

EXERCISE 7.9

Factorize each of the following quadratic polynomials by using the method of completing the square:

1. $p^2 + 6p + 8$

Solution:

We have,

$$p^2 + 6p + 8$$

Coefficient of p^2 is unity. So, we add and subtract square of half of coefficient of p .

$$\begin{aligned} p^2 + 6p + 8 &= p^2 + 6p + 3^2 - 3^2 + 8 \text{ (Adding and subtracting } 3^2\text{)} \\ &= (p + 3)^2 - 1^2 \text{ (By completing the square)} \end{aligned}$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$\begin{aligned} &= (p + 3 - 1)(p + 3 + 1) \\ &= (p + 2)(p + 4) \end{aligned}$$

2. $q^2 - 10q + 21$

Solution:

We have,

$$q^2 - 10q + 21$$

Coefficient of q^2 is unity. So, we add and subtract square of half of coefficient of q .

$$\begin{aligned} q^2 - 10q + 21 &= q^2 - 10q + 5^2 - 5^2 + 21 \text{ (Adding and subtracting } 5^2\text{)} \\ &= (q - 5)^2 - 2^2 \text{ (By completing the square)} \end{aligned}$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$\begin{aligned} &= (q - 5 - 2)(q - 5 + 2) \\ &= (q - 3)(q - 7) \end{aligned}$$

3. $4y^2 + 12y + 5$

Solution:

We have,

$$4y^2 + 12y + 5$$

$$4(y^2 + 3y + 5/4)$$

Coefficient of y^2 is unity. So, we add and subtract square of half of coefficient of y .

$$\begin{aligned} 4(y^2 + 3y + 5/4) &= 4 [y^2 + 3y + (3/2)^2 - (3/2)^2 + 5/4] \text{ (Adding and subtracting } (3/2)^2\text{)} \\ &= 4 [(y + 3/2)^2 - 1^2] \text{ (Completing the square)} \end{aligned}$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$\begin{aligned} &= 4 (y + 3/2 + 1)(y + 3/2 - 1) \\ &= 4 (y + 1/2)(y + 5/2) \text{ (by taking LCM)} \\ &= 4 [(2y + 1)/2] [(2y + 5)/2] \\ &= (2y + 1)(2y + 5) \end{aligned}$$

4. $p^2 + 6p - 16$

Solution:

We have,

$$p^2 + 6p - 16$$

Coefficient of p^2 is unity. So, we add and subtract square of half of coefficient of p .

$$p^2 + 6p - 16 = p^2 + 6p + 3^2 - 3^2 - 16 \text{ (Adding and subtracting } 3^2)$$

$$= (p + 3)^2 - 5^2 \text{ (Completing the square)}$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$= (p + 3 + 5)(p + 3 - 5)$$

$$= (p + 8)(p - 2)$$

5. $x^2 + 12x + 20$

Solution:

We have,

$$x^2 + 12x + 20$$

Coefficient of x^2 is unity. So, we add and subtract square of half of coefficient of x .

$$x^2 + 12x + 20 = x^2 + 12x + 6^2 - 6^2 + 20 \text{ (Adding and subtracting } 6^2)$$

$$= (x + 6)^2 - 4^2 \text{ (Completing the square)}$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$= (x + 6 + 4)(x + 6 - 4)$$

$$= (x + 2)(x + 10)$$

6. $a^2 - 14a - 51$

Solution:

We have,

$$a^2 - 14a - 51$$

Coefficient of a^2 is unity. So, we add and subtract square of half of coefficient of a .

$$a^2 - 14a - 51 = a^2 - 14a + 7^2 - 7^2 - 51 \text{ (Adding and subtracting } 7^2)$$

$$= (a - 7)^2 - 10^2 \text{ (Completing the square)}$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$= (a - 7 + 10)(a - 7 - 10)$$

$$= (a - 17)(a + 3)$$

7. $a^2 + 2a - 3$

Solution:

We have,

$$a^2 + 2a - 3$$

Coefficient of a^2 is unity. So, we add and subtract square of half of coefficient of a .

$$a^2 + 2a - 3 = a^2 + 2a + 1^2 - 1^2 - 3 \text{ (Adding and subtracting } 1^2\text{)}$$

$$= (a + 1)^2 - 2^2 \text{ (Completing the square)}$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$= (a + 1 + 2)(a + 1 - 2)$$

$$= (a + 3)(a - 1)$$

8. $4x^2 - 12x + 5$

Solution:

We have,

$$4x^2 - 12x + 5$$

$$4(x^2 - 3x + 5/4)$$

Coefficient of x^2 is unity. So, we add and subtract square of half of coefficient of x .

$$4(x^2 - 3x + 5/4) = 4[x^2 - 3x + (3/2)^2 - (3/2)^2 + 5/4] \text{ (Adding and subtracting } (3/2)^2\text{)}$$

$$= 4[(x - 3/2)^2 - 1^2] \text{ (Completing the square)}$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$= 4(x - 3/2 + 1)(x - 3/2 - 1)$$

$$= 4(x - 1/2)(x - 5/2) \text{ (by taking LCM)}$$

$$= 4[(2x-1)/2][(2x-5)/2]$$

$$= (2x - 5)(2x - 1)$$

9. $y^2 - 7y + 12$

Solution:

We have,

$$y^2 - 7y + 12$$

Coefficient of y^2 is unity. So, we add and subtract square of half of coefficient of y .

$$y^2 - 7y + 12 = y^2 - 7y + (7/2)^2 - (7/2)^2 + 12 \text{ [Adding and subtracting } (7/2)^2\text{]}$$

$$= (y - 7/2)^2 - (7/2)^2 \text{ (Completing the square)}$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$= (y - (7/2 - 1/2))(y - (7/2 + 1/2))$$

$$= (y - 3)(y - 4)$$

10. $z^2 - 4z - 12$

Solution:

We have,

$$z^2 - 4z - 12$$

Coefficient of z^2 is unity. So, we add and subtract square of half of coefficient of z .

$$z^2 - 4z - 12 = z^2 - 4z + 2^2 - 2^2 - 12 \text{ [Adding and subtracting } 2^2\text{]}$$

$$= (z - 2)^2 - 4^2 \text{ (Completing the square)}$$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$$\begin{aligned} &= (z - 2 + 4)(z - 2 - 4) \\ &= (z - 6)(z + 2) \end{aligned}$$



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