^\circ$

(iii) $\sec 76^\circ + \operatorname{cosec} 52^\circ$

(iv) $\cos 78^\circ + \sec 78^\circ$

(v) $\operatorname{cosec} 54^\circ + \sin 72^\circ$

(vi) $\cot 85^\circ + \cos 75^\circ$

(vii) $\sin 67^\circ + \cos 75^\circ$

Solution:

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Using the below trigonometric ratios of complementary angles, we find the required

$$\sin(90 - \theta) = \cos \theta \quad \operatorname{cosec}(90 - \theta) = \sec \theta$$

$$\cos(90 - \theta) = \sin \theta \quad \sec(90 - \theta) = \operatorname{cosec} \theta$$

$$\tan(90 - \theta) = \cot \theta \quad \cot(90 - \theta) = \tan \theta$$

- (i) $\sin 59^\circ + \cos 56^\circ = \sin(90 - 31)^\circ + \cos(90 - 34)^\circ = \cos 31^\circ + \sin 34^\circ$
(ii) $\tan 65^\circ + \cot 49^\circ = \tan(90 - 25)^\circ + \cot(90 - 41)^\circ = \cot 25^\circ + \tan 41^\circ$
(iii) $\sec 76^\circ + \operatorname{cosec} 52^\circ = \sec(90 - 14)^\circ + \operatorname{cosec}(90 - 38)^\circ = \operatorname{cosec} 14^\circ + \sec 38^\circ$
(iv) $\cos 78^\circ + \sec 78^\circ = \cos(90 - 12)^\circ + \sec(90 - 12)^\circ = \sin 12^\circ + \operatorname{cosec} 12^\circ$
(v) $\operatorname{cosec} 54^\circ + \sin 72^\circ = \operatorname{cosec}(90 - 36)^\circ + \sin(90 - 18)^\circ = \sec 36^\circ + \cos 18^\circ$
(vi) $\cot 85^\circ + \cos 75^\circ = \cot(90 - 5)^\circ + \cos(90 - 15)^\circ = \tan 5^\circ + \sin 15^\circ$

4. Express $\cos 75^\circ + \cot 75^\circ$ in terms of angles between 0° and 30° .

Solution:

Given,

$$\cos 75^\circ + \cot 75^\circ$$

Since, $\cos(90 - \theta) = \sin \theta$ and $\cot(90 - \theta) = \tan \theta$

$$\cos 75^\circ + \cot 75^\circ = \cos(90 - 15)^\circ + \cot(90 - 15)^\circ = \sin 15^\circ + \tan 15^\circ$$

Hence, $\cos 75^\circ + \cot 75^\circ$ can be expressed as $\sin 15^\circ + \tan 15^\circ$

5. If $\sin 3A = \cos(A - 26^\circ)$, where $3A$ is an acute angle, find the value of A .

Solution:

Given,

$$\sin 3A = \cos(A - 26^\circ)$$

Using $\cos(90 - \theta) = \sin \theta$, we have

$$\sin 3A = \sin(90^\circ - (A - 26^\circ))$$

Now, comparing both L.H.S and R.H.S

$$3A = 90^\circ - (A - 26^\circ)$$

$$3A + (A - 26^\circ) = 90^\circ$$

$$4A - 26^\circ = 90^\circ$$

$$4A = 116^\circ$$

$$A = 116^\circ/4$$

$$\therefore A = 29^\circ$$

6. If A, B, C are the interior angles of a triangle ABC , prove that

(i) $\tan((C + A)/2) = \cot(B/2)$ (ii) $\sin((B + C)/2) = \cos(A/2)$

Solution:

We know that, in triangle ABC the sum of the angles i.e $A + B + C = 180^\circ$

$$\text{So, } C + A = 180^\circ - B \quad \Rightarrow (C + A)/2 = 90^\circ - B/2 \dots\dots (i)$$

$$\text{And, } B + C = 180^\circ - A \quad \Rightarrow (B + C)/2 = 90^\circ - A/2 \dots\dots (ii)$$

- (i) L.H.S = $\tan((C + A)/2)$
 $\Rightarrow \tan((C + A)/2) = \tan(90^\circ - B/2)$ [From (i)]
 $= \cot(B/2)$ [$\because \tan(90 - \theta) = \cot \theta$]

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$$= \text{R.H.S}$$

- Hence Proved

(ii) L.H.S = $\sin ((B + C)/2)$

$$\Rightarrow \sin ((B + C)/2) = \sin (90^\circ - A/2) \quad [\text{From (ii)}]$$

$$= \cos (A/2)$$

$$= \text{R.H.S}$$

- Hence Proved

7. Prove that:

(i) $\tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ = 1$

(ii) $\sin 48^\circ \sec 48^\circ + \cos 48^\circ \operatorname{cosec} 42^\circ = 2$

(iii) $\frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \operatorname{cosec} 42^\circ = 0$

(iv) $\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ = 2$

Solution:

(i) Taking L.H.S = $\tan 20^\circ \tan 35^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ$

$$= \tan (90^\circ - 70^\circ) \tan (90^\circ - 55^\circ) \tan 45^\circ \tan 55^\circ \tan 70^\circ$$

$$= \cot 70^\circ \cot 55^\circ \tan 45^\circ \tan 55^\circ \tan 70^\circ$$

$$= (\tan 70^\circ \cot 70^\circ)(\tan 55^\circ \cot 55^\circ) \tan 45^\circ$$

$$= 1 \times 1 \times 1 = 1$$

- Hence proved

(ii) Taking L.H.S = $\sin 48^\circ \sec 48^\circ + \cos 48^\circ \operatorname{cosec} 42^\circ$

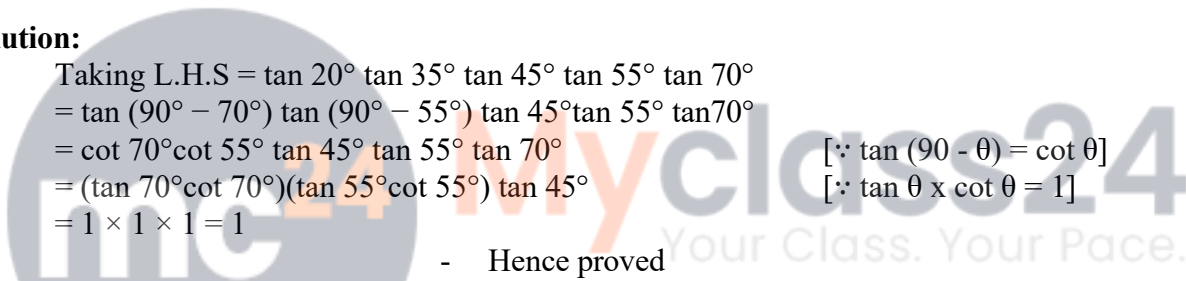
$$= \sin 48^\circ \sec (90^\circ - 48^\circ) + \cos 48^\circ \operatorname{cosec} (90^\circ - 48^\circ)$$

$$= \sin 48^\circ \operatorname{cosec} 48^\circ + \cos 48^\circ \sec 48^\circ$$

$$= 1 + 1 = 2$$

- Hence proved

(iii) Taking the L.H.S,



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$$\begin{aligned} & \frac{\sin 70^\circ}{\cos 20^\circ} + \frac{\operatorname{cosec} 20^\circ}{\sec 70^\circ} - 2 \cos 70^\circ \operatorname{cosec} 20^\circ \\ &= \frac{\sin(90^\circ - 20^\circ)}{\cos 20^\circ} + \frac{\operatorname{cosec}(90^\circ - 70^\circ)}{\sec 70^\circ} - 2 \cos(90^\circ - 20^\circ) \operatorname{cosec} 20^\circ \\ &= \frac{\cos 20^\circ}{\cos 20^\circ} + \frac{\sec 70^\circ}{\sec 70^\circ} - 2 \sin 20^\circ \times \frac{1}{\sin 20^\circ} \left[\begin{array}{l} \sin(90^\circ - \theta) = \cos \theta \\ \operatorname{cosec}(90^\circ - \theta) = \sec \theta \\ \cos(90^\circ - \theta) = \sin \theta \end{array} \right] \\ &= 1 + 1 - 2 \\ &= 2 - 2 \\ &= 0 \end{aligned}$$

- Hence proved

(iv) Taking L.H.S,

$$\frac{\cos 80^\circ}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec} 31^\circ$$

$$= \frac{\cos(90^\circ - 10^\circ)}{\sin 10^\circ} + \cos 59^\circ \operatorname{cosec}(90^\circ - 59^\circ)$$

$$= \frac{\sin 10^\circ}{\sin 10^\circ} + \cos 59^\circ \sec 59^\circ$$

$$= 1 + 1$$

$$= 2$$

- Hence proved

8. Prove the following:

(i) $\sin \theta \sin(90^\circ - \theta) - \cos \theta \cos(90^\circ - \theta) = 0$

Solution:

Taking the L.H.S,

$$\sin \theta \sin(90^\circ - \theta) - \cos \theta \cos(90^\circ - \theta)$$

$$= \sin \theta \cos \theta - \cos \theta \sin \theta$$

$$= 0$$

$$[\because \sin(90^\circ - \theta) = \cos \theta \text{ and } \cos(90^\circ - \theta) = \sin \theta]$$

(ii)

$$\frac{\cos(90^\circ - \theta) \sec(90^\circ - \theta) \tan \theta}{\operatorname{cosec}(90^\circ - \theta) \sin(90^\circ - \theta) \cot(90^\circ - \theta)} + \frac{\tan(90^\circ - \theta)}{\cot \theta} = 2$$

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Solution:

Taking the L.H.S,

$$\begin{aligned} & \frac{\cos(90^\circ-\theta) \sec(90^\circ-\theta) \tan \theta}{\operatorname{cosec}(90^\circ-\theta) \sin(90^\circ-\theta) \cot(90^\circ-\theta)} + \frac{\tan(90^\circ-\theta)}{\cot \theta} \\ &= \frac{\sin \theta \operatorname{cosec} \theta \tan \theta}{\sec \theta \cos \theta \tan \theta} + \frac{\cot \theta}{\cot \theta} \\ &= \frac{1 \times \tan \theta}{1 \times \tan \theta} + 1 = \frac{\tan \theta}{\tan \theta} + 1 \quad [\because \operatorname{cosec} \theta \times \sin \theta = 1 \text{ and } \cos \theta \times \sec \theta = 1] \\ &= 1 + 1 \\ &= 2 = \text{R.H.S} \end{aligned}$$

- Hence Proved

(iii) $\frac{\tan(90^\circ-A) \cot A}{\operatorname{cosec}^2 A} - \cos^2 A = 0$

Solution:

Taking the L.H.S,

$$\begin{aligned} \text{L.H.S.} &= \frac{\tan(90^\circ-A) \cot A}{\operatorname{cosec}^2 A} - \cos^2 A \\ &= \frac{\cot A \cot A}{\operatorname{cosec}^2 A} - \cos^2 A \\ &= \frac{\cot^2 A}{\operatorname{cosec}^2 A} - \cos^2 A = \frac{\frac{\cos^2 A}{\sin^2 \theta}}{\frac{1}{\sin^2 A}} - \cos^2 A \end{aligned}$$

$$\begin{aligned} &= \frac{\cos^2 A \times \sin^2 A}{\sin^2 A \times 1} - \cos^2 A = \cos^2 A - \cos^2 A \\ &= 0 = \text{R.H.S} \end{aligned}$$

- Hence Proved

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$$(iv) \frac{\cos(90^\circ - A) \sin(90^\circ - A)}{\tan(90^\circ - A)} = \sin^2 A$$

Solution:

Taking L.H.S,

[$\because \sin(90 - \theta) = \cos \theta$ and $\cos(90 - \theta) = \sin \theta$]

$$\text{L.H.S.} = \frac{\cos(90^\circ - A) \sin(90^\circ - A)}{\tan(90^\circ - A)}$$

$$= \frac{\sin A \cos A}{\cot A} = \frac{\sin A \cos A}{\frac{\cos A}{\sin A}}$$

$$= \frac{\sin A \cos A \times \sin A}{\cos A} = \sin A \times \sin A$$

$$= \sin^2 A = \text{R.H.S}$$

- Hence Proved

$$(v) \sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 70^\circ \tan 80^\circ \tan 89^\circ = 1$$

Solution:

Taking the L.H.S,

$$= \sin(50^\circ + \theta) - \cos(40^\circ - \theta) + \tan 1^\circ \tan 10^\circ \tan 20^\circ \tan 70^\circ \tan 80^\circ \tan 89^\circ$$

$$= [\sin(90^\circ - (40^\circ - \theta))] - \cos(40^\circ - \theta) + \tan(90 - 89)^\circ \tan(90 - 80)^\circ \tan(90 - 70)^\circ \tan 70^\circ \tan 80^\circ \tan 89^\circ$$

[$\because \sin(90 - \theta) = \cos \theta$]

$$= \cos(40^\circ - \theta) - \cos(40^\circ - \theta) + \cot 89^\circ \cot 80^\circ \cot 70^\circ \tan 70^\circ \tan 80^\circ \tan 89^\circ$$

[$\because \tan(90^\circ - \theta) = \cot \theta$]

$$= 0 + (\cot 89^\circ \times \tan 89^\circ) (\cot 80^\circ \times \tan 80^\circ) (\cot 70^\circ \times \tan 70^\circ)$$

$$= 0 + 1 \times 1 \times 1$$

[$\because \tan \theta \times \cot \theta = 1$]

$$= 1 = \text{R.H.S}$$

- Hence Proved