CHAPTER 11

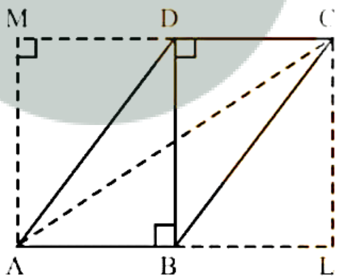
AREA OF PARALLELOGRAMS AND TRIANGLES

EXERCISE11

Answer 1:

(i) No, it does not lie on the same base and between the same parallels.
(ii) No, it does not lie on the same base and between the same parallels.
(iii) Yes, it lies on the same base and between the same parallels. The same base is AB and the parallels are AB and DE.
(iv) No, it does not lie on the same base and between the same parallels.
(v) Yes, it lies on the same base and between the same parallels. The same base is BC and the parallels are BC and AD.
(vi) Yes, it lies on the same base and between the same parallels. The same base is CD and the parallels are CD and BP.

Answer 2:



Given: A quadrilateral $ABCD$ and BD is a diagonal.
To prove: $ABCD$ is a parallelogram.
Construction: Draw AM ⊥ DC and CL ⊥ AB (extend DC and AB). Join AC, the other diagonal of ABCD.

Proof: (quad. ABCD) = area(ΔABD) + area(ΔDCB)
= 2 area(ΔABD) [area(ΔABD) = area(ΔDCB)]

$\text{Area}(\Delta ABD) = \frac{1}{2} \times \text{area(quad. } ABCD)$ ...(i)

Again, area(quad. $ABCD$) = area(ΔABC) + area(ΔCDA)
= 2 area(Δ ABC) [area(ΔABC) = area(ΔCDA)]

$\therefore$ area(ΔABC) = $\frac{1}{2} \times$ area(quad. $ABCD$) ...(ii)

From (i) and (ii), we have:

$\text{area}(\Delta ABD) = \text{area}(\Delta ABC) = \frac{1}{2} \times AB \times BD = \frac{1}{2} \times AB \times CL$

$\Rightarrow CL = BD$

$\Rightarrow DC \parallel AB$

Similarly, $AD \parallel BC$.

Hence, $ABCD$ is a parallelogram.

$\therefore$ area(| | gm $ABCD$) = base height = $5 \times 7 = 35 \text{ cm}^2$

Answer 3:

area(parallelogram $ABCD$) = base $\times$ height

$\Rightarrow AB \times DL = AD \times BM$

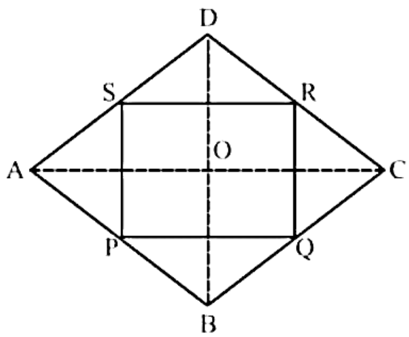
$\Rightarrow 10 \times 6 = AD \times BM$

$\Rightarrow AD \times 8 = 60 \text{ cm}^2$

$\Rightarrow AD =$ 7.5 cm

$\therefore AD = 7.5$ cm

Answer 4:



Let $ABCD$ be a rhombus and P, Q, R and S be the midpoints of AB, BC, CD and DA,

respectively.
Join the diagonals, AC and BD.
In $\Delta\, ABC$, we have:
$PQ \mid\mid AC$ and $PQ = \frac{1}{2} AC$ [By midpoint theorem]
$PQ = \frac{1}{2} \times 16 = 8$ cm
Again, in ΔDAC, the points S and R area the midpoints of AD and DC, respectively.
$\therefore SR \mid\mid AC$ and $SR = \frac{1}{2} AC$ [By midpoint theorem]
$SR = \frac{1}{2} * 12 = 6$
Area of PQRS= length × breadth $= 6 \times 8 = 48$ cm^2

Answer 5:

$\text{area(trapezium)} = \frac{1}{2} \times \text{(sum of parallel sides)} \times \text{(distance between them)}$
$= \frac{1}{2} \times (9 + 6) \times 8$
$= 60$ cm^2
Hence, the area of the trapezium is 60 cm^2.

Answer 6:

(i) In ΔBCD,
$DB^2 + BC^2 = DC^2 \Rightarrow DB^2 = 17^2 - 8^2 = 225 \Rightarrow DB = 15$ cm
Area (ΔBCD) $= \frac{1}{2} \times b \times h = \frac{1}{2} \times 8 \times 15 = 60$ cm^2
In ΔBAD,
$DA^2 + AB^2 = DB^2 \Rightarrow AB^2 = 15^2 - 9^2 = 144 \Rightarrow AB = 12$ cm
Area(ΔDAB) $= \frac{1}{2} \times b \times h = \frac{1}{2} \times 9 \times 12 = 54$ cm^2

Area of quad. $ABCD$ = Area(ΔDAB) + Area(ΔBCD) = 54 + 60 = 114 cm^2.

(ii) Area of trap(PQRS) $= \frac{1}{2} \times (8+16) \times 8 = 96$ cm^2

Answer 7:

ΔADL is a right angle triangle.
So, $DL = \sqrt{5^2 + 4^2} = \sqrt{9} = 3$ cm

Similarly, in ΔBMC, we have,
$MC = \sqrt{5^2 + 4^2} = \sqrt{9} = 3$ cm
$\therefore DC = DL + LM + MC = 3 + 7 + 3 = 13$ cm

Now, area(trapezium. $ABCD$) = $\frac{1}{2}$ × (sum of parallel sides) × (distance between them)

$= \frac{1}{2} \times (7 + 13) \times 4$

$= 40 \text{ cm}^2$

Hence, $DC = 13$ cm and area of trapezium = 40 cm^2

.

Answer 8:

area(quad. $ABCD$) = area(ΔABD) + area (ΔDBC)

area(ΔABD) = $\frac{1}{2}$ × base × height = $\frac{1}{2} \times BD \times AL$ ...(i)

area(ΔDBC) = $\frac{1}{2} \times BD \times CL$ ...(ii)

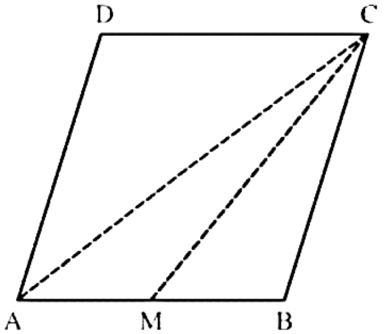
From (i) and (ii), we get:

area(quad $ABCD$) = $\frac{1}{2} \times BD \times AL + \frac{1}{2} \times BD \times CL$

⇒area(quad $ABCD$) = $\frac{1}{2} \times BD \times (AL + CL)$

Hence, proved.

Answer 9:



Join AC.
AC divides parallelogram ABCD into two congruent triangles of equal area.

$\text{area}(\Delta ABC) = \text{area}(\Delta ACD) = \frac{1}{2} \times \text{area}(ABCD)$

M is the midpoint of AB. So, CM is the median.
CM divides ΔABC in two triangles with equal area.

$\text{area}(\Delta AMC)=\text{area}(\Delta BMC)= \frac{1}{2} \times \text{area}(\Delta ABC)$

$\text{area}(AMCD) = \text{area}(\Delta ACD) + \text{area}(\Delta AMC)$

$= \text{area}(\Delta ABC) + \text{area}(\Delta AMC)$

$= \text{area}(\Delta ABC) + \frac{1}{2} \times \text{area}(\Delta ABC)$

$\Rightarrow 24 = \frac{3}{2} \times \text{area}(\Delta ABC)$

$\Rightarrow \text{area}(\Delta ABC)=16 \text{ cm}^2$

Answer 10:

$\text{area}(\text{quad } ABCD) = \text{area}(\Delta ABD) + \text{area}(\Delta BDC)$

$= \frac{1}{2} \times BD \times AL \ +12 \times BD \times CM$

$= \frac{1}{2} \times BD \times (AL + CM)$

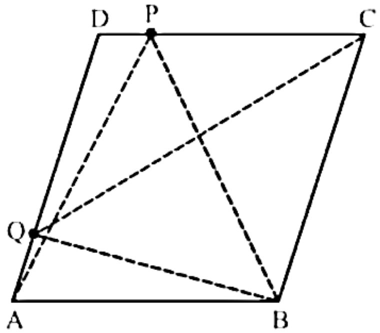
By substituting the values, we have;

$\text{area}(\text{quad } ABCD) = \frac{1}{2} \times 14 \times (8 + 6)$

$= 7 \times 14$

$= 98 \text{ cm}^2$

Answer 11:



We know

area(ΔAPB) = $\frac{1}{2}$ × area(ABCD) ....(1)

Similarly,

area(ΔBQC) = $\frac{1}{2}$ × area(ABCD) .....(2)

From (1) and (2)

area(ΔAPB) = area(ΔBQC)

Hence Proved

Answer 12:

(i) We know that parallelograms on the same base and between the same parallels area equal in area

So, area($MNPQ$) = area($ABPQ$) (Same base PQ and MB || PQ) .....(1)

(ii) If a parallelogram and a triangle area on the same base and between the same parallels then the area of the triangle is equal to half the area of the parallelogram.

So, area(ΔATQ) = $\frac{1}{2}$ area($ABPQ$) (Same base AQ and AQ || BP) .....(2)

From (1) and (2)

area(ΔATQ) = $\frac{1}{2}$ area($MNPQ$)

Answer 13:

ΔCDA and ΔCBD lies on the same base and between the same parallel lines.
So, area(ΔCDA) = area(ΔCDB) ...(i)
Subtracting area(ΔOCD) from both sides of equation (i), we get:
area(ΔCDA) −area(ΔOCD) = area(ΔCDB) −area (ΔOCD)
$\Rightarrow$ area(ΔAOD) = area(ΔBOC)

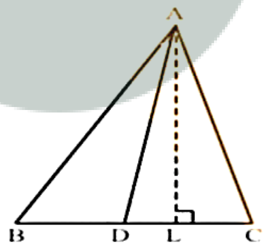
Answer 14:

ΔDEC and ΔDEB lies on the same base and between the same parallel lines.
So, area(ΔDEC) = area(ΔDEB) ...(1)

(i) On adding area(ΔADE) in both sides of equation (1), we get:
area(ΔDEC) + area(ΔADE) = area(ΔDEB) + area(ΔADE)
$\Rightarrow$ area(ΔACD) = area(ΔABE)

(ii) On subtracting area(ODE) from both sides of equation (1), we get:
area(ΔDEC) −area(ΔODE) = area(ΔDEB) −area(ΔODE)
$\Rightarrow$ area(ΔOCE) = area(ΔOBD)

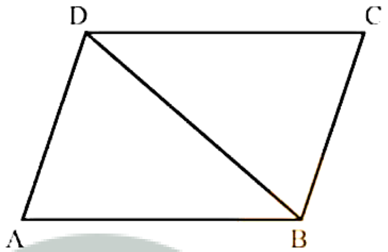
Answer 15:



Let AD is a median of ΔABC and D is the midpoint of BC. AD divides ΔABC in two triangles: ΔABD and ΔADC.
To prove: area(ΔABD) = area(ΔADC)
Construction: Draw $AL \perp BC$.
Proof:
Since D is the midpoint of BC, we have:
$BD = DC$
Multiplying with $\frac{1}{2}$, AL on both sides, we get:

$\frac{1}{2} \times BD \times AL = \frac{1}{2} \times DC \times AL$
$\Rightarrow$ area(ΔABD) = area(ΔADC)

Answer 16:



Let $ABCD$ be a parallelogram and BD be its diagonal.
To prove: area(ΔABD) = area(ΔCDB)

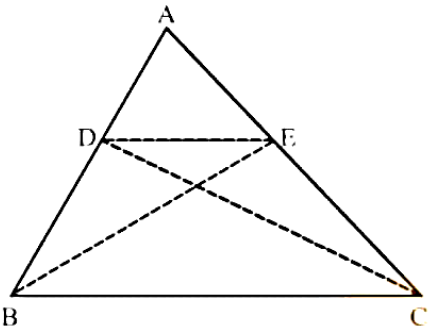
Proof:
In ΔABD and ΔCDB, we have:
$AB = CD$ [Opposite sides of a parallelogram]
$AD = CB$ [Opposite sides of a parallelogram]

$BD = DB$ [Common]
i.e., $\Delta ABD \cong \Delta CDB$ [SSS criteria]
$\therefore$ area(ΔABD) = area(ΔCDB)

Answer 17:

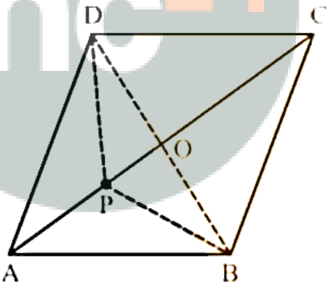
Line segment CD is bisected by AB at O (Given)
CO = OD .....(1)
In ΔCAO,
AO is the median. (From (1))
So, area(ΔCAO) = area(ΔDAO) .....(2)
Similarly,
In ΔCBD,
BO is the median (From (1))
So, area(ΔCBO) = area(ΔDBO) .....(3)
From (2) and (3) we have
area(ΔCAO) + area(ΔCBO) = area(ΔDBO) + area(ΔDAO)
⇒area(ΔABC) = area(ΔABD)

Answer 18:



area(ΔBCD) = area(ΔBCE) (Given)
We know, triangles on the same base and having equal area lie between the same parallels.
Thus, DE || BC.

Answer 19:



Join BD.
Let BD and AC intersect at point O.
O is thus the midpoint of DB and AC.
PO is the median of ΔDPB,
So,
area(ΔDPO)=area(ΔBPO) .....(1)

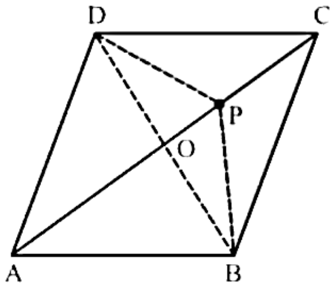
area(ΔADO)=area(ΔABO) .....(2)

Case 1:

(2)−(1)

$\Rightarrow$area(ΔADO)−area(ΔDPO)=area(ΔABO)−area(ΔBPO)
Thus, area(ΔADP) = area(ΔABP)

Case II:



area(ΔADO)+area(ΔDPO)=area(ΔABO)+area(ΔBPO)

Thus, area(ΔADP) = area(ΔABP)

Answer 20:

Given: $BO = OD$
To prove: area(ΔABC) = area(ΔADC)
Proof:
Since $BO = OD$, O is the mid point of BD.
We know that a median of a triangle divides it into two triangles of equal area.
CO is a median of ΔBCD.
i.e., area(ΔCOD) = area (ΔCOB) ...(i)

AO is a median of ΔABD.
i.e., area(ΔAOD) = area(ΔAOB) ...(ii)

From (i) and (ii), we have:
area(ΔCOD) + area(ΔAOD) = area(ΔCOB) + area(ΔAOB)
$\therefore$ area(ΔADC) = area(ΔABC)

Answer 21:

Given: D is the midpoint of BC and E is the midpoint of AD.

To prove: area(ΔBEC)= $\frac{1}{2}$ × area(ΔABC)

Proof:
Since E is the midpoint of AD, BE is the median of ΔABD.
We know that a median of a triangle divides it into two triangles of equal areas.

i.e., area(ΔBED) = $\frac{1}{2}$ × area(ΔABD) ...(i)

Also, area(ΔCDE) = $\frac{1}{2}$ × area(ΔADC) ...(ii)

From (i) and (ii), we have:

area(ΔBED) + area(ΔCDE) = $\frac{1}{2}$ × area(ΔABD) + $\frac{1}{2}$ × area(ΔADC)

⇒ area(ΔBEC) = $\frac{1}{2}$ × [area(ΔABD) + area(ΔADC)]

⇒ area(ΔBEC) = $\frac{1}{2}$ × area(ΔABC)

Answer 22:

D is the midpoint of side BC of ΔABC.
⇒ AD is the median of ΔABC.

⇒area(ΔABD)=area(ΔACD)= $\frac{1}{2}$ × area(ΔABC)

E is the midpoint of BD of ΔABD,
⇒AE is the median of ΔABD

⇒area(ΔABE)=area(ΔAED)= $\frac{1}{2}$ × area(ΔABD)= $\frac{1}{4}$ × area(ΔABC)

Also, O is the midpoint of AE,
⇒BO is the median of ΔABE,

⇒area(ΔABO)=area(ΔBOE)= $\frac{1}{2}$ × area(ΔABE)= $\frac{1}{4}$ × area(ΔABD)= $\frac{1}{8}$ × area(ΔABC)

Thus, area(ΔBOE) = $\frac{1}{8}$ × area(ΔABC)

Answer 23:

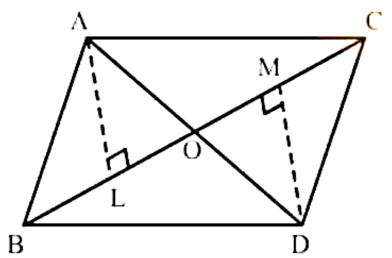
In ΔMQC and ΔMPB,
MC = MB (M is the midpoint of BC)
∠CMQ = ∠BMP (Vertically opposite angles)
∠MCQ = ∠MBP (Alternate interior angles on the parallel lines AB and DQ)
Thus, ΔMQC ≅ ΔMPB (ASA congruency)
⇒area(ΔMQC) = area(ΔMPB)
⇒area(ΔMQC) + area(APMCD) = area(ΔMPB) + area(APMCD)
⇒area(APQD) = area(ABCD)

Answer 24:

We have:
area(quad. $ABCD$) = area(ΔACD) + area(ΔABC)
area(ΔABP) = area(ΔACP) + area(ΔABC)

ΔACD and ΔACP area on the same base and between the same parallels AC and DP.
∴ area(ΔACD) = area(ΔACP)
By adding area(ΔABC) on both sides, we get:
area(ΔACD) + area(ΔABC) = area(ΔACP) + area(ΔABC)
⇒area (quad. $ABCD$) = area(ΔABP)
Hence, proved.

Answer 25:



Given: ΔABC and ΔDBC area on the same base BC.
area(ΔABC) = area(ΔDBC)
To prove: BC bisects AD

Construction: Draw AL ⊥ BC and DM ⊥ BC.

Proof:

Since ΔABC and ΔDBC area on the same base BC and they have equal area, their altitudes must be equal.

i.e., $AL = DM$

Let AD and BC intersect at O.

Now, in ΔALO and ΔDMO, we have:

$AL = DM$

$\angle ALO = \angle DMO = 90°$

$\angle AOL = \angle DOM$ (Vertically opposite angles)

i.e., $\Delta ALO \cong \Delta DMO$

$OA = OD$

Hence, BC bisects AD.

Answer 26:

In ΔMDA and Δ MCP,

∠DMA = ∠CMP (Vertically opposite angles)

∠MDA = ∠MCP (Alternate interior angles)

AD = CP (Since AD = CB and CB = CP)

So, Δ MDA ≅ Δ MCP (ASA congruency)

⇒DM = MC (CPCT)

⇒M is the midpoint of DC

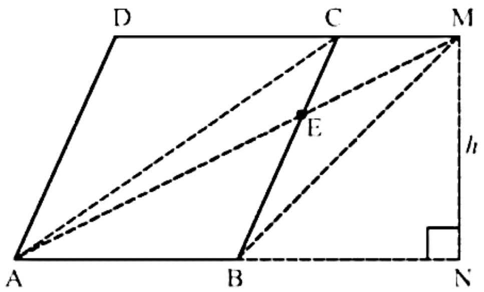
⇒BM is the median of Δ BDC

⇒area(Δ BMC) = area(Δ DMB) = 7 cm^2

area(Δ BMC) + area(Δ DMB) = area(Δ DBC) = 7 + 7 = 14 cm^2

Area of parallelogram ABCD = 2 × area(Δ DBC) = 2 × 14 = 28 cm^2

Answer 27:



Join BM and AC.

area(Δ ADC) = $\frac{1}{2}$ × b × h

= $\frac{1}{2}$ ×DC × h

area(Δ ABM) = $\frac{1}{2}$ ×AB × h

AB = DC (Since ABCD is a parallelogram)

So, area(Δ ADC) = area(Δ ABM)

⇒area(Δ ADC) + area(Δ AMC) = area(Δ ABM) + area(Δ AMC)

⇒area(Δ ADM) = area(ABMC)

Hence Proved

Answer 28:

Given: $ABCD$ is a parallelogram and P, Q, R *and* S area the midpoints of sides AB, BC, CD and DA, respectively.

To prove: area(parallelogram $PQRS$) = $\frac{1}{2}$ × area(parallelogram $ABCD$)

Proof:

In ΔABC, $PQ \parallel AC$ and $PQ = \frac{1}{2} \times AC$ [By midpoint theorem]

Again, in ΔDAC, the points S and R area the mid points of AD and DC, respectively.

∴ SR || AC and $SR = \frac{1}{2} \times AC$ [By midpoint theorem]

Now, $PQ \parallel AC$ and $SR \parallel AC$

⇒ $PQ \parallel SR$

Also, $PQ = SR = \frac{1}{2} \times AC$

∴ $PQ \parallel SR$ and $PQ = SR$

Hence, $PQRS$ is a parallelogram.

Now, area(parallelogram $PQRS$) = area(ΔPSQ) + area(ΔSRQ) ...(i)
also,

area(parallelogram $ABCD$) = area(parallelogram $ABQS$)+area(parallelogram $SQCD$) ...(ii)

ΔPSQ and parallelogram $ABQS$ area on the same base and between the same parallel lines.
So, area(ΔPSQ) =12 × area(parallelogram $ABQS$) ...(iii)
Similarly, ΔSRQ and parallelogram $SQCD$ area on the same base and between the same parallel lines.
So, area(ΔSRQ) = 12 × area(parallelogram $SQCD$) ...(iv)
Putting the values from (iii) and (iv) in (i), we get:

area(parallelogram $PQRS$) $= \frac{1}{2}$ × area(parallelogram $ABQS$) + $\frac{1}{2}$ × area(parallelogram $SQCD$)

From (ii), we get:

area(parallelogram($PQRS$) $= \frac{1}{2}$ × area(parallelogram $ABCD$)

Answer 29:

CF is median of ΔABC.

⇒area(ΔBCF) = $\frac{1}{2}$ (ΔABC) .....(1)

Similarly, BE is the median of ΔABC,

⇒area(ΔABE) = $\frac{1}{2}$ (ΔABC) .....(2)

From (1) and (2) we have
area(ΔBCF) = area(ΔABE)
⇒area(ΔBCF) − area(ΔBFG) = area(ΔABE) − area(ΔBFG)
⇒area(ΔBCG) = area(AFGE)

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Answer 30:

Given: D is a point on BC of ΔABC, such that BD = $\frac{1}{2}DC$

To prove: area(ΔABD) = 13area(ΔABC)
Construction: Draw AL ⊥ BC.
Proof:
In ΔABC, we have:
$BC = BD + DC$
⇒ BD + 2 BD = 3 × BD
Now, we have:

area(ΔABD) $= \frac{1}{2}$ × BD × AL area(ΔABC) = $\frac{1}{2}$ × BC × AL

⇒ area(ΔABC) = $\frac{1}{2}$ × $3BD \times AL = 3 \times (\frac{1}{2} \times BD \times AL)$

⇒ area(ΔABC) = 3 × area(ΔABD)

∴ area(ΔABD) = $\frac{1}{3}$ area(ΔABC)

Answer 31:

E is the midpoint of CA.
So, AE = EC .....(1)
Also, BD = $\frac{1}{2}$ CA (Given)
So, BD = AE .....(2)
From (1) and (2) we have
BD = EC
BD || CA and BD = EC so, BDEC is a parallelogram
BE acts as the median of ΔABC
so, area(ΔBCE) = area(ΔABE) = 12area(ΔABC) .....(1)
area(ΔDBC) = area(ΔBCE) .....(2)

From (1) and (2)
area(ΔABC) = 2area(ΔDBC)

Answer 32:

Given: *ABCDE* is a pentagon. *EG* || *DA* and *CF* || *DB*.
To prove: area(pentagon *ABCDE*) = area(Δ*DGF)*
Proof:
area(pentagon *ABCDE*) = area(Δ*DBC*) + area(Δ*ADE*) + area(Δ*ABD*) ...(i)
Also, area(Δ*DGF)* = area(Δ*DBF*) + area(Δ*ADG*) + area(Δ*ABD*) ...(ii)

Now, ΔDBC and Δ*DBF* lie on the same base and between the same parallel lines.
∴ area(Δ*DBC*) = area(Δ*DBF*) ...(iii)
Similarly, ΔADE and Δ*ADG* lie on same base and between the same parallel lines.
∴ area(Δ*ADE*) = area(Δ*ADG*) ...(iv)

From (iii) and (iv), we have:
area(Δ*DBC*) + area(Δ*ADE*) = area(Δ*DBF*) + area(Δ*ADG*)
Adding area(Δ*ABD*) on both sides, we get:
area(Δ*DBC*) + area(Δ*ADE*) + area(Δ*ABD*) = area (Δ*DBF*) + area(Δ*ADG*) + area(Δ*ABD*)

By substituting the values from (i) and (ii), we get:
area(pentagon $ABCDE$) = area(ΔDGF)

Answer 33:

area(ΔCFA)=area(ΔCFB)

(Triangles on the same base CF and between the same parallels CF || BA will be equal in area)
⇒area(ΔCFA)−area(ΔCFG)
=area(ΔCFB)−area(ΔCFG)
⇒area(ΔAFG)=area(ΔCBG)

Hence Proved

Answer 34:

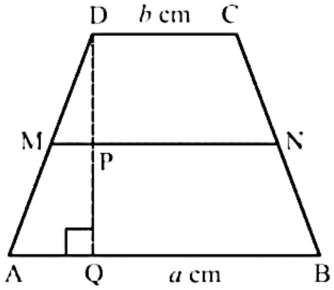
Given: D is a point on BC of Δ ABC, such that BD : DC = $m : n$
To prove: area(ΔABD) : area(ΔADC) = $m : n$
Construction: Draw $AL \perp BC$.
Proof:
area(ΔABD) = $\frac{1}{2} \times BD \times AL$ ...(i)
area(ΔADC) = $\frac{1}{2} \times DC \times AL$ ...(ii)
Dividing (i) by (ii), we get:

$$\frac{\text{area}(\Delta ABD)}{\text{area}(\Delta ADC)} = \frac{\frac{1}{2} \times BD \times AL}{\frac{1}{2} \times DC \times AL}$$

$$\frac{\text{BD}}{\text{DC}} = \frac{m}{n}$$

∴ area(ΔABD) : area(ΔADC) = $m : n$

Answer 35:



Construction: Draw a perpendicular from point D to the opposite side AB, meeting AB at Q and MN at P.

Let length DQ = h

Given, M and N area the midpoints of AD and BC respectively.

So, MN || AB || DC and MN = $\frac{1}{2}$ (AB+DC)=($\frac{a+b}{2}$)

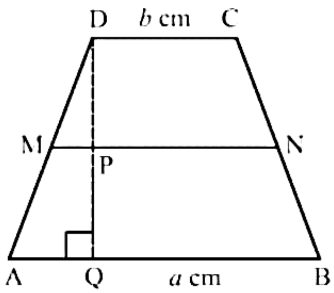
M is the mid point of AD and MP || AB so by converse of mid point theorem, MP || AQ and P will be the mid point of DQ.

area(DCNM)= $\frac{1}{2}$ ×DP(DC+MN)= $\frac{1}{2}$ $h(b+\frac{a+b}{2})$= $\frac{h}{4}$ $(a+3b)$

area(MNBA)= $\frac{1}{2}$ ×PQ(AB+MN)= $\frac{1}{2}$ $h(a+\frac{a+b}{2})$= $\frac{h}{4}$ $(b+3a)$

area(DCNM) : area(MNBA) = $(a + 3b)$: $(3a + b)$

Answer 36:



Construction: Draw a perpendicular from point D to the opposite side CD, meeting CD at Q and EF at P.

Let length AQ = h
Given, E and F area the midpoints of AD and BC respectively.
So, EF || AB || DC and EF = $\frac{1}{2}$ (AB+DC)=($\frac{a+b}{2}$)

E is the mid point of AD and EP || AB so by converse of mid point theorem,
EP || DQ and P will be the mid point of AQ.
area(ABFE)= $\frac{1}{2}\times AP(AB+EF)=12h(b+\frac{a+b}{2})=\frac{h}{4}(a+3b)$
area(EFCD)= $\frac{1}{2}\times PQ(CD+EF)=12h(a+\frac{a+b}{2})=\frac{h}{4}(b+3a)$
area(ABEF) : area(EFCD) = $(a + 3b) : (3a + b)$
Here a = 24 cm and b = 16 cm
So,

$$\frac{area(ABEF)}{area(EFCD)}=\frac{24+3\times16}{16+3\times24}=\frac{9}{11}$$

Answer 37:

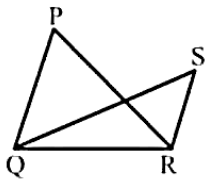
In ΔPAC,
PA || DE and E is the midpoint of AC
So, D is the midpoint of PC by converse of midpoint theorem.
Also, DE=$\frac{1}{2}$ PA .....(1)
Similarly, DE=$\frac{1}{2}$ AQ .....(2)
From (1) and (2) we have
PA = AQ
ΔABQ and ΔACP area on same base PQ and between same parallels PQ and BC
area(ΔABQ) = area(ΔACP)

Answer 38:

In ΔRSC and ΔPQB
∠CRS = ∠BPQ (CD || AB) so, corresponding angles area equal)
∠CSR = ∠BQP (SC || QB so, corresponding angles area equal)
SC = QB (BQSC is a parallelogram)
So, ΔRSC ≅ΔPQB (AAS congruency)
Thus, area(ΔRSC) = area(ΔPQB)

MULTIPLE-CHOICE QUESTIONS

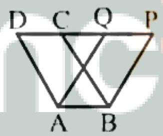
Answer 1:



(b)

In this figure, both the triangles area on the same base (QR) but not on the same parallels.

Answer 2:



(c)

In this figure, the following polygons lie on the same base and between the same parallel lines:
a) Parallelogram $ABCD$
b) Parallelogram $ABPQ$

Answer 3:

(a) triangles of equal areas

Answer 4:

(c)114 cm^2

area (quad. $ABCD$) = area ($\Delta\, ABC$) + area ($\Delta\, ACD$)
In right angle triangle ACD, we have:

$AC = \sqrt{17^2 + 8^2} = \sqrt{225} = 15$ cm
In right angle triangle ABC, we have:
$BC = \sqrt{15^2 + 9^2} = \sqrt{144} = 12$ cm
Now, we have the following:
$\text{area}(\Delta ABC) = \frac{1}{2} \times 12 \times 9 = 54 \text{ cm}^2$
$\text{area}(\Delta ADC) = \frac{1}{2} \times 15 \times 8 = 60 \text{ cm}^2$
$\text{area}(\text{quad. } ABCD) = 54 + 60 = 114 \text{ cm}^2$

Answer 5:

(c)124 cm^2

In the right angle triangle BEC, we have:
$EC = \sqrt{17^2 + 15^2} = \sqrt{64} = 8$ cm
$\text{area}(\text{trapez. } ABCD) = \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{distance between them}$

$= \frac{1}{2} \times 31 \times 8 = 124 \text{cm}^2$

Answer 6:

(c) 34 cm^2

$\text{area}(\text{parallelogram } ABCD) = \text{base} \times \text{height} = 5 \times 6.8 = 34 \text{ cm}^2$

Answer 7:

(d) 13 cm^2
The diagonals of a parallelogram divides it into four triangles of equal areas.
$\therefore$ Area of $\Delta OAB = \frac{1}{4} \times \text{area}(\|\text{gm } ABCD)$
$\Rightarrow \text{area}(\Delta OAB) = \frac{1}{4} \times 52 = 13 \text{ cm}^2$

Answer 8:

(a) 40 cm^2
area(||gm $ABCD$) = base × height = $10 \times 4 = 40$ cm^2

Answer 9 (c)

Area of a parallelogram is base into height.
Height = DL = NB
Base = AB = CD
So, area of parallelogram ABCD = DC × DL
Hence, the correct answer is option (c).

Answer 10: (b)

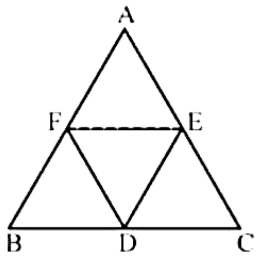
Parallelograms on the same base and between the same parallels area equal in area.
So, the ratio of their areas will be 1 : 1.
Hence, the correct answer is option (b).

Answer 11: (a)

We know parallelogram on the same base and between the same parallels area equal in area.
Here, AB is the common base and AB || PD
Hence, area(ABCD) = area(ABPQ) .....(1)
Also, when a triangle and a parallelogram area on the same base and between the same parallels then the
area of triangle is half the area of the parallelogram.
Here, for the ΔBMP and parallelogram ABPQ, BP is the common base and they area between the common parallels BP and AQ

So, area(ΔBMP) = $\frac{1}{2}$ area(||gm ABPQ) .....(2)
From (1) and (2) we have
area(ΔBMP) = $\frac{1}{2}$ area(||gm ABCD)
Thus, the given statement is true.
Hence, the correct answer is option (a).

Answer 12: (a)



D, E and F area the midpoints of sides BC, AC and AB respectively.
On joining FE, we divide ΔABC into 4 triangles of equal area.
Also, median of a triangle divides it into two triangles with equal area
area(AFDE)=area(ΔAFE)+area(ΔFED)

$=2\text{area}(\Delta AFE)=2\times\frac{1}{4}\text{ area}(\Delta ABC)=\frac{1}{2}\text{ area}(\Delta ABC)$

Hence, the correct answer is option (a).

Answer 13:

(b) 96 cm^2

Area of the rhombus $=\frac{1}{2}\times$ product of diagonals

$=\frac{1}{2}\times 12\times 16 = 96\text{ cm}^2$

Answer 14:

(c) 65 cm^2

Area of the trapezium $=\frac{1}{2}\times$ (sum of parallel sides) × distance between them

$=\frac{1}{2}\times(12+8)\times 6.5$

$=65\text{ cm}^2$

Answer 15:

(b) 40 cm^2

In right angled triangle *MBC*, we have:
$MC = \sqrt{5^2 + 4^2} = \sqrt{9} = 3$ cm

In right angled triangle *ADL*, we have:
$DL = \sqrt{5^2 + 4^2} = \sqrt{9} = 3$ cm
Now, $CD = ML + MC + LD = 7 + 3 + 3 = 13$ cm
$\therefore$ Area of the trapezium $= \frac{1}{2} \times$ (sum of parallel sides) $\times$ distance between them
$= \frac{1}{2} \times (13 + 7) \times 4$
$= 40$ cm^2

Answer 16:

(b)128 cm^2

area(quad *ABCD*) = area (Δ *ABD*) + area (Δ *DBC*)

We have the following:
area(ΔABD) $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 16 \times 9 = 72$ cm^2
area(ΔDBC) $= \frac{1}{2} \times$ base $\times$ height $= \frac{1}{2} \times 16 \times 7 = 56$ cm^2

$\therefore$ area(quad *ABCD*) $= 72 + 56 = 128$ cm^2

Answer 17:

(a)$\sqrt{3}$:1

ABCD is a rhombus. So all of its sides area equal.
Now, $BC = DC$
$\Rightarrow \angle BDC = \angle DBC = x°$ (Angles opposite to equal sides area equal)

Also, $\angle BCD = 60°$
$\therefore\ x° + x° + 60° = 180°$
$\Rightarrow 2x° = 120°$
$\Rightarrow x° = 60°$
i.e., $\angle BCD = \angle BDC = \angle DBC = 60°$
So, ΔBCD is an equilateral triangle.
$\therefore\ BD = BC = a$
Also, $OB = \frac{a}{2}$
Now, in ΔOAB, we have:
$AB^2 = OA^2 + OB^2$
$\Rightarrow OA^2 = AB^2 - OB^2 = a^2 - (\frac{a}{2})^2 = a^2 - \frac{a^2}{4} = \frac{3a}{4}$

$\Rightarrow OA^2 = \frac{3a}{4}$

$\Rightarrow OA = \frac{\sqrt{3}a}{2}$

Now, $AC = 2\times OA = 2\times\frac{\sqrt{3}a}{2} = \sqrt{3}a \therefore\ AC:BD = \sqrt{3}a:a = \sqrt{3}:1$

Answer 18:

(d) 8 cm^2

Since ||gm $ABCD$ and ||gm $ABFE$ area on the same base and between the same parallel lines, we have:
area(||gm $ABFE$) = area(||gm $ABCD$) = 25 cm^2
$\Rightarrow$ area(ΔBCF) = area(||gm $ABFE$) − area(quad $EABC$) = (25 −17) = 8 cm^2

Answer 19:

(b) 1:4

ΔABC and ΔBDE area two equilateral triangles and D is the midpoint of BC.
Let $AB = BC = AC = a$
Then $BD = BE = ED = \frac{a}{2}$

$$\frac{\text{area}(\Delta BDE)}{\text{area}(\Delta ABC)} = \frac{\frac{\sqrt{3}}{4} \times AB^2}{\frac{\sqrt{3}}{4} \times BE^2} = \frac{(\frac{a}{2})^2}{a^2} = \frac{1}{4}$$

So, required ratio = 1 : 4

Answer 20:

(a) 8 cm^2

Let the distance between *AB* and *CD* be *h* cm.
Then area(||gm *APQD*) = *AP* × *h*

$= \frac{1}{2} \times AB \times h \qquad (AP = \frac{1}{2} AB)$

$= \frac{1}{2} \times \text{area}(\|\text{gm } ABCD) \qquad [\text{ area}(\|\text{ gm } ABCD) = AB \times h\ )$

$\therefore$ area (||gm *APQD*) = $\frac{1}{2} \times 16 = 8\ cm^2$

Answer 21:

(d) rhombus of 24 cm^2

We know that the figure formed by joining the midpoints of the adjacent sides of a rectangle is a rhombus.
So, *PQRS* is a rhombus and *SQ* and *PR* area its diagonals.
i.e., *SQ* = 8 cm and *PR* = 6 cm

$\therefore$ area(rhombus *PQRS*) = $\frac{1}{2}$ × product of diagonals

$= \frac{1}{2} \times 8 \times 6 = 24\ cm^2$

Answer 22:

(c) $\frac{1}{4}$ area (Δ *ABC*)

Since *D* is the mid point of *BC*, *AD* is a median of ΔABC and *BE* is the median of ΔABD.
We know that a median of a triangle divides it into two triangles of equal areas.

i.e., area(ΔABD) = $\frac{1}{2}$ area(ΔABC) ...(i)

$\Rightarrow$ area(ΔBED) = $\frac{1}{2}$ area(ΔABD) ...(ii)

From (i) and (ii), we have:

area(ΔBED) = $\frac{1}{2} \times \frac{1}{2}$ × area(ΔABC)

$\therefore$ area(ΔBED) = $\frac{1}{4}$ × area(ΔABC)

Answer 23:

(a) $\frac{1}{2}$ area(ΔABC)

Since E is the midpoint of AD, BE is a median of ΔABD.
We know that a median of a triangle divides it into two triangles of equal areas.

i.e., area(ΔBED) = $\frac{1}{2}$ × area(ΔABD) ...(i)

Since E is the midpoint of AD, CE is a median of ΔADC.
We know that a median of a triangle divides it into two triangles of equal areas.

i.e., area(ΔCED) = $\frac{1}{2}$ × area(ΔADC) ...(ii)

Adding (i) and (ii), we have:

area(ΔBED) + area(ΔCED) = $\frac{1}{2}$ × area(ΔABD) + $\frac{1}{2}$ × area(ΔADC)

$\Rightarrow$ area (Δ BEC) = $\frac{1}{2}$ × ($\Delta ABD+ADC$) = $\frac{1}{2}$ ΔABC

Answer 24:

(d) $\frac{1}{8}$area (Δ ABC)

Given: D is the midpoint of BC, E is the midpoint of BD and O is the mid point of AE.
Since D is the midpoint of BC, AD is the median of ΔABC.
E is the midpoint of BC, *so* AE is the median of ΔABD. O is the midpoint of AE, *so* BO is median of ΔABE.

We know that a median of a triangle divides it into two triangles of equal areas.

i.e., $\text{area}(\Delta ABD) = \frac{1}{2} \times \text{area}(\Delta ABC)$ ...(i)

$\text{area}(\Delta ABE) = \frac{1}{2} \times \text{area}(\Delta ABD)$ ...(ii)

$\text{area}(\Delta BOE) = \frac{1}{2} \times \text{area}(\Delta ABE)$ ...(iii)

From (i), (ii) and (iii), we have:

$\text{area}(\Delta BOE) = \frac{1}{2} \times \text{area}(\Delta ABE)$

$\text{area}(\Delta BOE) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \text{area}(\Delta ABC)$

$\therefore \text{area}(\Delta BOE) = \frac{1}{8} \text{area}(\Delta ABC)$

Answer 25:

(a) 1:2

If a triangle and a parallelogram area on the same base and between the same parallels, then the area of the triangles is half the area of the parallelogram.

i.e., area of triangle = $\frac{1}{2}$ ×area of parallelogram

$\therefore$ Required ratio = area of triangle : area of parallelogram = $\frac{1}{2} : 1 = 1 : 2$

Answer 26:

(c) $(3a + b) : (a + 3b)$

Clearealy, $EF = \frac{1}{2}(a + b)$ [Mid point theorem]

Let d be the distance between AB and EF.
Then d is the distance between DC and EF.
Now, we have:

$\text{area(trap } ABEF) = \frac{1}{2} \times (a + \frac{a+b}{2})d = \frac{(3a+b)d}{4}$

area(trap $EFCD$)= $\frac{1}{2} \times (b+\frac{a+b}{2})d = \frac{(3b+a)d}{4}$

∴ Required ratio= $\frac{(3a+b)d}{4} : \frac{(3b+a)d}{4} = (3a+b) : (a+3b)$

Answer 27:

(d) all of these
In all the mentioned quadrilaterals, a diagonal divides them into two triangles of equal areas.

Answer 28:

(c) perimeter of $ABCD$ > perimeter of $ABEF$

Parallelogram $ABCD$ and rectangle $ABEF$ lie on the same base AB, i.e., one side is common in both the figures.
In ||gm $ABCD$, we have:
AD is the hypotenuse of right angled triangle ADF.
So, $AD > AF$
∴ Perimeter of $ABCD$ > perimeter of $ABEF$

Answer 29:

(b) 40 cm^2

Radius of the circle, AC = 10 cm
Diagonal of the rectangle, AC = 10 cm

Now, $AB=\sqrt{AC^2 + BC^2} = \sqrt{80} = 4\sqrt{5}$ cm

∴ Area of the rectangle=$AB \times AD = 2\sqrt{5} \times 4\sqrt{5} = 40\ cm^2$

Answer 30:

(d) In a trap. $ABCD$, it is given that $AB \parallel DC$ and the diagonals AC and BD intersect at O. Then area(ΔAOB) = area(ΔCOD).

Consider ΔADB and ΔADC, which do not lie on the same base but lie between same parallel lines.

i.e., area(ΔADB)≠area(ΔADC)

Subtracting area(ΔAOD) from both sides, we get:

area(ΔADB) − area(ΔAOD) ≠ area(ΔADC) − area(ΔAOD)

Or area(Δ AOB)≠area(Δ COD)

Answer 31:

(b) Area of a parallelogram $= \frac{1}{2} \times$ base × corresponding height

Area of a parallelogram = base × corresponding height

Answer 32:

(c) I and II

Statement I is true, because if a parallelogram and a rectangle lie on the same base and between the same parallel lines, then they have the same altitude and therefore equal areas.

Statement II is also true as area of a parallelogram = base × height

$\Rightarrow AB \times DE = AD \times BF$

$\Rightarrow 10 \times 6 = 8 \times AD$

$\Rightarrow AD = 60 \div 8 = 7.5$ cm

Hence, statements I and II area true.

Answer 33:

(a) Both Assertion and Reason area true and Reason is a correct explanation of Assertion.

In trapezium $ABCD$, ΔABC and ΔABD area on the same base and between the same parallel lines.

∴ area(ΔABC) = area(ΔABD)

⇒ area(ΔABC) −area(ΔAOB) = area(ΔABD) − area(ΔAOB)

$\Rightarrow$ area(ΔBOC) = area(ΔAOD)

$\therefore$ Assertion (A) is true and, clearly, reason (R) gives (A).

Answer 34:

(b) Both Assertion and Reason area true but Reason is not a correct explanation of Assertion.

Reason (R) is clearly true.
The explanation of assertion (A) is as follows:
ABCD is a rhombus. So, all of its sides area equal.
Now, $BC = DC$
$\Rightarrow \angle BDC = \angle DBC = x°$

Also, $\angle BCD = 60°$
$\therefore x° + x° + 60° = 180°$
$\Rightarrow 2x° = 120°$
$\Rightarrow x° = 60°$
$\therefore \angle BCD = \angle BDC = \angle DBC = 60°$
So, ΔBCD is an equilateral triangle.
i.e., $BD = BC = a$
$\therefore OB = \frac{a}{2}$
Now, in Δ OAB, we have:
$OA^2 = AB^2 - OB^2 = \frac{3a^2}{4}$
$\Rightarrow OA = \frac{\sqrt{3}}{2} a$
$\Rightarrow AC = 2 \times \frac{\sqrt{3}}{2} a = \sqrt{3}a$
$\therefore AC:BD = \sqrt{3}a : a = \sqrt{3}:1$

Thus, assertion (A) is also true, but reason (R) does not give (A).
Hence, the correct answer is (b).

Answer 35:

(a) Both Assertion and Reason area true and Reason is a correct explanation of Assertion.

Answer 36:

(b) Both Assertion (A) and Reason (R) area true, but Reason (R) is not a correct explanation of Assertion (A).

Explanation:
Reason (R):

$\therefore$ area(ΔABC) = $\frac{\sqrt{3}}{4}\times(side)^2$=($\frac{\sqrt{3}}{4}\times 8\times 8$) = $16\sqrt{3}$ cm^2

Thus, reason (R) is true.

Assertion (A):

Area of trapezium = $\frac{1}{2}\times(25+15)\times 6 = 120\ cm^2$

Thus, assertion (A) is true, but reason (R) does not give assertion (A).

Answer 37:

(d) Assertion is false and Reason is true.

Clearly, reason (R) is true.
Assertion: Area of a parallelogram = base × height
$\Rightarrow AB \times DE = AD \times$ BF
$\Rightarrow$AD = (16 × 8) ÷ 10 = 12.8 cm

So, the assertion is false.