NCERT Solutions for Class-XI Maths

Chapter-11 Exercise-11.4

NCERT Math Class 11

1. Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$

1. The given equation is $\frac{x^2}{16}-\frac{y^2}{9}=1$ or $\frac{x^2}{4^2}-\frac{y^2}{3^2}=1$

On comparing this equation with the standard equation of hyperbola

$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, we get,

a = 4 and b = 3,

We know, $a^2 + b^2 = c^2$

Thus,

$c^2 = 4^2 + 3^2 = 25$

=> c = 5

Therefore,

The coordinates of the foci are (±5,0).

The coordinates of the vertices are (±4, 0).

Eccentricity, $e = \frac{c}{a} = \frac{5}{4}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times9}{4} = \frac{9}{2}$

2. Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $\frac{y^2}{9}-\frac{x^2}{27}=1$

2. The given equation is $\frac{y^2}{9}-\frac{x^2}{27}=1$ or $\frac{y^2}{3^2}-\frac{x^2}{(\sqrt{27})^2}=1$

On comparing this equation with the standard equation of hyperbola $\frac{y^2}{a^2}-\frac{x^2}{b^2}=1$ i.e.,

we obtain $a = 3$ and $b = \sqrt{27}$.

We know that $a^2 + b^2 = c^2$

$\therefore c^2 = 3^2 + (\sqrt{27})^2 = 9 + 27 = 36$

$\Rightarrow c = 6$

Therefore,

The coordinates of the foci are $(0, \pm 6)$.

The coordinates of the vertices are $(0, \pm 3)$.

Eccentricity, $e = \frac{c}{a} = \frac{6}{3} = 2$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 27}{3} = 18$

3. $9y^2 - 4x^2 = 36$

3. The given equation is $9y^2 - 4x^2 = 36$

We can re-write the given as

$\frac{y^2}{4} - \frac{x^2}{9} = 1$ or $\frac{y^2}{2^2} - \frac{x^2}{3^2} = 1$..........(1)

On comparing this equation (1) with the standard equation of hyperbola

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we get,

a = 2 and b =3

We know, $a^2 + b^2 = c^2$

Thus,

$c^2 = 4 + 9 = 13$

$\Rightarrow c = \sqrt{13}$

Therefore,

The coordinates of the foci are $(0, \pm\sqrt{13})$.

The coordinates of the vertices are $(0, \pm 2)$.

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{13}}{2}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 9}{2} = 9$

4. Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $16x^2 - 9y^2 = 576$

4. The given equation is $16x^2 - 9y^2 = 576$.

It can be written as

$16x^2 - 9y^2 = 576$

$\Rightarrow \frac{x^2}{36} - \frac{y^2}{64} = 1$

$\Rightarrow \frac{x^2}{6^2} - \frac{y^2}{8^2} = 1$

On comparing equation (1) with the standard equation of hyperbola i.e., $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$,

we obtain $a = 6$ and $b = 8$.

We know that $a^2 + b^2 = c^2$.

$\therefore c^2 = 36 + 64 = 100$

Therefore,

The coordinates of the foci are $(\pm 10, 0)$.

The coordinates of the vertices are $(\pm 6,0)$.

Eccentricity, $e = \frac{c}{a} = \frac{10}{6} = \frac{5}{3}$

5. Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $5y^2 - 9x^2 = 36$

5. The given equation is $5y^2 - 9x^2 = 36$

We can re-write the given as

$\frac{y^2}{\left(\frac{36}{5}\right)} - \frac{x^2}{4} = 1$ Or $\frac{y^2}{\left(\frac{6}{\sqrt{5}}\right)^2} - \frac{x^2}{2^2} = 1$………. (1)

On comparing this equation (1) with the standard equation of hyperbola

$\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we get,

$a = \frac{6}{\sqrt{5}}$ and $b = 2$

We know, $a^2 + b^2 = c^2$

Thus,

$c^2 = \frac{36}{5} + 4 = \frac{56}{5}$

$\Rightarrow c = \sqrt{\frac{56}{5}} = \frac{2\sqrt{14}}{\sqrt{5}}$

Therefore,

The coordinates of the foci are $\left(0,, \pm \frac{2\sqrt{14}}{\sqrt{5}}\right)$

The coordinates of the vertices are $\left(0, \pm \frac{6}{\sqrt{5}}\right)$.

Eccentricity, $e = \frac{c}{a} = \frac{\frac{2\sqrt{14}}{\sqrt{5}}}{\frac{6}{\sqrt{5}}} = \frac{\sqrt{14}}{3}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 4}{\left(\frac{6}{\sqrt{5}}\right)} = \frac{4\sqrt{5}}{3}$

6. Find the coordinates of the foci and the vertices, the eccentricity, and the length of the latus rectum of the hyperbola $49y^2 - 16x^2 = 784$

6. The given equation is $49y^2 - 16x^2 = 784$.

It can be written as

$49y^2 - 16x^2 = 784$

Or, $\frac{y^2}{16} - \frac{x^2}{49} = 1$

Or, $\frac{y^2}{4^2} - \frac{x^2}{7^2} = 1$

On comparing equation (1) with the standard equation of hyperbola i.e., $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$, we obtain $a = 4$ and $b = 7$.

We know that $a^2 + b^2 = c^2$.

$\therefore c^2 = 16 + 49 = 65$

$\Rightarrow c = \sqrt{65}$

Therefore,

The coordinates of the foci are $(0, \pm\sqrt{65})$.

The coordinates of the vertices are $(0, \pm 4)$.

Eccentricity, $e = \frac{c}{a} = \frac{\sqrt{65}}{4}$

Length of latus rectum $= \frac{2b^2}{a} = \frac{2\times 49}{4} = \frac{49}{2}$

7. Vertices (± 2, 0), foci (± 3, 0)

7. Vertices (± 2, 0), foci (± 3, 0)

Here, the vertices are on the x-axis.

Thus,

The equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Since, the vertices are (±2,0), a = 2

Since, the foci are (±3, 0), c= 3

We know that, $a^2 + b^2 = c^2$

Thus, $2^2 + b^2 = 3^2$

=> $b^2 = 9 - 4 = 5$

Hence, the equation of the hyperbola is $\frac{x^2}{4} - \frac{y^2}{5} = 1$

8. Find the equation of the hyperbola satisfying the give conditions: Vertices $(0, \pm 5)$, foci $(0, \pm 8)$

8. Vertices $(0, \pm 5)$, foci $(0, \pm 8)$

Here, the vertices are on the y-axis.

Therefore, the equation of the hyperbola is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$.

Since the vertices are $(0, \pm 5)$, $a = 5$.

Since the foci are $(0, \pm 8)$, $c = 8$.

We know that $a^2 + b^2 = c^2$.

$\therefore 5^2 + b^2 = 8^2$

$b^2 = 64 - 25 = 39$

Thus, the equation of the hyperbola is $\frac{y^2}{25} - \frac{x^2}{39} = 1$.

9. Vertices (0, ± 3), foci (0, ± 5)

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Here, the vertices are on the y-axis.
Thus,
The equation of the hyperbola is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Since, the vertices are (0, ± 3), a = 3
Since, the foci are (0, ±5), c= 5
We know that, $a^2 + b^2 = c^2$
Thus, $3^2 + b^2 = 5^2$
$\Rightarrow b^2 = 25 - 9 = 16$
Hence, the equation of the hyperbola is $\frac{y^2}{9} - \frac{x^2}{16} = 1$

10. Find the equation of the hyperbola satisfying the give conditions: Foci $(\pm 5,0)$, the transverse axis is of length 8.

10. Foci $(\pm 5,0)$, the transverse axis is of length 8 .
Here, the foci are on the x-axis.
Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.
Since the foci are $(\pm 5,0), c = 5$.
Since the length of the transverse axis is $8, 2a = 8 \Rightarrow a = 4$.
We know that $a^2 + b^2 = c^2$.
$\therefore 4^2 + b^2 = 5^2$
$\Rightarrow b^2 = 25 - 16 = 9$
Thus, the equation of the hyperbola is $\frac{x^2}{16} - \frac{y^2}{9} = 1$.

11. Foci (0, ±13), the conjugate axis is of length 24.

11. Foci (0, ±13), the conjugate axis is of length 24.
Here, the foci are on y-axis.
Thus,
The equation of the hyperbola is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
Since, the foci are (0, ±13), c= 13
Since, the length of the conjugate axis is 24,
$2b = 24 \Rightarrow b = 12$
We know that, $a^2 + b^2 = c^2$
$a^2 + 12^2 = 13^2$
$\Rightarrow a^2 = 169 - 144 = 25$
Hence, the equation of the hyperbola is $\frac{y^2}{25} - \frac{x^2}{144} = 1$

12. Find the equation of the hyperbola satisfying the give conditions: Foci $(\pm 3\sqrt{5}, 0)$, the latus rectum is of length 8 .

12. Foci $(\pm 3\sqrt{5}, 0)$, the latus rectum is of length 8 .

Here, the foci are on the x-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Since the foci are $(\pm 3\sqrt{5}, 0) c = \pm 3\sqrt{5}$.

Length of latus rectum = 8

$\Rightarrow \dfrac{2b^2}{a} = 8$

$\Rightarrow b^2 = 4a$

We know that $a^2 + b^2 = c^2$.

$\therefore a^2 + 4a = 45$

$\Rightarrow a^2 + 4a - 45 = 0$

$\Rightarrow a^2 + 9a - 5a - 45 = 0$

$\Rightarrow (a + 9)(a - 5) = 0$

$\Rightarrow a = -9,5$

Since a is non-negative, $a = 5$.

$\therefore b^2 = 4a = 4 \times 5 = 20$

Thus, the equation of the hyperbola is $\frac{x^2}{25} - \frac{y^2}{20} = 1$.

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13. Foci (± 4, 0), the latus rectum is of length 12

13. Foci (± 4, 0), the latus rectum is of length 12

Here, the foci are on x-axis.

Thus,

The equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Since, the foci are (± 4, 0), c = 4

Length of latus rectum = 12

$\Rightarrow \frac{2b^2}{a} = 12$

$\Rightarrow b^2 = 6a$

We know that, $a^2 + b^2 = c^2$

$a^2 + 6a = 16$

$\Rightarrow a^2 + 6a - 16 = 0$

$\Rightarrow a^2 + 8a - 2a - 16 = 0$

$\Rightarrow (a + 8)(a - 2) = 0$

$\Rightarrow A = -8, 2$

Since, a is non – negative, a = 2

Thus, $b^2 = 6a = 6 \times 2 = 12$

Hence, the equation of the hyperbola is $\frac{x^2}{4} - \frac{y^2}{12} = 1$

14. Find the equation of the hyperbola satisfying the give conditions: Vertices $(\pm 7,0)$, $e = \frac{4}{3}$

14. Vertices $(\pm 7,0)$, $e = \frac{4}{3}$

Here, the vertices are on the x-axis.

Therefore, the equation of the hyperbola is of the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$.

Since the vertices are $(\pm 7,0), a = 7$.

It is given that $e = \frac{4}{3}$

$\therefore \frac{c}{a} = \frac{4}{3} \left[e = \frac{c}{a}\right]$

$\Rightarrow \frac{c}{7} = \frac{4}{3}$

$\Rightarrow c = \frac{28}{3}$

We know that $a^2 + b^2 = c^2$.

$\therefore 7^2 + b^2 = \left(\frac{28}{3}\right)^2$

$\Rightarrow b^2 = \frac{784}{9} - 49$

$\Rightarrow b^2 = \frac{784 - 441}{9} = \frac{343}{9}$

Thus, the equation of the hyperbola is $\frac{x^2}{49} - \frac{9y^2}{343} = 1$.

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15. Foci $(0, \pm\sqrt{10})$, passing through (2, 3)

15. Foci $(0, \pm\sqrt{10})$, passing through (2, 3)

Here, the foci are on y-axis.

Thus,

The equation of the hyperbola is of the form $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

Since, the foci are $(\pm\sqrt{10}, 0)$, $c = \sqrt{10}$

We know that, $a^2 + b^2 = c^2$

$\Rightarrow b^2 = 10 - a^2$ …………..(1)

Since, the hyperbola passes through point (2, 3)

$\frac{9}{a^2} - \frac{4}{b^2} = 1$……………(2)

From equations (1) and (2), we get,

$\frac{9}{a^2} - \frac{4}{(10-a^2)^2} = 1$

$\Rightarrow$ $9(10 - a^2) - 4a^2 = a^2(10 - a^2)$

$\Rightarrow$ $90 - 9a^2 - 4a^2 = 10a^2 - a^4$

$\Rightarrow$ $a^4 - 23a^2 + 90 = 0$

$\Rightarrow$ $a^4 - 18a^2 - 5a^2 + 90 = 0$

$\Rightarrow$ $a^2(a^2 - 18) - 5(a^2 - 18) = 0$

$\Rightarrow$ $(a^2 - 18)(a^2 - 5) = 0$

$\Rightarrow$ $a^2 = 18$ or 5

In hyperbola, $c>a$ that is $c^2 > a^2$

Thus, $a^2 = 5$

$\Rightarrow$ $b^2 = 10 - a^2 = 10 - 5 = 5$

Hence, the equation of the hyperbola is $\frac{y^2}{5} - \frac{x^2}{5} = 1$

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