EXERCISE 23.12

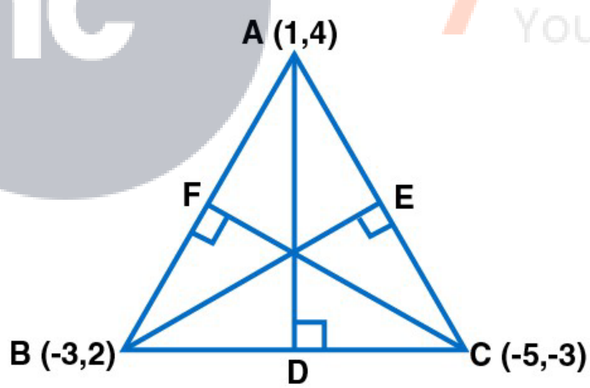
1. Find the equation of a line passing through the point (2, 3) and parallel to the line 3x– 4y + 5 = 0.
Solution:
Given:
The equation is parallel to $3x - 4y + 5 = 0$ and pass through (2, 3)
The equation of the line parallel to $3x - 4y + 5 = 0$ is
$3x - 4y + \lambda = 0$,
Where, λ is a constant.
It passes through (2, 3).
Substitute the values in above equation, we get
$3(2) - 4(3) + \lambda = 0$
$6 - 12 + \lambda = 0$
$\lambda = 6$
Now, substitute the value of $\lambda = 6$ in $3x - 4y + \lambda = 0$, we get
$3x - 4y + 6$
∴ The required line is $3x - 4y + 6 = 0$.

2. Find the equation of a line passing through (3, -2) and perpendicular to the line x – 3y + 5 = 0.
Solution:
Given:
The equation is perpendicular to $x - 3y + 5 = 0$ and passes through (3,-2)
The equation of the line perpendicular to $x - 3y + 5 = 0$ is
$3x + y + \lambda = 0$,
Where, λ is a constant.
It passes through (3, − 2).
Substitute the values in above equation, we get
$3(3) + (-2) + \lambda = 0$
$9 - 2 + \lambda = 0$
$\lambda = -7$
Now, substitute the value of $\lambda = -7$ in $3x + y + \lambda = 0$, we get
$3x + y - 7 = 0$
∴ The required line is $3x + y - 7 = 0$.

3. Find the equation of the perpendicular bisector of the line joining the points (1, 3) and (3, 1).
Solution:

Given:
A (1, 3) and B (3, 1) be the points joining the perpendicular bisector
Let C be the midpoint of AB.
So, coordinates of C = [(1+3)/2, (3+1)/2]
= (2, 2)
Slope of AB = [(1-3) / (3-1)]
= -1
Slope of the perpendicular bisector of AB = 1
Thus, the equation of the perpendicular bisector of AB is given as,
y – 2 = 1(x – 2)
y = x
x – y = 0
∴ The required equation is y = x.

4. Find the equations of the altitudes of a ΔABC whose vertices are A (1, 4), B (-3, 2) and C (-5, -3).
Solution:
Given:
The vertices of ΔABC are A (1, 4), B (− 3, 2) and C (− 5, − 3).
Now let us find the slopes of ΔABC.



Slope of AB = [(2 - 4) / (-3-1)]
= ½

Slope of BC = [(-3 - 2) / (-5+3)]
= 5/2

Slope of CA = [(4 + 3) / (1 + 5)]
= 7/6
Thus, we have:

Slope of CF = -2
Slope of AD = -2/5
Slope of BE = -6/7
Hence,
Equation of CF is:
$y + 3 = -2(x + 5)$
$y + 3 = -2x - 10$
$2x + y + 13 = 0$

Equation of AD is:
$y – 4 = (-2/5) (x - 1)$
$5y – 20 = -2x + 2$
$2x + 5y – 22 = 0$

Equation of BE is:
$y – 2 = (-6/7) (x + 3)$
$7y – 14 = -6x - 18$
$6x + 7y + 4 = 0$
∴ The required equations are $2x + y + 13 = 0$, $2x + 5y – 22 = 0$, $6x + 7y + 4 = 0$.

5. Find the equation of a line which is perpendicular to the line $\sqrt{3}x – y + 5 = 0$ and which cuts off an intercept of 4 units with the negative direction of y-axis.
Solution:
Given:
The equation is perpendicular to $\sqrt{3}x – y + 5 = 0$ equation and cuts off an intercept of 4 units with the negative direction of y-axis.
The line perpendicular to $\sqrt{3}x – y + 5 = 0$ is $x + \sqrt{3}y + \lambda = 0$

It is given that the line $x + \sqrt{3}y + \lambda = 0$ cuts off an intercept of 4 units with the negative direction of the y-axis.
This means that the line passes through (0,-4).
So,
Let us substitute the values in the equation $x + \sqrt{3}y + \lambda = 0$, we get
$0 - \sqrt{3} (4) + \lambda = 0$
$\lambda = 4\sqrt{3}$
Now, substitute the value of λ back, we get
$x + \sqrt{3}y + 4\sqrt{3} = 0$
∴ The required equation of line is $x + \sqrt{3}y + 4\sqrt{3} = 0$.