NCERT Solutions for Class-XII Maths

Chapter-10

Exercise- Miscellaneous

NCERT Chemistry Class 12

1. Write down a unit vector in XY-plane, making an angle of 30° with the positive direction of x-axis.

1. If $\vec{r}$ is a unit vector in the XY-plane, then $\vec{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$.

Here, θ is the angle made by the unit vector with the positive direction of the x-axis.

Therefore, for $\theta = 30^{o}$:

$$\vec{r} = \cos 30^{o}\,\hat{i} + \sin 30^{o}\,\hat{j} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$$

Hence, the required unit vector is $\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

2. Find the scalar components and magnitude of the vector joining the points P(x_1, y_1, z_1) and Q(x_2, y_2, z_2).

2. Given: points P(x_1, y_1, z_1) and Q(x_2, y_2, z_2) are given.

The vector obtained by joining the given points P and Q:

$\overrightarrow{PQ}$ = position vector of Q – position vector of P

$$= (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$

$$|\overrightarrow{PQ}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Hence the scalar component of the vector obtained by joining the points are

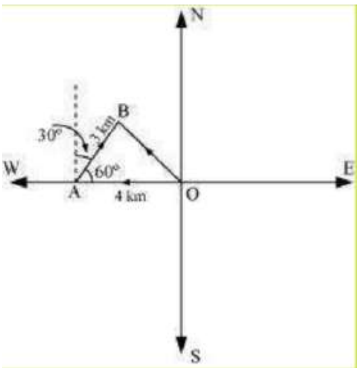
[(x_2-x_1), (y_2-y_1), (z_2-z_1)]

And the magnitude of the vector obtained by joining the points is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

3. A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl’s displacement from her initial point of departure.

3. Let O and B be the initial and final positions of the girl respectively.
Then, the girl's position can be shown as:



Now, we have:

$\overrightarrow{OA} = -4\hat{i}$

$\overrightarrow{AB} = \hat{i}\left|\overrightarrow{AB}\right|\cos 60^{o} + \hat{j}\left|\overrightarrow{AB}\right|\sin 60^{o}$

$= \hat{i}3\times\frac{1}{2} + \hat{j}3\times\frac{\sqrt{3}}{2}$

$= \frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

By the triangle law of vector addition, we have:

$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$

$= \left(-4\hat{i}\right) + \left(\frac{3}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}\right)$

$= \left(-4 + \frac{3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

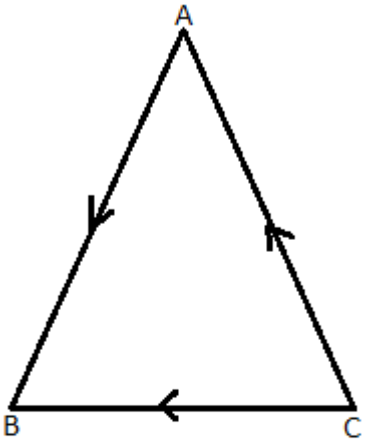
$= \left(\frac{-8+3}{2}\right)\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

$= \frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

Hence, the girl's displacement from her initial point of departure is $\frac{-5}{2}\hat{i} + \frac{3\sqrt{3}}{2}\hat{j}$

4. If $\left|\vec{a}\right| = \left|\vec{b}\right| + \left|\vec{c}\right|$, then is it true that $\left|\vec{a}\right| = \left|\vec{b}\right| + \left|\vec{c}\right|$? Justify your answer.

4. Let in given triangle $\overrightarrow{CB} = \vec{a}, \overrightarrow{CA} = \vec{b}$ and $\overrightarrow{AB} = \vec{c}$



Now, by using triangle law of vector addition,

$$\overrightarrow{CB} = \overrightarrow{CA} + \overrightarrow{AB}$$

$$\Rightarrow \vec{a} = \vec{b} + \vec{c}$$

As we can see that $|\vec{a}|$, $|\vec{b}|$ and $|\vec{c}|$ represent the sides of triangle.

Also we know that sum of two sides of a triangle must be greater than its third side.

$$\Rightarrow |\vec{b}| + |\vec{c}| > |\vec{a}|$$

$$\Rightarrow |\vec{a}| \neq |\vec{b}| + |\vec{c}|$$

$\therefore |\vec{a}| = |\vec{b}| + |\vec{c}|$ is not true.

5. Find the value of x for which $x(\hat{i} + \hat{j} + \hat{k})$ is a unit vector.

5. $x\left(\hat{i} + \hat{j} + \hat{k}\right)$ is a unit vector if $\left|x\left(\hat{i} + \hat{j} + \hat{k}\right)\right| = 1$.

Now,

$$\left|x\left(\hat{i} + \hat{j} + \hat{k}\right)\right| = 1$$

$$\Rightarrow \sqrt{x^2 + x^2 + x^2} = 1$$

$$\Rightarrow \sqrt{3x^2} = 1$$

$$\Rightarrow \sqrt{3}x = 1$$

$$\Rightarrow x = \pm\frac{1}{\sqrt{3}}$$

Hence, the required value of x is $\pm\frac{1}{\sqrt{3}}$.

6. Find a vector of magnitude 5 units, and parallel to the resultant of the vectors

$\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + \hat{k}$

6. Given: $\vec{a} = \left(2\hat{i} + 3\hat{j} - \hat{k}\right)$ and $\vec{b} = \left(\hat{i} - 2\hat{j} + \hat{k}\right)$

Let the resultant of $\vec{a}$ and $\vec{b}$ is $\vec{c}$.

Then $\vec{c} = \vec{a} + \vec{b}$

$\Rightarrow \vec{c} = (2+1)\hat{i} + (3-2)\hat{j} + (-1+1)\hat{k}$

$\Rightarrow \vec{c} = (3)\hat{i} + (1)\hat{j} + (0)\hat{k}$

$\Rightarrow \vec{c} = 3\hat{i} + \hat{j}$

Then

$|\vec{c}| = \sqrt{(3)^2 + (1)^2}$

$\Rightarrow |\vec{c}| = \sqrt{9+1}$

$\Rightarrow |\vec{c}| = \sqrt{10}$

$\therefore \hat{c} = \dfrac{\vec{c}}{|\vec{c}|} = \dfrac{3\hat{i} + \hat{j}}{\sqrt{10}}$

Now the vector of magnitude 5 units and parallel to $\hat{c}$ is:

$\pm 5\hat{c} = \pm 5.\dfrac{3\hat{i} + \hat{j}}{\sqrt{10}}$

$= \pm\sqrt{5}\sqrt{5}.\dfrac{3\hat{i} + \hat{j}}{\sqrt{2}\sqrt{5}}$

$= \pm\sqrt{5}.\dfrac{3\hat{i} + \hat{j}}{\sqrt{2}} \times \dfrac{\sqrt{2}}{\sqrt{2}}$

$. = \pm\sqrt{10}.\dfrac{3\hat{i} + \hat{j}}{2}.$

$= \pm\dfrac{3\sqrt{10}\hat{i}}{2} \pm \dfrac{\sqrt{10}}{2}\hat{j}$

7. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$, and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$, find a unit vector parallel to the vector $2\vec{a} - \vec{b} + 3\vec{c}$.

7. We have,

$\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{c} = \hat{i} - 2\hat{j} + \hat{k}$

$2\vec{a} - \vec{b} + 3\vec{c} = 2\left(\hat{i} + \hat{j} + \hat{k}\right) - \left(2\hat{i} - \hat{j} + 3\hat{k}\right) + 3\left(\hat{i} - 2\hat{j} + \hat{k}\right)$

$= 2\hat{i} + 2\hat{j} + 2\hat{k} - 2\hat{i} + \hat{j} - 3\hat{k} + 3\hat{i} - 6\hat{j} + 3\hat{k}$

$= 3\hat{i} - 3\hat{j} + 2\hat{k}$

$\left|2\vec{a} - \vec{b} + 3\vec{c}\right| = \sqrt{3^2 + (-3)^2 + 2^2} = \sqrt{9+9+4} = \sqrt{22}$

Hence, the unit vector along $2\vec{a} - \vec{b} + 3\vec{c}$ is

$$\frac{2\vec{a} - \vec{b} + 3\vec{c}}{\left|2\vec{a} - \vec{b} + 3\vec{c}\right|} = \frac{3\hat{i} - 3\hat{j} + 2\hat{k}}{\sqrt{22}} = \frac{3}{\sqrt{22}}\hat{i} - \frac{3}{\sqrt{22}}\hat{j} + \frac{2}{\sqrt{22}}\hat{k}$$

8. Show that the points A(1, – 2, – 8), B(5, 0, –2) and C(11, 3, 7) are collinear, and find the ratio in which B divides AC.

8. A (1, – 2, – 8), B (5, 0, –2) and C (11, 3, 7)

Then

$\overrightarrow{AB} = (5-1)\hat{i} + (0-(-2))\hat{j} + (-2-(-8))\hat{k}$

$\Rightarrow \overrightarrow{AB} = (4)\hat{i} + (2)\hat{j} + (6)\hat{k}$

$\overrightarrow{BC} = (11-5)\hat{i} + (3-0)\hat{j} + (7-(-2))\hat{k}$

$\Rightarrow \overrightarrow{BC} = (6)\hat{i} + (3)\hat{j} + (9)\hat{k}$

$\overrightarrow{AC} = (11-1)\hat{i} + (3-(-2))\hat{j} + (7-(-8))\hat{k}$

$\Rightarrow \overrightarrow{AB} = (10)\hat{i} + (5)\hat{j} + (15)\hat{k}$

Then

$\left|\overrightarrow{AB}\right| = \sqrt{(4)^2 + (2)^2 + (6)^2}$

$= \sqrt{16+4+36}$

$\Rightarrow \left|\overrightarrow{AB}\right| = \sqrt{56} = 2\sqrt{14}$

$\left|\overrightarrow{BC}\right| = \sqrt{(6)^2 + (3)^2 + (9)^2}$

$= \sqrt{36+9+81}$

$\Rightarrow \left|\overrightarrow{BC}\right| = \sqrt{126} = 3\sqrt{14}$

$\left|\overrightarrow{AC}\right| = \sqrt{(10)^2 + (5)^2 + (15)^2}$

$= \sqrt{100+25+225}$

$\Rightarrow \left|\overrightarrow{AC}\right| = \sqrt{350} = 5\sqrt{14}$

$\therefore \left|\overrightarrow{AC}\right| = \left|\overrightarrow{BC}\right| + \left|\overrightarrow{AB}\right|$

Thus the given points are collinear.

Now to find the ratio in which B divides AC. Let it be $\lambda : 1$

$$\overrightarrow{OB}=\frac{\lambda.\overrightarrow{OC}+1.\overrightarrow{OA}}{\lambda+1}$$

$$\Rightarrow \overrightarrow{OB}=\frac{\lambda.\overrightarrow{OC}+1.\overrightarrow{OA}}{\lambda+1}$$

$$\Rightarrow 5\hat{i}-2\hat{k}=\frac{\lambda.\left(11\hat{i}+3\hat{j}+7\hat{k}\right)+1.\left(\hat{i}-2\hat{j}-8\hat{k}\right)}{\lambda+1}$$

$$\Rightarrow\left(5\hat{i}-2\hat{k}\right)(\lambda+1)=\lambda.\left(11\hat{i}+3\hat{j}+7\hat{k}\right)+1.\left(\hat{i}-2\hat{j}-8\hat{k}\right)$$

$$\Rightarrow\left(5(\lambda+1)\hat{i}-2(\lambda+1)\hat{k}\right)=\left(11\lambda+1\right)\hat{i}+(3\lambda-2)\hat{j}+(7\lambda-8)\hat{k}\right)$$

On equating the terms, we get:

$5(\lambda + 1) = 11\lambda + 1$

$\Rightarrow 5\lambda + 5 = 11\lambda + 1$

$\Rightarrow 4 = 6\lambda$

$\Rightarrow \lambda = 4/6 = 2/3$

Hence, B divides AC in the ratio 2:3.

9. Find the position vector of a point R which divides the line joining two points P and Q whose position vectors are $\left(2\vec{a}+\vec{b}\right)$ and $\left(\vec{a}-3\vec{b}\right)$ externally in the ratio 1 : 2. Also, show that P is the midpoint of the line segment RQ.

9. It is given that $\overrightarrow{OP}=2\vec{a}+\vec{b}, \overrightarrow{OQ}=\vec{a}-3\vec{b}$.

It is given that point R divides a line segment joining two points P and Q externally in the ratio 1: 2 . Then, on using the section formula, we get:

$$\overrightarrow{OR}=\frac{2\left(2\vec{a}+\vec{b}\right)-\left(\vec{a}-3\vec{b}\right)}{2-1}=\frac{4\vec{a}+2\vec{b}-\vec{a}+3\vec{b}}{1}=3\vec{a}+5\vec{b}$$

Therefore, the position vector of point R is $3\vec{a}+5\vec{b}$

Position vector of the mid-point of $RQ=\dfrac{\overrightarrow{OQ}+\overrightarrow{OR}}{2}$

$$=\frac{\left(\vec{a}-3\vec{b}\right)+\left(3\vec{a}+5\vec{b}\right)}{2}$$

$$=2\vec{a}+\vec{b}$$

$$=\overrightarrow{OP}$$

Hence, P is the mid-point of the line segment RQ.

10. The two adjacent sides of a parallelogram are $2\hat{i}-4\hat{j}+5\hat{k}$ and $\hat{i}-2\hat{j}-3\hat{k}$ Find the unit vector parallel to its diagonal. Also, find its area.

10. Given: Two adjacent sides of a parallelogram are $\vec{a}=\left(2\hat{i}-4\hat{j}+5\hat{k}\right)$ and $\vec{b}=\left(\hat{i}-2\hat{j}-3\hat{k}\right)$

Then the diagonal of parallelogram is given by the resultant of $\vec{a}$ and $\vec{b}$.

Let the diagonal is $\vec{c}$.

Then $\vec{c}=\vec{a}+\vec{b}$

$\Rightarrow \vec{c}=(2+1)\hat{i}+(-4-2)\hat{j}+(5-3)\hat{k}$

$\Rightarrow \vec{c}=(3)\hat{i}+(-6)\hat{j}+(2)\hat{k}$

$\Rightarrow \vec{c}=3\hat{i}-6\hat{j}+2\hat{k}$

Then

$|\vec{c}|=\sqrt{(3)^2+(-6)^2+(2)^2}$

$\Rightarrow |\vec{c}|=\sqrt{9+36+4}$

$\Rightarrow |\vec{c}|=\sqrt{49}=7$

$\therefore \hat{c}=\dfrac{\vec{c}}{|\vec{c}|}=\dfrac{3\hat{i}-6\hat{j}+2\hat{k}}{7}$

$\therefore$ unit vector parallel to its diagonal is $\hat{c}=\frac{3}{7}\hat{i}-\frac{6}{7}\hat{j}+\frac{2}{7}\hat{k}$

And area of parallelogram ABCD is $|\vec{a}\times\vec{b}|$

$$\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -4 & 5\\ 1 & -2 & -3\end{vmatrix}$$

$=\hat{i}(12+10)-\hat{j}(-6-5)+\hat{k}(-4+4)$

$=22\hat{i}+11\hat{j}$

$=11\left(2\hat{i}+\hat{j}\right)$

$\therefore |\vec{a}\times\vec{b}|=11\sqrt{(2)^2+(1)^2}=11\sqrt{5}$

Hence, area of parallelogram ABCD is $11\sqrt{5}$.

11. Show that the direction cosines of a vector equally inclined to the axes OX, OY and OZ are

$\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}},\frac{1}{\sqrt{3}}.$

11. Let a vector be equally inclined to axes OX, OY, and OZ at angle a.

Then, the direction cosines of the vector are $\cos a, \cos a$, and $\cos a$.

Now,

$$\cos^2\alpha + \cos^2\alpha + \cos^2\alpha = 1$$

$$\Rightarrow 3\cos^2\alpha = 1$$

$$\Rightarrow \cos\alpha = \frac{1}{\sqrt{3}}$$

Hence, the direction cosines of the vector which are equally inclined to the axes are

$$\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

12. Let $\vec{a} = \hat{i} + 4\hat{j} + 2\hat{k}, \vec{b} = 3\hat{i} - 2\hat{j} + 7\hat{k}$ and $\vec{c} = 2\hat{i} - \hat{j} + 4\hat{k}$. Find a vector $\vec{d}$ which is perpendicular to both $\vec{a}$ a and $\vec{b}$, and $\vec{c}.\vec{d} = 15$

12. Given: $\vec{a} = \left(\hat{i} + 4\hat{j} + 2\hat{k}\right)$, $\vec{b} = \left(3\hat{i} - 2\hat{j} + 7\hat{k}\right)$and $\vec{c} = \left(2\hat{i} - \hat{j} + 4\hat{k}\right)$

Let $\vec{d} = \left(d_1\hat{i} + d_2\hat{j} + d_3\hat{k}\right)$

Because $\vec{d}$ is perpendicular to both $\vec{a}$ and $\vec{b}$.

$\Rightarrow \vec{d}.\vec{a} = 0$ and $\vec{d}.\vec{b} = 0$

Then $\vec{d}.\vec{a} = \left(d_1\hat{i} + d_2\hat{j} + d_3\hat{k}\right).\left(\hat{i} + 4\hat{j} + 2\hat{k}\right) = 0$

$= d_1 + 4d_2 + 2d_3 = 0$ ….(1)

And $\vec{d}.\vec{b} = \left(d_1\hat{i} + d_2\hat{j} + d_3\hat{k}\right).\left(3\hat{i} - 2\hat{j} + 7\hat{k}\right) = 0$

$= 3d_1 - 2d_2 + 7d_3 = 0$ ….(2)

And $\vec{c}.\vec{d} = 15$ (given)

$\Rightarrow \vec{c}.\vec{d} = \left(d_1\hat{i} + d_2\hat{j} + d_3\hat{k}\right).\left(2\hat{i} - \hat{j} + 4\hat{k}\right) = 15$

$= 2d_1 - d_2 + 4d_3 = 0$ ….(3)

After solving (1), (2) and (3) we get:

$d_1 = \frac{160}{3}$, $d_2 = -\frac{5}{3}$ and $d_3 = -\frac{70}{3}$

Hence $\vec{d} = \left(\frac{160}{3}\hat{i} - \frac{5}{3}\hat{j} - \frac{70}{3}\hat{k}\right)$

$$\Rightarrow \vec{d} = \frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$$

Hence, the required vector is $\frac{1}{3}\left(160\hat{i} - 5\hat{j} - 70\hat{k}\right)$

13. The scalar product of the vector $\hat{i}+\hat{j}+\hat{k}$ a unit vector along the sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ is equal to one. Find the value of λ.

13. $\left(2\hat{i}+4\hat{j}-5\hat{k}\right)+\left(\lambda\hat{i}+2\hat{j}+3\hat{k}\right)$

$=(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}$

Therefore, unit vector along $\left(2\hat{i}+4\hat{j}-5\hat{k}\right)+\left(\lambda\hat{i}+2\hat{j}+3\hat{k}\right)$ is given as:

$$\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{(2+\lambda)^2+6^2+(-2)^2}}=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{4+4\lambda+\lambda^2+36+4}}=\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}$$

Scalar product of $\left(\hat{i}+\hat{j}+\hat{k}\right)$ with this unit vector is 1 .

$$\Rightarrow\left(\hat{i}+\hat{j}+\hat{k}\right)\cdot\frac{(2+\lambda)\hat{i}+6\hat{j}-2\hat{k}}{\sqrt{\lambda^2+4\lambda+44}}=1$$

$$\Rightarrow\frac{(2+\lambda)+6-2}{\sqrt{\lambda^2+4\lambda+44}}=1$$

$$\Rightarrow\sqrt{\lambda^2+4\lambda+44}=\lambda+6$$

$$\Rightarrow\lambda^2+4\lambda+44=(\lambda+6)^2$$

$$\Rightarrow\lambda^2+4\lambda+44=\lambda^2+12\lambda+36$$

$$\Rightarrow 8\lambda=8$$

$$\Rightarrow\lambda=1$$

Hence, the value of λ is 1 .

14. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are mutually perpendicular vectors of equal magnitudes, show that the vector $\vec{a}+\vec{b}+\vec{c}$ is equally inclined to $\vec{a}$, $\vec{b}$ and $\vec{c}$

14. Given: vectors $\vec{a},\vec{b}$ and $\vec{c}$ are mutually perpendicular to each other and are of equal magnitude.

$\Rightarrow\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$

And $|\vec{a}|=|\vec{b}|=|\vec{c}|$

Let the vector $\vec{a}+\vec{b}+\vec{c}$ be inclined to $\vec{a},\vec{b}$ and $\vec{c}$ at angles α , β and γ respectively.

Then, we have

$$\cos\alpha = \frac{\left(\vec{a}+\vec{b}+\vec{c}\right).\vec{a}}{\left|\vec{a}+\vec{b}+\vec{c}\right|.\left|\vec{a}\right|}$$

$$= \frac{\left(\vec{a}.\vec{a}+\vec{b}.\vec{a}+\vec{c}.\vec{a}\right)}{\left|\vec{a}+\vec{b}+\vec{c}\right|.\left|\vec{a}\right|}$$

$$= \frac{\left(\left|\vec{a}\right|^2+0+0\right)}{\left|\vec{a}+\vec{b}+\vec{c}\right|.\left|\vec{a}\right|} = \frac{\left|\vec{a}\right|}{\left|\vec{a}+\vec{b}+\vec{c}\right|} \quad ..(1)$$

$$\cos\beta = \frac{\left(\vec{a}+\vec{b}+\vec{c}\right).\vec{b}}{\left|\vec{a}+\vec{b}+\vec{c}\right|.\left|\vec{b}\right|}$$

$$= \frac{\left(\vec{a}.\vec{b}+\vec{b}.\vec{b}+\vec{c}.\vec{b}\right)}{\left|\vec{a}+\vec{b}+\vec{c}\right|.\left|\vec{b}\right|}$$

$$= \frac{\left(0+\left|\vec{b}\right|^2+0\right)}{\left|\vec{a}+\vec{b}+\vec{c}\right|.\left|\vec{b}\right|} = \frac{\left|\vec{b}\right|}{\left|\vec{a}+\vec{b}+\vec{c}\right|} \quad ..(2)$$

$$\cos\gamma = \frac{\left(\vec{a}+\vec{b}+\vec{c}\right).\vec{a}}{\left|\vec{a}+\vec{b}+\vec{c}\right|.\left|\vec{a}\right|}$$

$$= \frac{\left(\vec{a}.\vec{c}+\vec{b}.\vec{c}+\vec{c}.\vec{c}\right)}{\left|\vec{a}+\vec{b}+\vec{c}\right|.\left|\vec{a}\right|}$$

$$= \frac{\left(0+0+\left|\vec{c}\right|^2\right)}{\left|\vec{a}+\vec{b}+\vec{c}\right|.\left|\vec{a}\right|} = \frac{\left|\vec{c}\right|}{\left|\vec{a}+\vec{b}+\vec{c}\right|} \quad ..(3)$$

From (1), (2) and (3)

As $\left|\vec{a}\right| = \left|\vec{b}\right| = \left|\vec{c}\right|$

Hence, $\cos\alpha = \cos\beta = \cos\gamma$

$\Rightarrow \alpha = \beta = \gamma$

Hence, the vector $\vec{a}+\vec{b}+\vec{c}$ are equal inclined to $\vec{a}, \vec{b}$ and $\vec{c}$.

15. Prove that $\left(\vec{a}+\vec{b}\right).\left(\vec{a}+\vec{b}\right)=\left|\vec{a}\right|^2+\left|\vec{b}\right|^2$, if and only if $\vec{a}, \vec{b}$ are perpendicular, given $\vec{a}\neq\vec{0}, \vec{b}\neq\vec{0}$.

15. $\left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right)=|\vec{a}|^2+|\vec{b}|^2$

$\Leftrightarrow \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b}=|\vec{a}|^2+|\vec{b}|^2$ [Distributivity of scalar products over addition]

$\Leftrightarrow |\vec{a}|^2+2\vec{a}\cdot\vec{b}+|\vec{b}|^2=|\vec{a}|^2+|\vec{b}|^2$ [$\vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{a}$ (Scalar product is commutative)]

$\Leftrightarrow 2\vec{a}\cdot\vec{b}=0$

$\Leftrightarrow \vec{a}\cdot\vec{b}=0$

$\therefore \vec{a}$ and $\vec{b}$ are perpendicular. $\left[\vec{a}\neq 0, \vec{b}\neq 0 \text{ (Given)}\right]$

16. If θ is the angle between two vectors $\vec{a}$ and $\vec{b}$, then $\vec{a}\,.\,\vec{b}\geq 0$ only when

(A) $0<\theta<\dfrac{\pi}{2}$

(B) $0\leq\theta<\dfrac{\pi}{2}$

(C) $0\leq\theta<\pi$

(D) $0\leq\theta<\pi$

16. let θ is the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then $\vec{a}$ and $\vec{b}$ are non-zero vectors so that $|a|$ and $|b|$ are positive.

As we know $\vec{a}.\vec{b}=|\vec{a}|\,|\vec{b}|\cos\theta$

For $\vec{a}.\vec{b}>0$

$\Rightarrow |\vec{a}|\,|\vec{b}|\cos\theta>0$

As $|\vec{a}|$ and $|\vec{b}|$ are positive.

$\Rightarrow \cos\theta>0$

$\Rightarrow 0\leq\theta\leq\dfrac{\pi}{2}$

Hence, $\vec{a}.\vec{b}>0$ when $0\leq\theta\leq\dfrac{\pi}{2}$

The correct answer is (B).

17. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and q is the angle between them. Then $\vec{a}+\vec{b}$ is a unit vector if

(A) $\theta=\dfrac{\pi}{4}$

(B) $\theta=\dfrac{\pi}{3}$

(C) $\theta = \frac{\pi}{2}$ (D) $\theta = \frac{2\pi}{3}$

17. Let $\vec{a}$ and $\vec{b}$ be two unit vectors and θ be the angle between them.

Then, $|\vec{a}| = |\vec{b}| = 1$.

Now, $\vec{a}+\vec{b}$ is a unit vector if $|\vec{a}+\vec{b}| = 1$.

$$|\vec{a}+\vec{b}| = 1$$

$$\Rightarrow (\vec{a}+\vec{b})^2 = 1$$

$$\Rightarrow \left(\vec{a}+\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right) = 1$$

$$\Rightarrow \vec{a}\cdot\vec{a}+\vec{a}\cdot\vec{b}+\vec{b}\cdot\vec{a}+\vec{b}\cdot\vec{b} = 1$$

$$\Rightarrow |\vec{a}|^2 + 2\vec{a}\cdot\vec{b} + |\vec{b}|^2 = 1$$

$$\Rightarrow 1^2 + 2|\vec{a}||\vec{b}|\cos\theta + 1^2 = 1$$

$$\Rightarrow 1 + 2\cdot 1\cdot 1\cos\theta + 1 = 1$$

$$\Rightarrow \cos\theta = -\frac{1}{2}$$

$$\Rightarrow \theta = \frac{2\pi}{3}$$

Hence, $\vec{a}+\vec{b}$ is a unit vector if $\theta = \frac{2\pi}{3}$

The correct answer is D.

18. The value of $\hat{i}.\left(\hat{j}\times\hat{k}\right)+\hat{j}.\left(\hat{i}\times\hat{k}\right)+\hat{k}.\left(\hat{i}\times\hat{j}\right)$ is

(A) 0 (B) –1

(C) 1 (D) 3

18. given: $\hat{i}.\left(\hat{j}\times\hat{k}\right)+\hat{j}.\left(\hat{i}\times\hat{k}\right)+\hat{k}.\left(\hat{i}\times\hat{j}\right)$

$$\hat{i}.\left(\hat{j}\times\hat{k}\right)+\hat{j}.\left(\hat{i}\times\hat{k}\right)+\hat{k}.\left(\hat{i}\times\hat{j}\right) = \hat{i}.\hat{i}+\hat{j}.\left(-\hat{j}\right)+\hat{k}.\hat{k}$$

$$= 1 - \hat{j}.\hat{j} + 1$$

=1-1+1

=1

The correct answer is (C).

19. If θ is the angle between any two vectors $\vec{a}$ and $\vec{b}$, then $\left|\vec{a}\,.\,\vec{b}\right| = \left|\vec{a}\times\vec{b}\right|$ when θ is equal to

(A) 0 (B) $\frac{\pi}{4}$

(C) $\frac{\pi}{2}$ (D) π

19. Let θ be the angle between two vectors $\vec{a}$ and $\vec{b}$.

Then, without loss of generality, $\vec{a}$ and $\vec{b}$ are non-zero vectors, so that $|\vec{a}|$ and $|\vec{b}|$ are positive.

$$\left|\vec{a}\cdot\vec{b}\right| = \left|\vec{a}\times\vec{b}\right|$$

$$\Rightarrow |\vec{a}||\vec{b}|\cos\theta = |\vec{a}||\vec{b}|\sin\theta$$

$$\Rightarrow \cos\theta = \sin\theta \left[|\vec{a}| \text{ and } |\vec{b}| \text{ are positive}\right]$$

$$\Rightarrow \tan\theta = 1$$

$$\Rightarrow \theta = \frac{\pi}{4}$$

Hence, $\left|\vec{a}\cdot\vec{b}\right| = \left|\vec{a}\times\vec{b}\right|$ when θ Is equal to $\frac{\pi}{4}$

The correct answer is B.