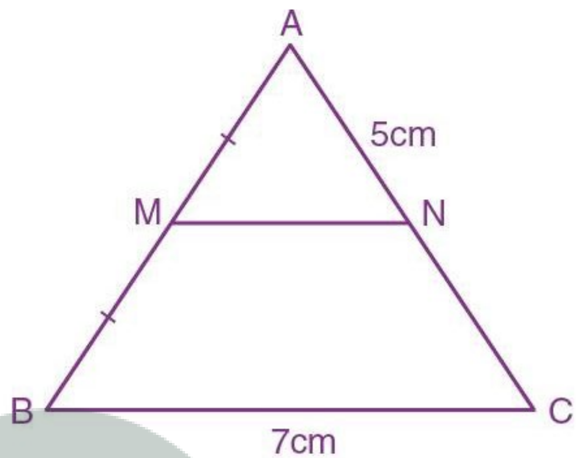
Exercise 12(A)

1. In triangle ABC, M is mid-point of AB and a straight line through M and parallel to BC cuts AC in N. Find the lengths of AN and MN if BC = 7 cm and AC = 5 cm.

Solution:

The triangle is shown as below:



Since M is the midpoint of AB and MN || BC

Then, by mid-point theorem N is the midpoint of AC.

Therefore,

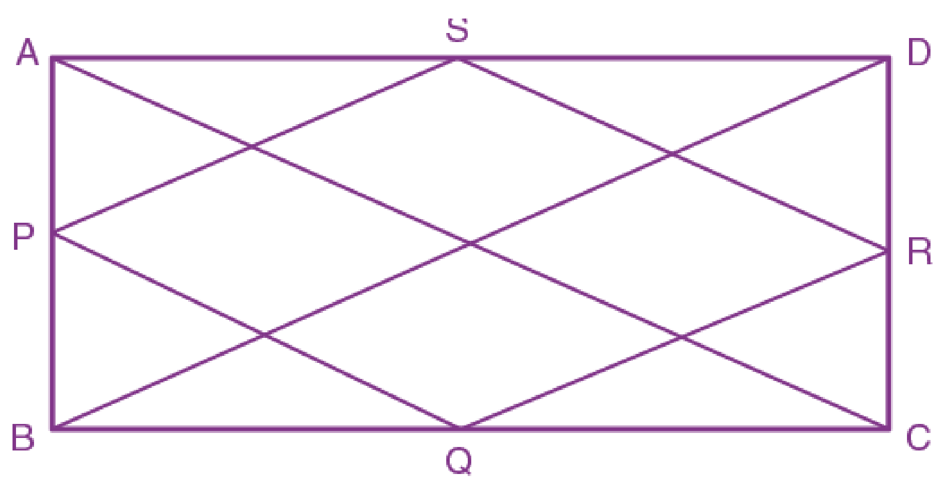
MN = ½ BC = ½ × 7 = 3.5cm

And, AN = ½ AC = ½ × 5 = 2.5cm

2. Prove that the figure obtained by joining the mid-points of the adjacent sides of a rectangle is a rhombus.

Solution:

The figure is shown as below:

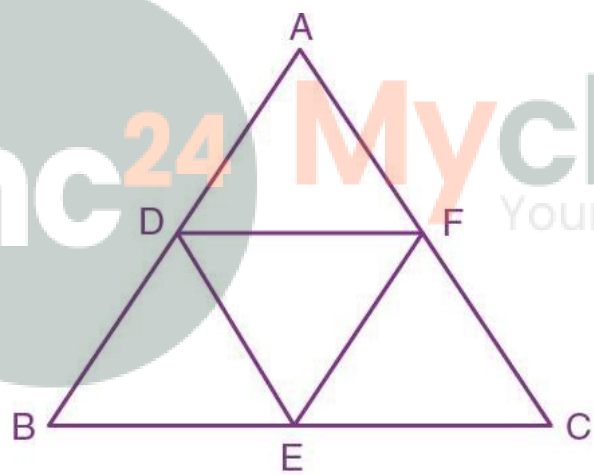


Let ABCD be a rectangle where P, Q, R, S are the midpoint of AB, BC, CD, DA. Then, we need to show that PQRS is a rhombus.

Let’s draw two diagonals BD and AC as shown in figure
And, BD = AC [Since diagonals of rectangle are equal]
Proof:
From △ABD and △BCD, we have
PS = ½ BD = QR and PS || BD || QR
2PS = 2QR = BD and PS || QR … (1)
Similarly,
2PQ = 2SR = AC and PQ || SR … (2)
From (1) and (2) we get
PQ = QR = RS = PS
Therefore, PQRS is a rhombus.
- Hence proved

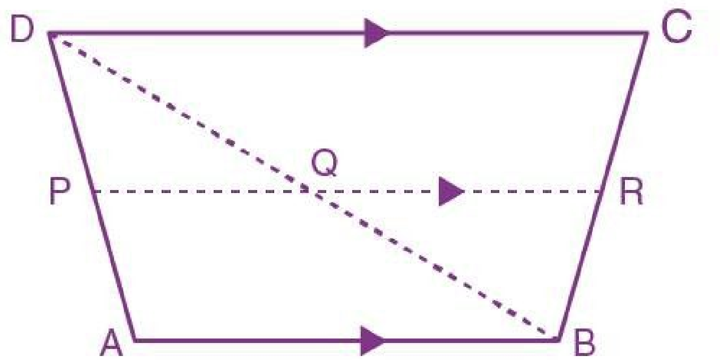
3. D, E and F are the mid-points of the sides AB, BC and CA of an isosceles △ABC in which AB = BC. Prove that △DEF is also isosceles.
Solution:

The figure is shown as below:



Given, ABC is an isosceles triangle and AB = AC
Since D, E and F are midpoints of AB, BC and CA respectively
Therefore, by mid-point theorem
2DE = AC and 2EF = AB
⇒ DE = EF
Therefore, DEF is an isosceles triangle where DE = EF.
- Hence proved

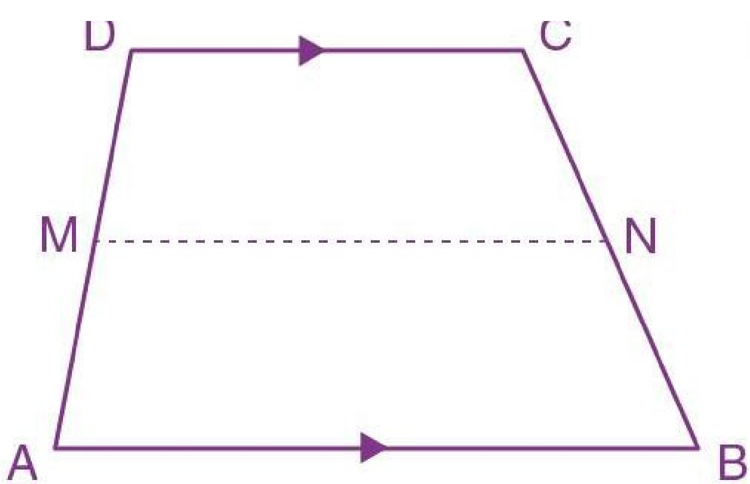
4. The following figure shows a trapezium ABCD in which AB // DC. P is the mid-point of AD and PR // AB. Prove that:
PR = ½ (AB + CD)



Solution:

Given,
In ∆ABD, P is the midpoint of AD and PR || AB
Therefore, Q is the midpoint of BD [By mid-point theorem]
Similarly, R is the midpoint of BC as PR || CD || AB
Now, from ∆ABD
2PQ = AB … (1)
And, from ∆BCD
2QR = CD … (2)
Adding (1) and (2), we get
2(PQ + QR) = AB + CD
2PR = AB + CD
PR = ½ (AB + CD)
- Hence proved.

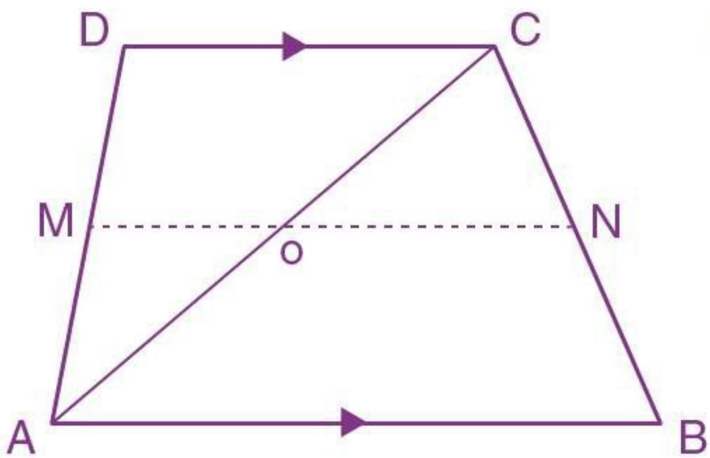
5. The figure, given below, shows a trapezium ABCD. M and N are the mid-point of the non-parallel sides AD and BC respectively. Find:



(i) MN, if AB = 11 cm and DC = 8 cm.
(ii) AB, if DC = 20 cm and MN = 27 cm.

(iii) DC, if MN = 15 cm and AB = 23 cm.
Solution:

Let's draw a diagonal AC as shown in the figure below,



(i) Given, AB = 11cm and CD = 8cm
From ∆ABC, we have
ON = ½ AB = ½ × 11 = 5.5cm
From ∆ACD, we have
OM = ½ CD = ½ × 8 = 4cm
Hence, MN = OM +ON
= (4 + 5.5)
= 9.5cm

(ii) Given, CD = 20cm and MN = 27cm
From ∆ACD, we have
OM = ½ CD = ½ × 20 = 10cm
Therefore, ON = 27 – 10 = 17cm
Then from ∆ABC, we have
AB = 2 ON
= 2 × 17
= 34cm

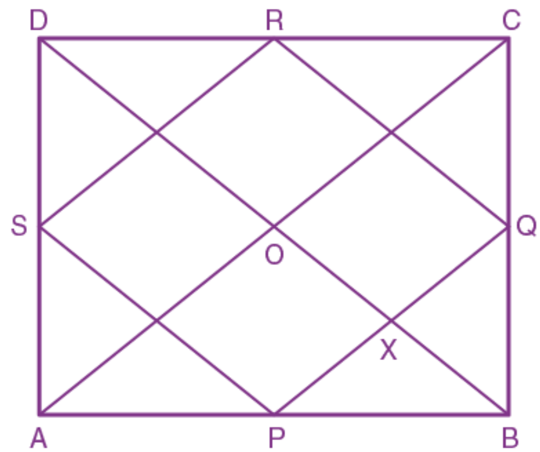
(iii) Given, AB = 23cm and MN = 15cm
From ∆ABC, we have
ON = ½ AB = ½ × 23 = 11.5cm
Therefore, OM = 15 - 11.5 = 3.5cm
Then from ∆ACD, we have
CD = 2 OM
= 2 × 3.5
= 7cm

6. The diagonals of a quadrilateral intersect at right angles. Prove that the figure

obtained by joining the mid-points of the adjacent sides of the quadrilateral is rectangle.

Solution:

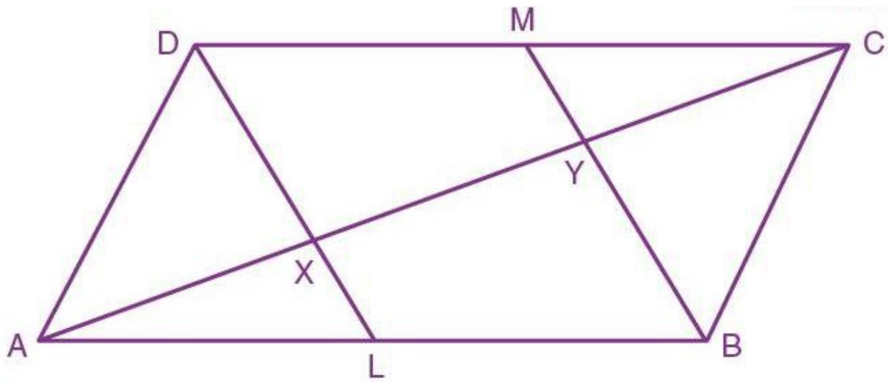
The figure is shown as below:



Let ABCD be a quadrilateral where P, Q, R, S are the midpoints of AB, BC, CD, DA.
Diagonals AC and BD intersects at right angles at point O.
Required to prove: PQRS is a rectangle
<u>Proof:</u>
From ∆ABC and ∆ADC, we have
2PQ = AC and PQ || AC … (1)
2RS = AC and RS || AC … (2)
From (1) and (2) we get,
PQ = RS and PQ || RS
Similarly,
PS = RQ and PS||RQ
Therefore, PQRS is a parallelogram.
Now as PQ||AC, we have
∠AOD = ∠PXO = 90° [Corresponding angles]
Again, as BD||RQ, we have
∠PXO = ∠RQX = 90° [Corresponding angle]
Similarly,
∠QRS = ∠RSP = ∠SPQ = 90°
Therefore, PQRS is a rectangle.
- Hence proved

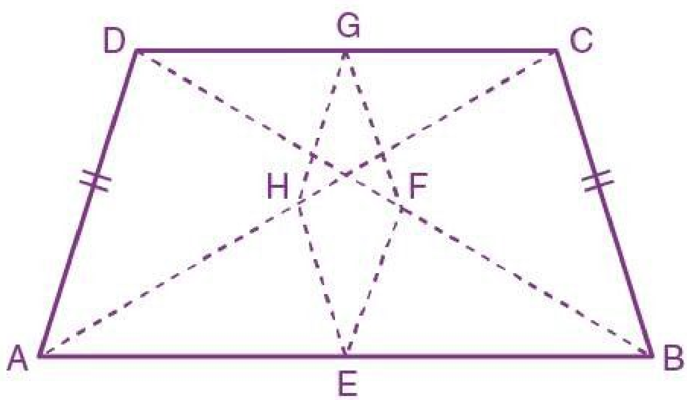
7. L and M are the mid-point of sides AB and DC respectively of parallelogram ABCD. Prove that segments DL and BM trisect diagonal AC.
Solution:

The required figure is shown as below:



We have,
BL = DM and BL || DM and BLMD is a parallelogram
⇒ BM || DL
Now, in ∆ABY, we have
L is the midpoint of AB and XL || BY,
Therefore, x is the midpoint of AY
⇒ AX = XY ... (1)
Similarly for triangle CDX
⇒ CY = XY … (2)
From (1) and (2), we get
AX = XY = CY and AC = AX + XY + CY
Thus, segments DL and BM trisect the diagonal AC of parallelogram ABCD
- Hence proved

8. ABCD is a quadrilateral in which AD = BC. E, F, G and H are the mid-points of AB, BD, CD and Ac respectively. Prove that EFGH is a rhombus.

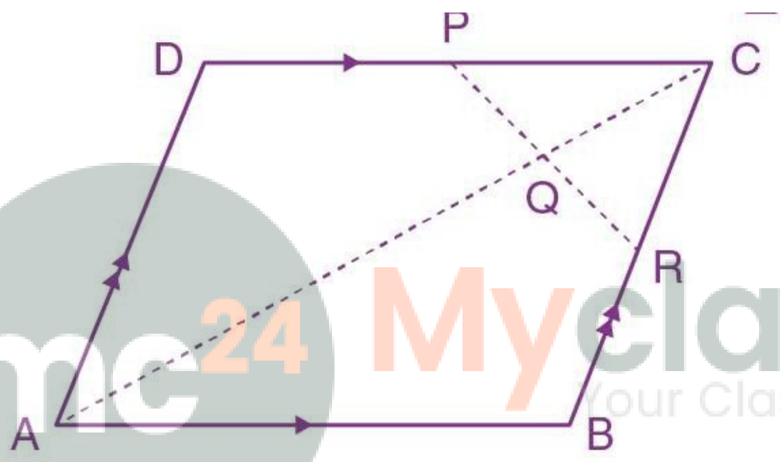


Solution:

Given, AD = BC ... (1)
From the figure,
In ∆ADC and ∆ABD, we have

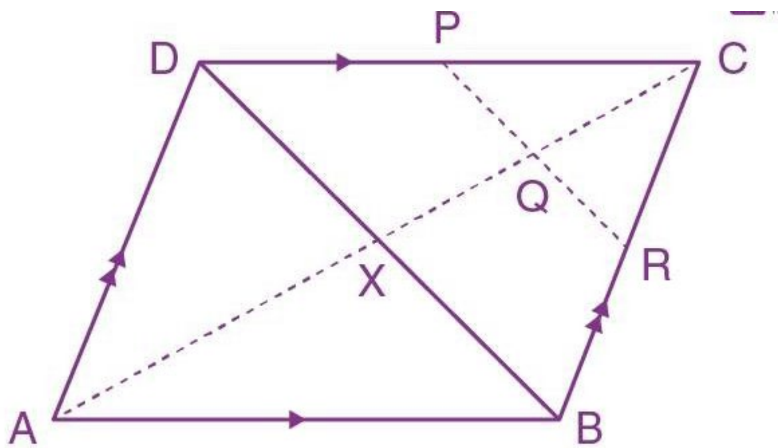
2GH = AD and 2EF = AD,
⇒ 2GH = 2EF = AD ... (2)
Now, in ∆BCD and ∆ABC, we have
2GF = BC and 2EH = BC
⇒ 2GF = 2EH = BC ... (3)
From (1), (2), (3) we get
2GH = 2EF = 2GF = 2EH
⇒ GH = EF = GF = EH
Therefore, EFGH is a rhombus.
- Hence proved

9. A parallelogram ABCD has P the mid-point of DC and Q a point of AC such that CQ = ¼ AC. PQ produced meets BC at R.



Prove that:
(i) R is the midpoint of BC
(ii) PR = ½ DB

Solution:
Let's draw the diagonal BD as shown below.



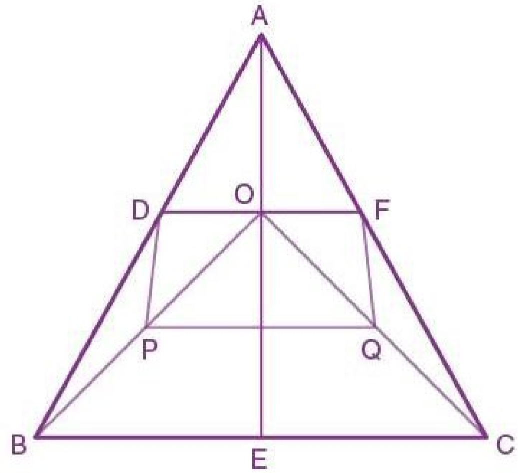
The diagonal AC and BD cuts at point X.
We know that the diagonals of a parallelogram bisect each other.
Therefore, AX = CX and BX = DX
Given,
CQ = ¼ AC
CQ = ¼ × 2CX
CQ = ½ CX
Therefore, Q is the midpoint of CX.

(i) For ∆CDX, PQ || DX or PR || BD
And in ∆CBX, Q is the midpoint of CX and QR||BX
Therefore, R is the midpoint of BC

(ii) In ∆BCD,
As P and R are the midpoint of CD and B, we have
Thus, PR = ½ DB

10. D, E and F are the mid-points of the sides AB, BC and CA respectively of ∆ABC. AE meets DF at O. P and Q are the mid-points of OB and OC respectively. Prove that DPQF is a parallelogram.
Solution:

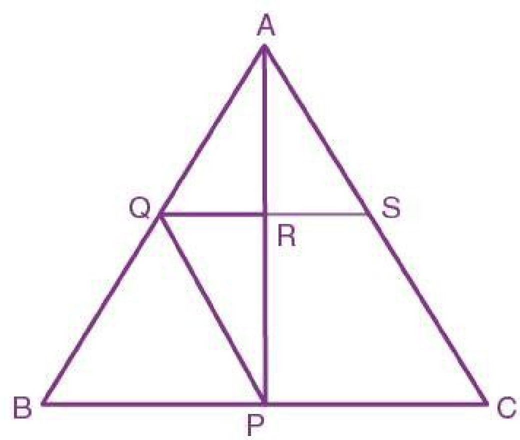
The required figure is shown as below:



In ΔABC and ΔOBC, we have
2DE = BC and 2PQ = BC,
Therefore, DE = PQ ... (1)
In ΔABO and ΔACO, we have
2PD = AO and 2FQ = AO,
Therefore, PD = FQ ... (2)
From (1) and (2), we get that PQFD is a parallelogram.
- Hence proved.

11. In triangle ABC, P is the mid-point of side BC. A line through P and parallel to CA meets AB at point Q; and a line through Q and parallel to BC meets median AP at point R. Prove that:
(i) AP = 2AR
(ii) BC = 4QR
Solution:

The required figure is shown as below:



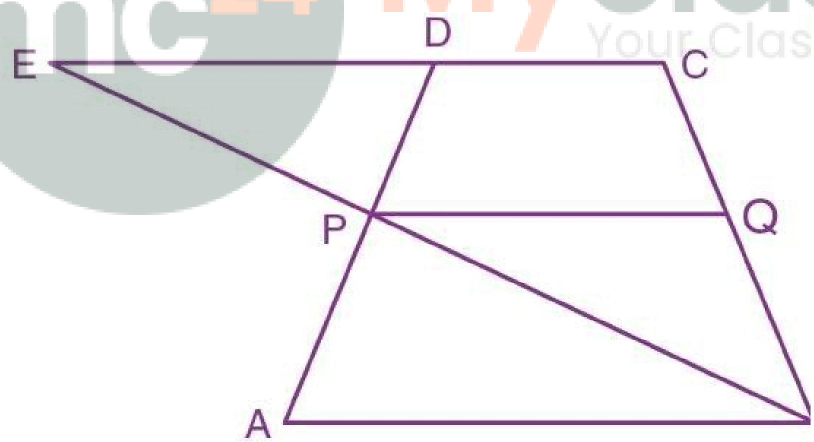
It's seen that P is the midpoint of BC, PQ || AC and QR || BC
Therefore, Q is the midpoint of AB and R is the midpoint of AP

(i) Thus, AP = 2AR

(ii) Let's extend QR such that it cuts AC at S as shown in the figure.
Now, in ∆PQR and ∆ARS, we have
∠PQR = ∠ARS (Opposite angles)
PR = AR
PQ = AS (Since, PQ = AS = ½ AC)
Thus, ∆PQR ≅ ∆ARS by SAS congruence criterion
Therefore, by CPCT
QR = RS
Now,
BC = 2QS
BC = 2 × 2QR
BC = 4QR
- Hence proved

12. In trapezium ABCD, AB is parallel to DC; P and Q are the mid-points of AD and BC respectively. BP produced meets CD produced at point E. Prove that:
(i) Point P bisects BE,
(ii) PQ is parallel to AB.
Solution:

The required figure is shown as below:



(i) In ∆PED and ∆ABP, we have
PD = AP [Since, P is the mid-point of AD]
∠DPE = ∠APB [Opposite angles]
∠PED = ∠PBA [Alternate angles as AB || CE]
∴ ∆PED ≅ ∆ABP by ASA congruence postulate
Thus, by CPCT
EP = BP

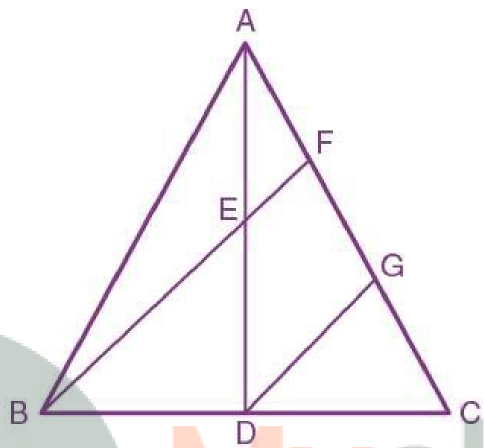
(ii) For ∆ECB, we have PQ||CE
Also, CE||AB

Therefore, PQ||AB

- Hence proved

13. In a triangle ABC, AD is a median and E is mid-point of median AD. A line through B and E meets AC at point F. Prove that AC = 3AF.
Solution:

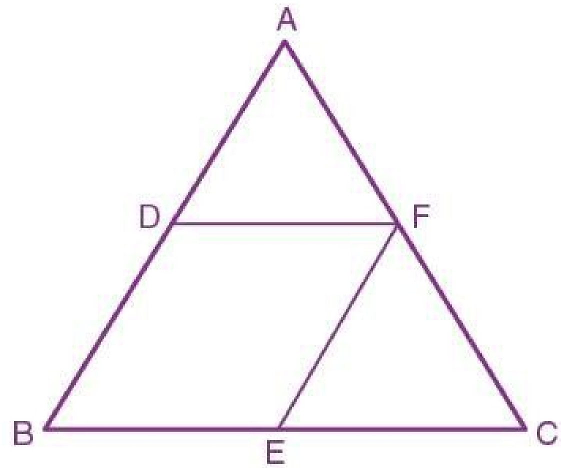
The required figure is shown as below:



Let's draw a line DG || BF
Now,
In ∆ADG, we have
DG || BF and E is the midpoint of AD
Therefore, F is the midpoint of AG
⇒ AF = GF ... (1)
And, in ∆BCF, we have
DG || BF and D is the midpoint of BC
Therefore, G is the midpoint of CF
⇒ GF=CF … (2)
AC = AF + GF + CF [From figure]
AC = 3AF [From (1) and (2)]
- Hence proved.

14. D and F are mid-points of sides AB and AC of a triangle ABC. A line through F and parallel to AB meets BC at point E.
(i) Prove that BDFE is parallelogram
(ii) Find AB, if EF = 4.8 cm.
Solution:

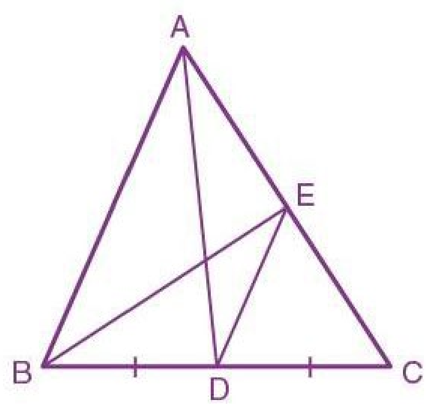
The required figure is shown as below:



(i) Since F is the midpoint and EF || AB
Therefore, E is the midpoint of BC
So, BE = ½ BC and EF = ½ AB … (1)
And,
Since D and F are the midpoint of AB and AC
Therefore, DE||BC
So, DF = ½ BC and DB = ½ AB … (2)
From (1) and (2), we get
BE = DF and BD = EF
Hence, BDEF is a parallelogram.

(ii) Now, AB = 2EF
= 2 × 4.8
= 9.6cm

15. In triangle ABC, AD is the median and DE, drawn parallel to side BA, meets AC at point E. Show that BE is also a median.
Solution:



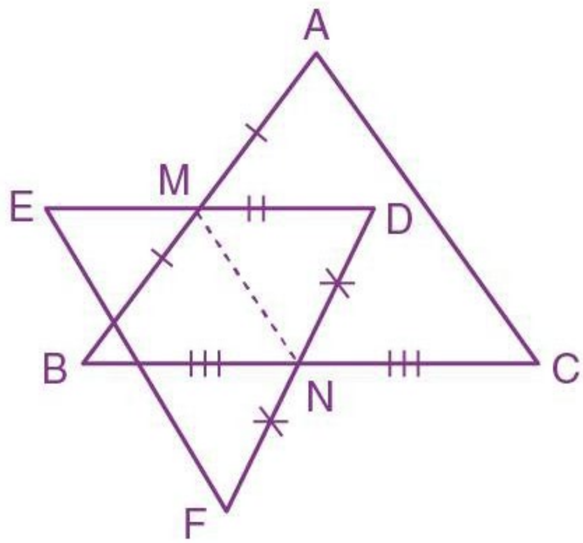
In △ABC, we have

AD is the median of BC
D is the mid-point of BC
Given that DE || BA
So, by the converse of the mid-point theorem,
DE bisects AC
⇒ E is the mid-Point of AC
And, BE is the median of AC
Hence, BE is also a median.

16. In △ABC, E is mid-point of the median AD and BE produced meets side AC at point Q. Show that BE: EQ = 3:1.
Solution:

Construction: Draw DY || BQ
In △BCQ and △DCY, we have
∠BCQ = DCY [Common]
∠BQC = ∠DYC [Corresponding angles]
So, △BCQ ~ △DCY by AA similarity criterion
Thus,
BQ/DY = BC/DC = CQ/CY [Corresponding sides are proportional]
BQ/DY = 2 … (i)
Similarly, △AEQ ~ △ADY
EQ/DY = AE/ED = ½ [E is the mid-point of AD]
⇒ EQ/DY = ½ … (ii)
On dividing (i) by(ii), we get
BQ/EQ = 4
BQ = 4 EQ
BE + EQ = 4EQ
BE = 3EQ
Therefore, BE/EQ = 3/1

17. In the given figure, M is mid-point of AB and DE, whereas N is mid-point of BC and DF. Show that: EF = AC.



Solution:

In △EDF, we have
M is the mid-point of AB and N is the mid-point of DE
So, MN = ½ EF (By mid-point theorem)
EF = 2MN … (i)
In △ABC, we have
M is the mid-point of AB and N is the mid-point of BC
MN = ½ AC (By mid-point theorem)
AC = 2MN … (ii)
From (i) and (ii), we get
EF = AC