Answer : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

$x + y = 9$ …(i)

$2x – 3y + 7 = 0$ …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (i) by 2, we get

$2x + 2y = 18$

or $2x + 2y – 18 = 0$ …(iii)

On subtracting eq. (iii) from (ii), we get

$2x – 3y + 7 – 2x – 2y + 18 = 0$

$\Rightarrow -5y + 25 = 0$

$\Rightarrow -5y = -25$

$\Rightarrow y = 5$

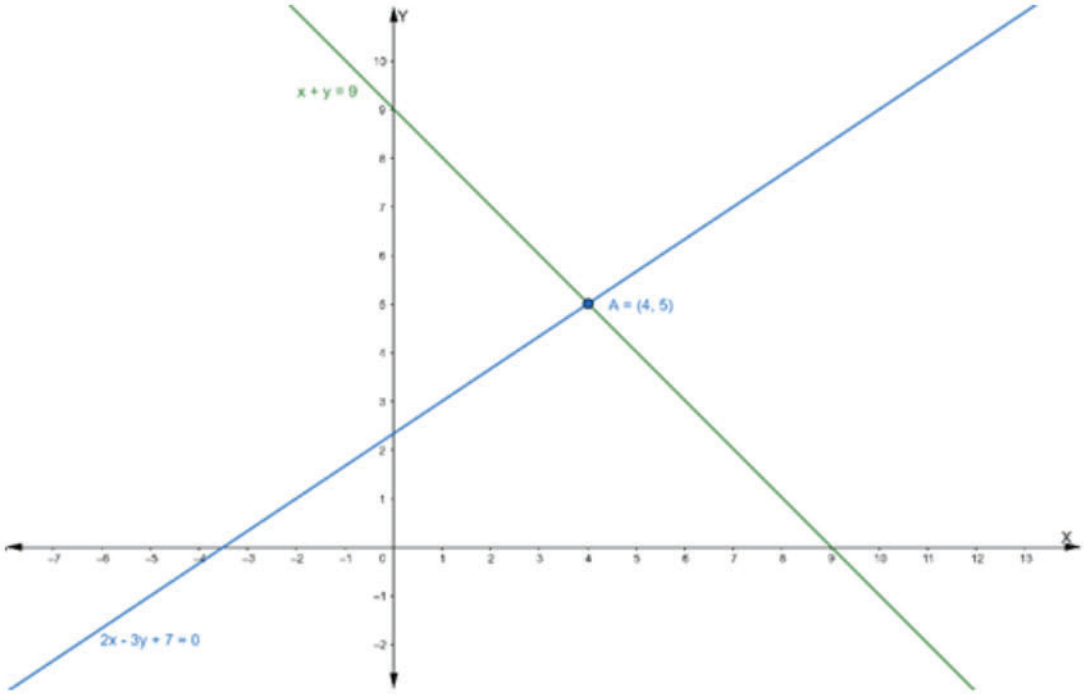
Putting the value of y in eq. (i), we get

$x + 5 = 9$

$\Rightarrow x = 9 – 5$

$\Rightarrow x = 4$

Hence, the point of intersection $P(x_1, y_1)$ is (4, 5)



Now, we have to find the equation of the line passing through the point (4, 5) and having slope = $-\frac{2}{3}$

Equation of line: $y - y_1 = m(x - x_1)$

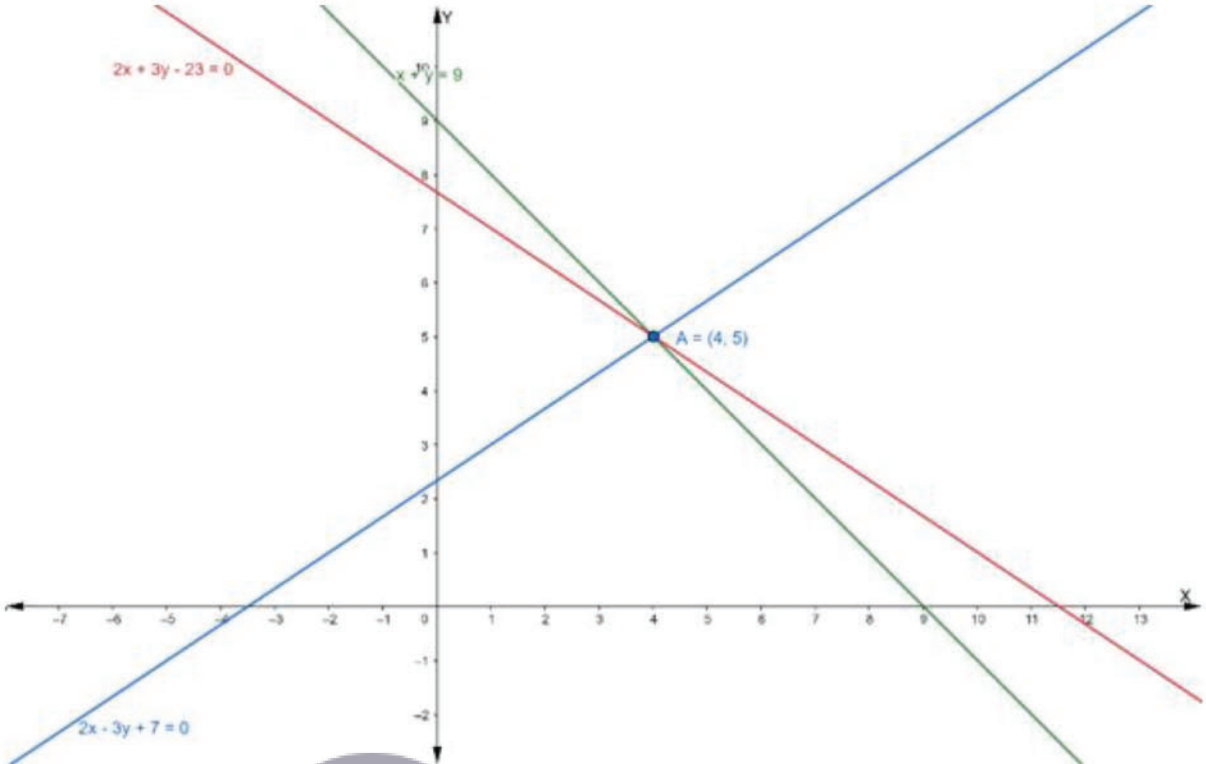
$$\Rightarrow y - 5 = -\frac{2}{3}(x - 4)$$

$$\Rightarrow 3y - 15 = -2x + 8$$

$\Rightarrow$ 2x + 3y – 15 – 8 = 0

$\Rightarrow$ 2x + 3y – 23 = 0

Hence, the equation of line having slope -2/3 is 2x + 3y – 23 = 0



Q. 4. Find the equation of the line drawn through the point of intersection of the lines x – y = 1 and 2x – 3y + 1 = 0 and which is parallel to the line 3x + 4y = 12.

Answer : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

$x - y = 1$ …(i)

$2x - 3y + 1 = 0$ …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (i) by 2, we get

$2x - 2y = 2$

or $2x - 2y - 2 = 0$ …(iii)

On subtracting eq. (iii) from (ii), we get

$2x - 3y + 1 - 2x + 2y + 2 = 0$

$\Rightarrow -y + 3 = 0$

$\Rightarrow y = 3$

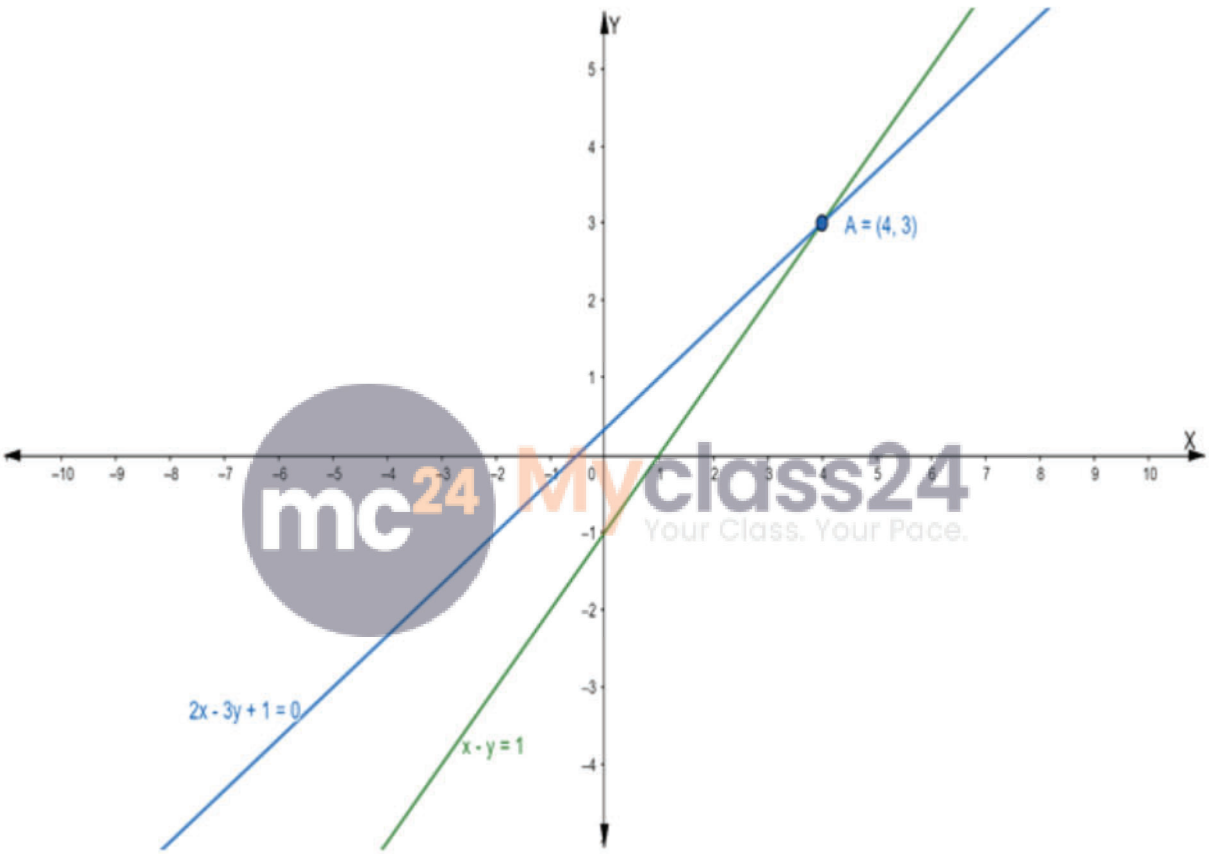
Putting the value of y in eq. (i), we get

x – 3 = 1

⇒ x = 1 + 3

⇒ x = 4

Hence, the point of intersection $P(x_1, y_1)$ is (4, 3)



Now, we find the slope of the given equation 3x + 4y = 12

We know that the slope of an equation is

$$m = -\frac{a}{b}$$

$$\Rightarrow m = -\frac{3}{4}$$

So, the slope of a line which is parallel to this line is also $-\frac{3}{4}$

Then the equation of the line passing through the point (4, 3) having a slope $-\frac{3}{4}$ is:

$y - y_1 = m (x - x_1)$

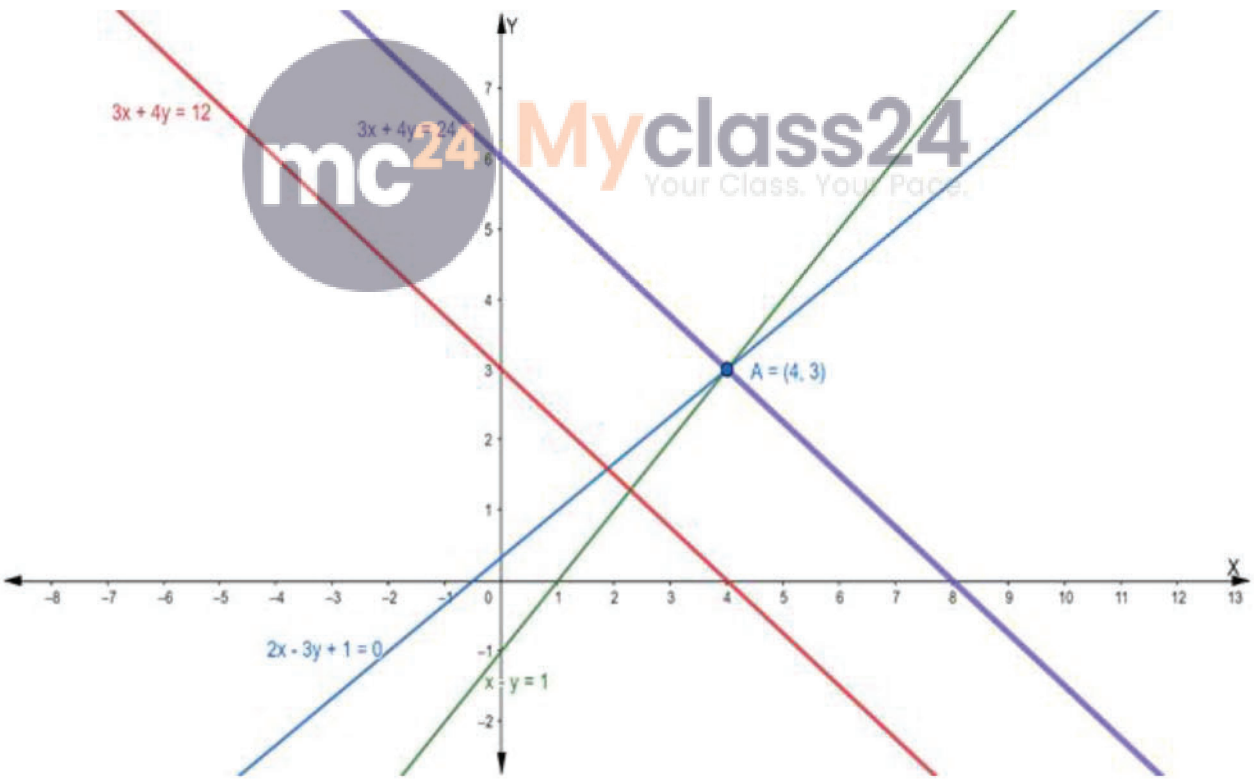
$$\Rightarrow y - (3) = -\frac{3}{4}(x - 4)$$

$\Rightarrow y - 3 = - 3x + 12$

$\Rightarrow 4y - 12 = -3x + 12$

$\Rightarrow 3x + 4y - 12 - 12 = 0$

$\Rightarrow 3x + 4y - 24 = 0$



Q. 5. Find the equation of the line through the intersection of the lines 5x – 3y = 1 and 2x + 3y = 23 and which is perpendicular to the line 5x – 3y = 1.

Answer : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

$5x - 3y = 1$ …(i)

$2x + 3y = 23$ …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Adding eq. (i) and (ii) we get

$5x - 3y + 2x + 3y = 1 + 23$

$\Rightarrow 7x = 24$

$$\Rightarrow x = \frac{24}{7}$$

Putting the value of x in eq. (i), we get

$$5\left(\frac{24}{7}\right) - 3y = 1$$

$$\Rightarrow \frac{120}{7} - 3y = 1$$

$$\Rightarrow -3y = 1 - \frac{120}{7}$$

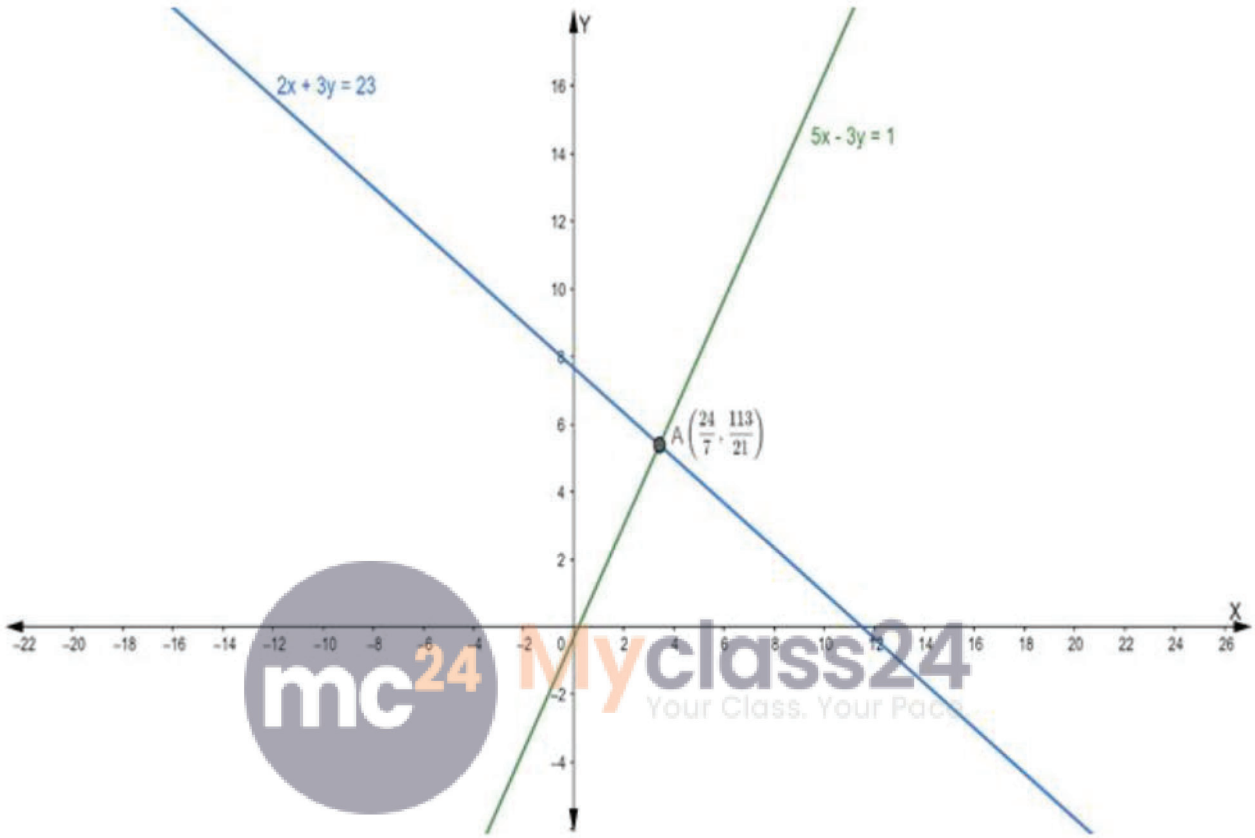
$$\Rightarrow -3y = \frac{7 - 120}{7}$$

$$\Rightarrow -3y = -\frac{113}{7}$$

$$\Rightarrow y = \frac{113}{21}$$

Hence, the point of intersection $P(x_1, y_1)$ is

$\left(\frac{24}{7}, \frac{113}{21}\right)$



Now, we know that, when two lines are perpendicular, then the product of their slope is equal to -1

$m_1 \times m_2 = -1$

⇒ Slope of the given line × Slope of the perpendicular line = -1

$\because \frac{5}{3} \times$ Slope of the perpendicular line = -1

⇒The slope of the perpendicular line $= -\frac{3}{5}$

So, the slope of a line which is perpendicular to the given line is $-\frac{3}{5}$

Then the equation of the line passing through the point $\left(\frac{24}{7}, \frac{113}{21}\right)$ having slope $-\frac{3}{5}$ is :

$y - y_1 = m(x - x_1)$

$$\Rightarrow y - \left(\frac{113}{21}\right) = -\frac{3}{5}\left(x - \frac{24}{7}\right)$$

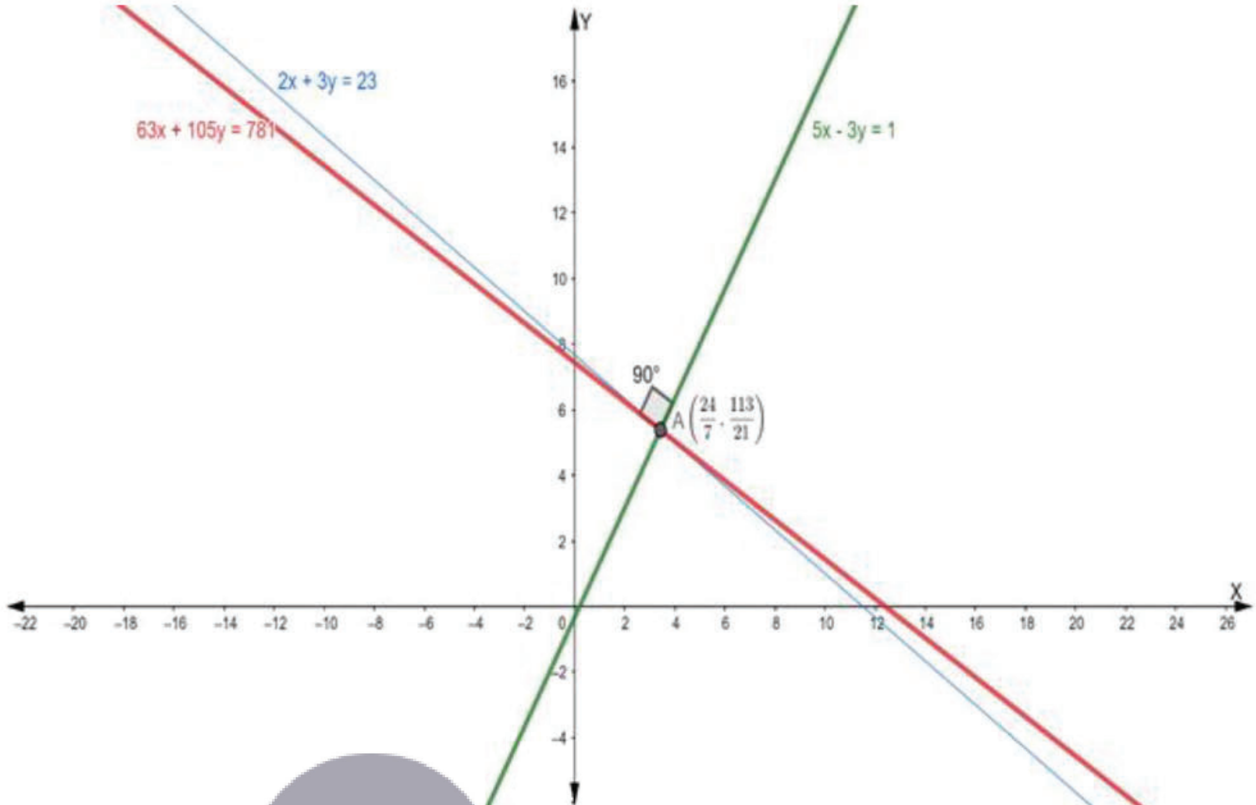
$$\Rightarrow 5y - 5 \times \frac{113}{21} = -3x + \frac{24}{7}$$

$$\Rightarrow 5y - \frac{565}{21} = -3x + \frac{72}{7}$$

$$\Rightarrow 3x + 5y - \frac{565}{21} - \frac{72}{7} = 0$$

$$\Rightarrow \frac{63x + 105y - 565 - 216}{21} = 0$$

$\Rightarrow 63x + 105y - 781 = 0$



Q. 6. Find the equation of the line through the intersection of the lines 2x – 3y = 0 and 4x – 5y = 2 and which is perpendicular to the line x + 2y + 1 = 0.

Answer : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

$2x - 3y = 0$ …(i)

$4x - 5y = 2$ …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (i) by 2, we get

$4x - 6y = 0$ …(iii)

On subtracting eq. (iii) from (ii), we get

$4x - 5y - 4x + 6y = 2 - 0$

$\Rightarrow y = 2$

Putting the value of y in eq. (i), we get

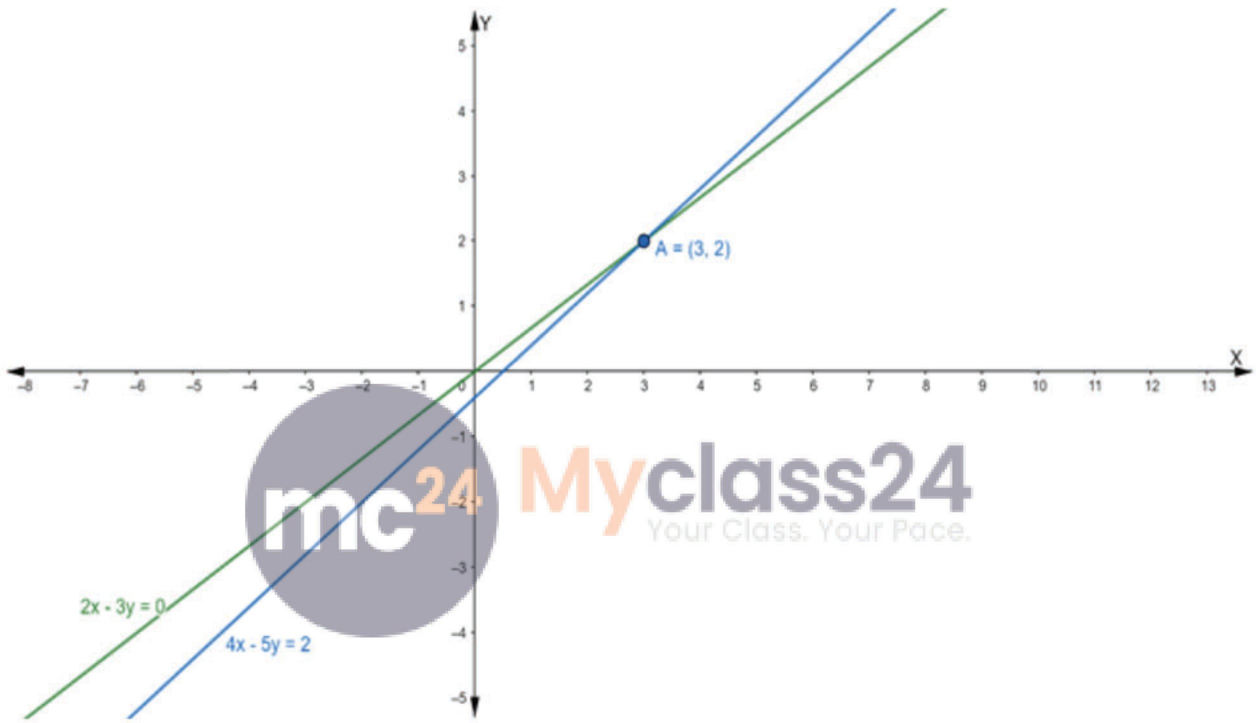
$2x - 3(2) = 0$

$\Rightarrow 2x - 6 = 0$

$\Rightarrow 2x = 6$

$\Rightarrow x = 3$

Hence, the point of intersection $P(x_1, y_1)$ is (3, 2)



Now, we know that, when two lines are perpendicular, then the product of their slope is equal to -1

$m_1 \times m_2 = -1$

$\Rightarrow$ Slope of the given line × Slope of the perpendicular line = -1

$$\therefore \left(-\frac{1}{2}\right) \times \text{Slope of the perpendicular line} = -1$$

$\Rightarrow$ The slope of the perpendicular line = 2

So, the slope of a line which is perpendicular to the given line is 2

Then the equation of the line passing through the point (3, 2) having slope 2 is:

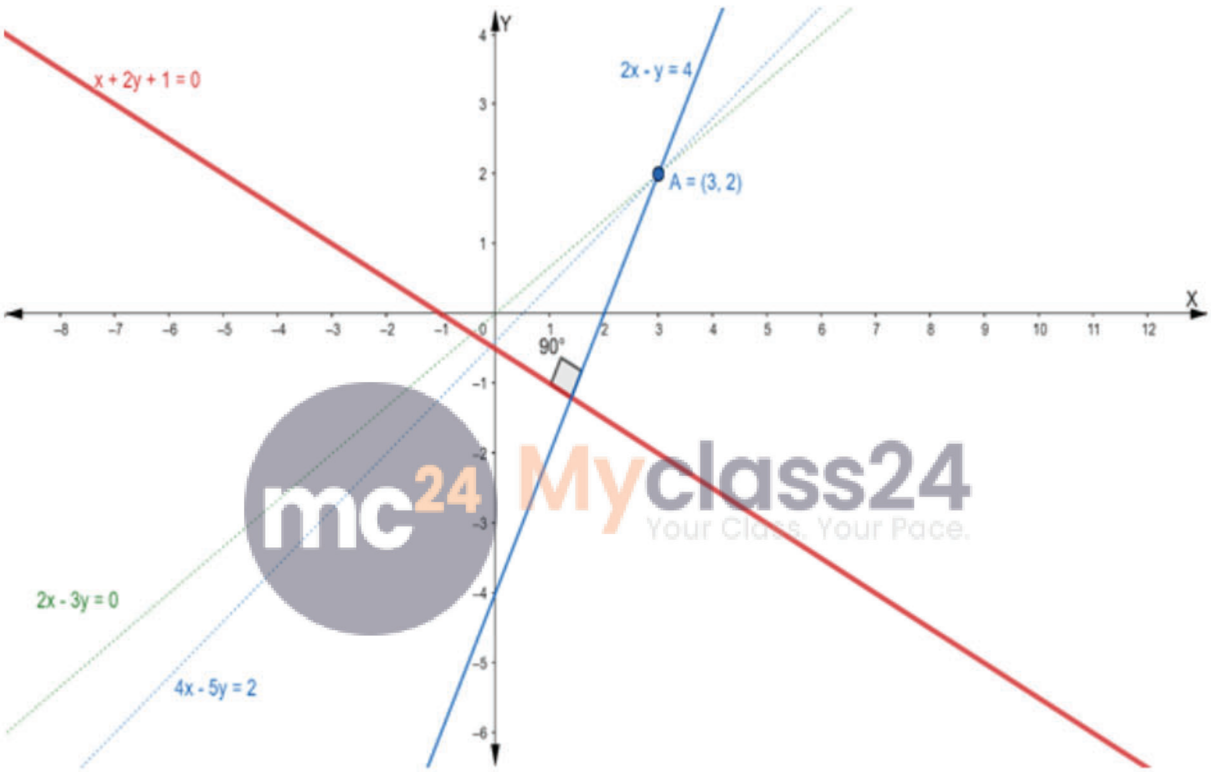
$y - y_1 = m(x - x_1)$

$\Rightarrow y - 2 = 2(x - 3)$

$\Rightarrow y - 2 = 2x - 6$

$\Rightarrow 2x - y - 6 + 2 = 0$

$\Rightarrow 2x - y - 4 = 0$



Q. 7. Find the equation of the line through the intersection of the lines x – 7y + 5 = 0 and 3x + y – 7 = 0 and which is parallel to x-axis.

Answer : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

$x - 7y + 5 = 0$ …(i)

$3x + y - 7 = 0$ …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (i) by 3, we get

$3x - 21y + 15 = 0$ …(iii)

On subtracting eq. (iii) from (ii), we get

3x + y – 7 – 3x + 21y – 15 = 0

⇒ 22y - 22 = 0

⇒ 22y = 22

⇒ y = 1

Putting the value of y in eq. (i), we get

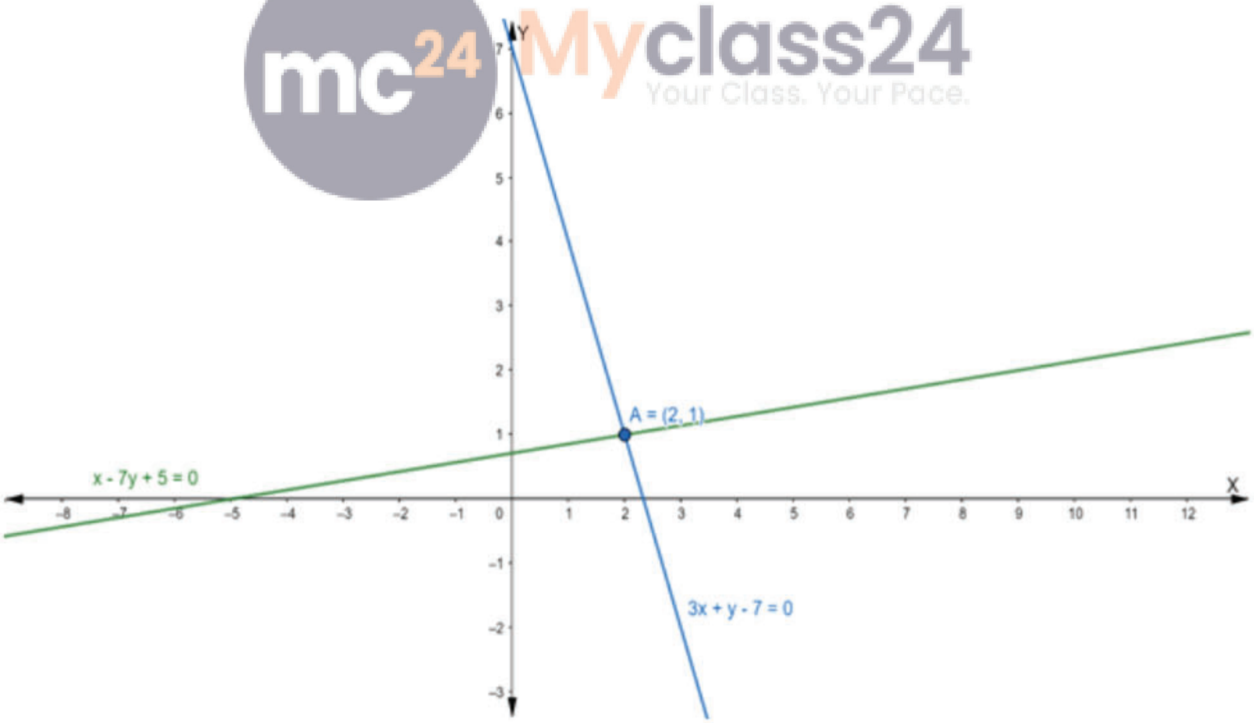
x – 7(1) + 5 = 0

⇒ x – 7 + 5 = 0

⇒ x – 2 = 0

⇒ x = 2

Hence, the point of intersection $P(x_1, y_1)$ is (2, 1)



The equation of line parallel to x – axis is of the form

y = b where b is some constant

Given that this equation of the line passing through the point of intersection (2, 1)

Hence, point (2, 1) will satisfy the equation of a line.

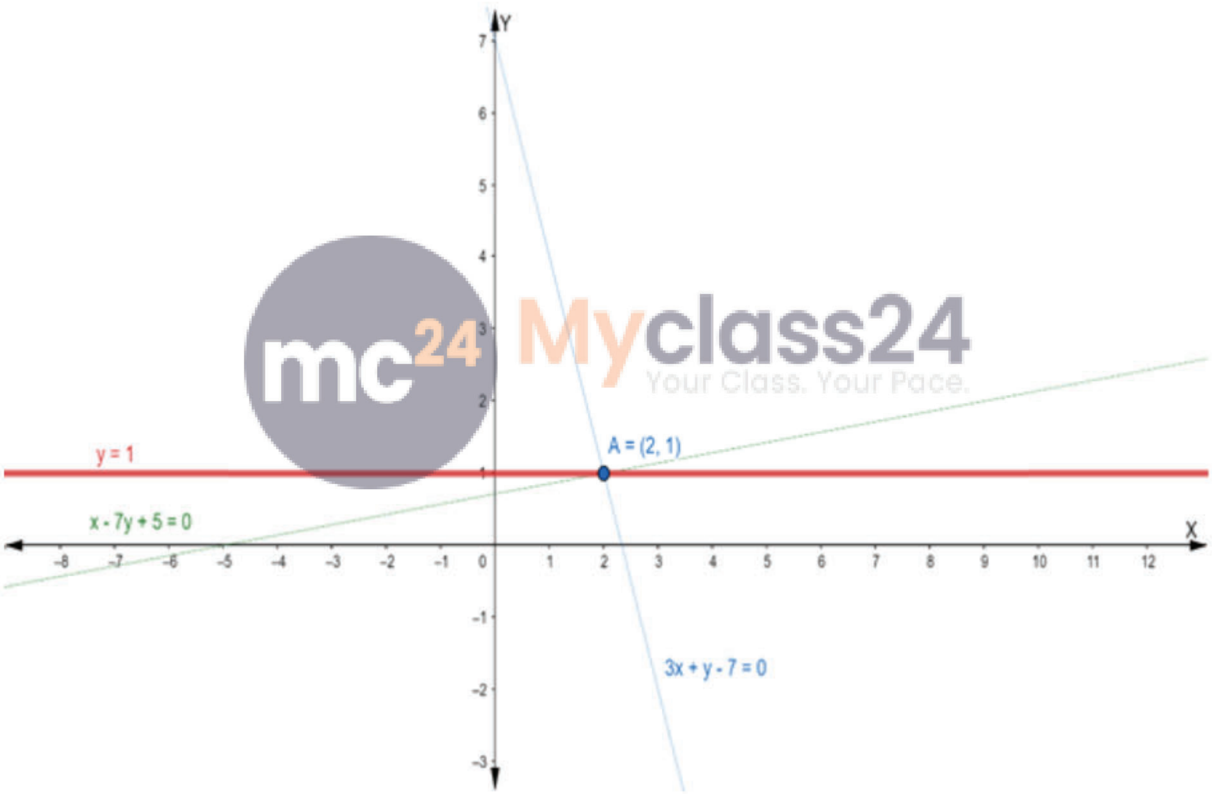
Putting y = 1 in the equation y = b, we get

$y = b$

$\Rightarrow 1 = b$

or $b = 1$

Now, the required equation of a line is y = 1



Q. 8. Find the equation of the line through the intersection of the lines 2x – 3y + 1 = 0 and x + y – 2 = 0 and drawn parallel to y-axis.

Answer : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

$2x – 3y + 1 = 0$ …(i)

$x + y – 2 = 0$ …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (ii) by 2, we get

$2x + 2y – 4 = 0$ …(iii)

On subtracting eq. (iii) from (i), we get

$2x – 3y + 1 – 2x – 2y + 4 = 0$

$\Rightarrow -5y + 5 = 0$

$\Rightarrow -5y = -5$

$\Rightarrow y = 1$

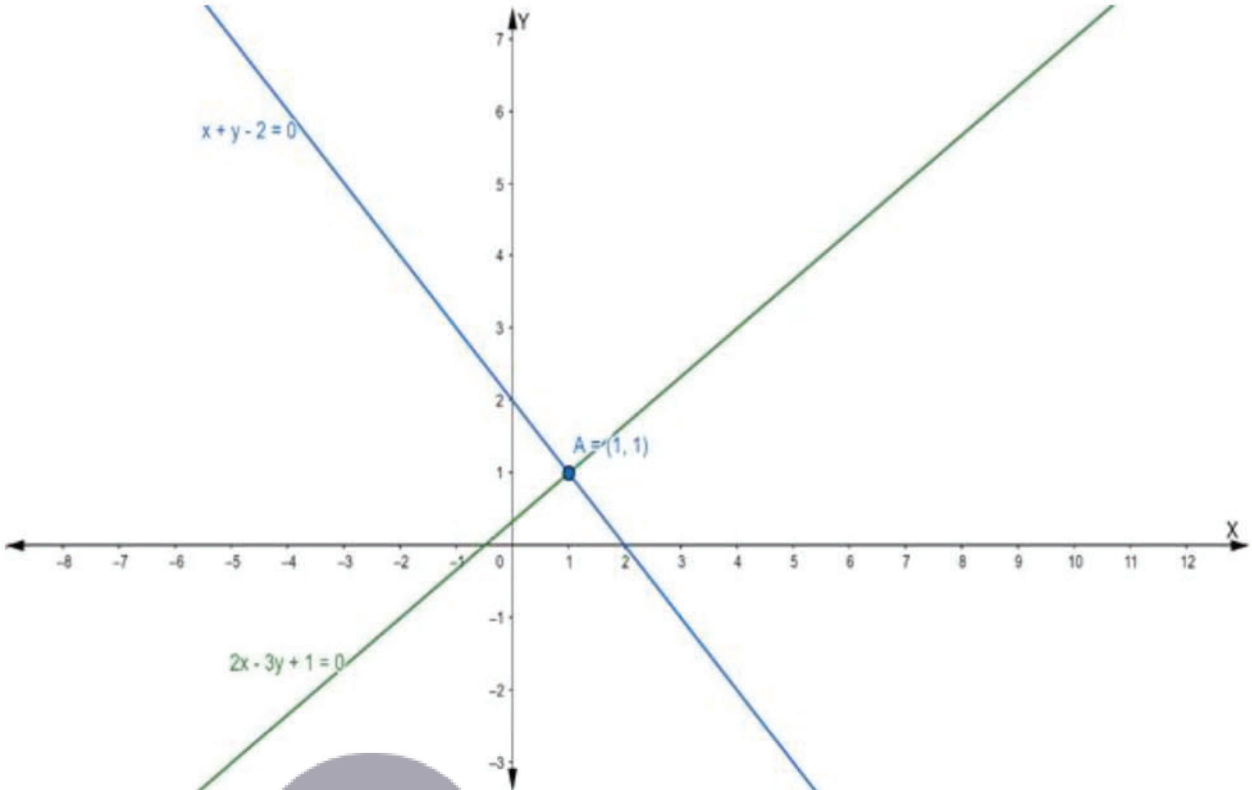
Putting the value of y in eq. (ii), we get

$x + 1 – 2 = 0$

$\Rightarrow x – 1 = 0$

$\Rightarrow x = 1$

Hence, the point of intersection $P(x_1, y_1)$ is (1, 1)



The equation of a line parallel to y – axis is of the form

x = a where a is some constant

Given that this equation of the line passing through the point of intersection (1, 1)

Hence, point (1, 1) will satisfy the equation of a line.

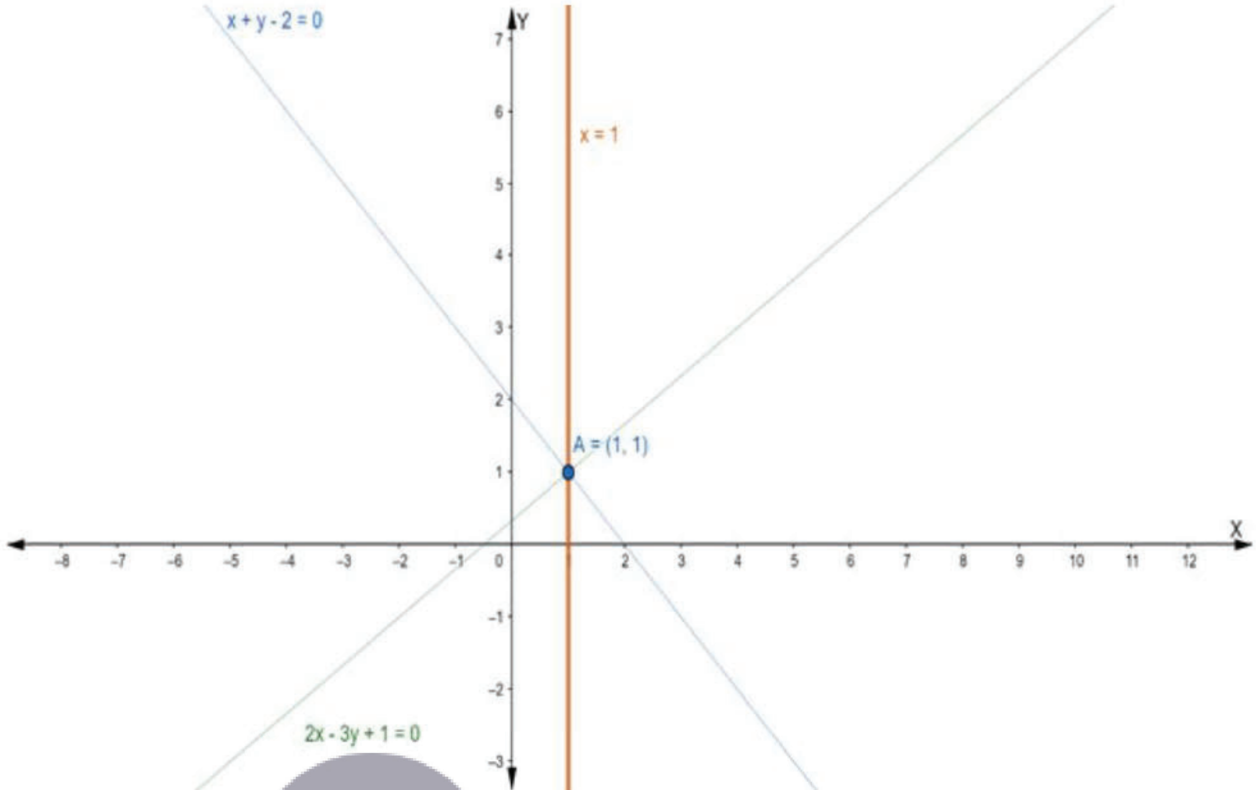
Putting x = 1 in the equation y = b, we get

x = a

⇒ 1 = a

or a = 1

Now, required equation of line is x = 1



Q. 9. Find the equation of the line through the intersection of the lines 2x + 3y – 2 = 0 and x – 2y + 1 = 0 and having x-intercept equal to 3.

Answer : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

2x + 3y – 2 = 0 …(i)

x – 2y + 1 = 0 …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (ii) by 2, we get

2x – 4y + 2 = 0 …(iii)

On subtracting eq. (iii) from (i), we get

2x + 3y – 2 – 2x + 4y – 2 = 0

⇒ 7y – 4 = 0

⇒ 7y = 4

$$\Rightarrow y = \frac{4}{7}$$

Putting the value of y in eq. (ii), we get

$$x - 2\left(\frac{4}{7}\right) + 1 = 0$$

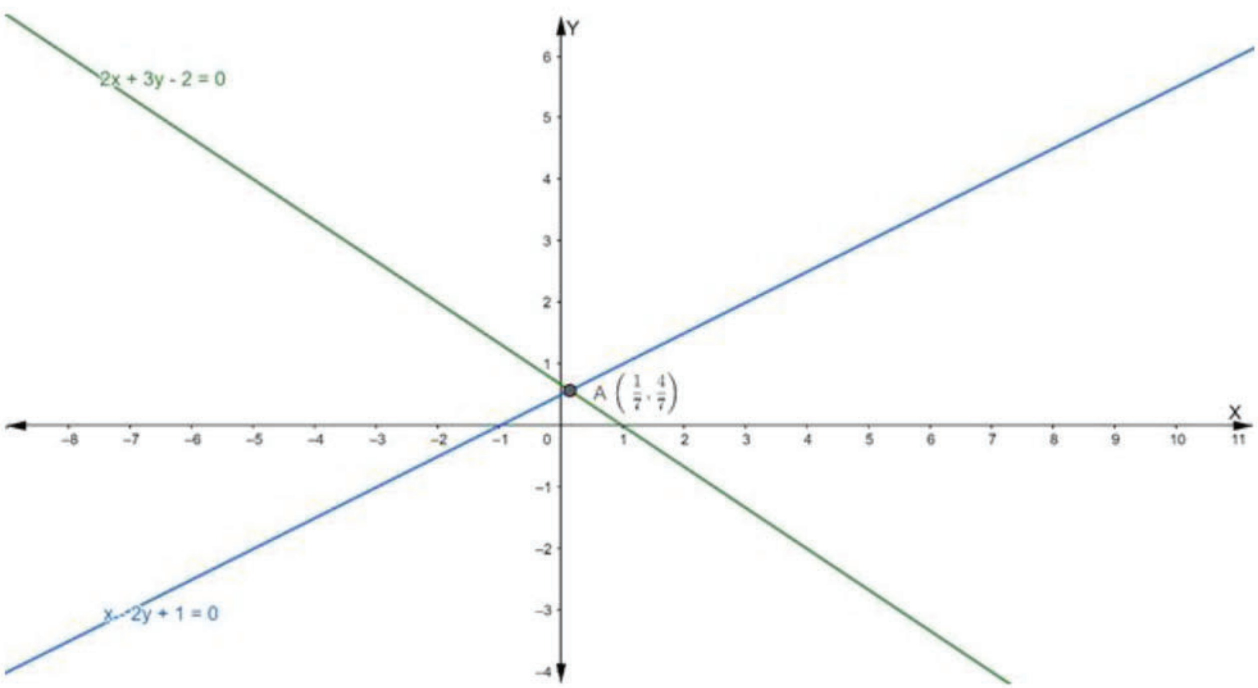
$$\Rightarrow x - \frac{8}{7} + 1 = 0$$

$$\Rightarrow x = \frac{8}{7} - 1$$

$$\Rightarrow x = \frac{1}{7}$$

Hence, the point of intersection $P(x_1, y_1)$ is

$$\left(\frac{1}{7}, \frac{4}{7}\right)$$



Now, the equation of a line in intercept form is:

$$\frac{x}{a}+\frac{y}{b}=1$$

where a and b are the intercepts on the axis.

Given that: a = 3

$$\Rightarrow \frac{x}{3}+\frac{y}{b}=1$$

$$\Rightarrow \frac{bx+3y}{3b}=1$$

⇒ bx + 3y = 3b …(i)

If eq. (i) passes through the point $\left(\frac{1}{7},\frac{4}{7}\right)$, we get

$$b\left(\frac{1}{7}\right)+3\left(\frac{4}{7}\right)=3b$$

$$\Rightarrow \frac{b+12}{7}=3b$$

⇒ b + 12 = 21b

⇒ b – 21b = - 12

⇒ 20b = 12

$$\Rightarrow b=\frac{12}{20}=\frac{3}{5}$$

Putting the value of ‘b’ in eq. (i), we get

$$\frac{3}{5}x + 3y = 3 \times \frac{3}{5}$$

$$\Rightarrow \frac{3}{5}x + 3y = \frac{9}{5}$$

⇒ 3x + 15y = 9

⇒ x + 5y = 3

Hence, the required equation of line is x + 5y = 3

Q. 10. Find the equation of the line passing through the intersection of the lines 3x – 4y + 1 = 0 and 5x + y – 1 = 0 and which cuts off equal intercepts from the axes.

Answer : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

3x – 4y + 1 = 0 …(i)

5x + y – 1 = 0 …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (ii) by 4, we get

20x + 4y – 4 = 0 …(iii)

On adding eq. (iii) and (i), we get

20x + 4y – 4 + 3x – 4y + 1 = 0

⇒ 23x – 3 = 0

⇒ 23x = 3

$$\Rightarrow x = \frac{3}{23}$$

Putting the value of x in eq. (ii), we get

$$5\left(\frac{3}{23}\right)+y-1=0$$

$$\Rightarrow \frac{15}{23}+y-1=0$$

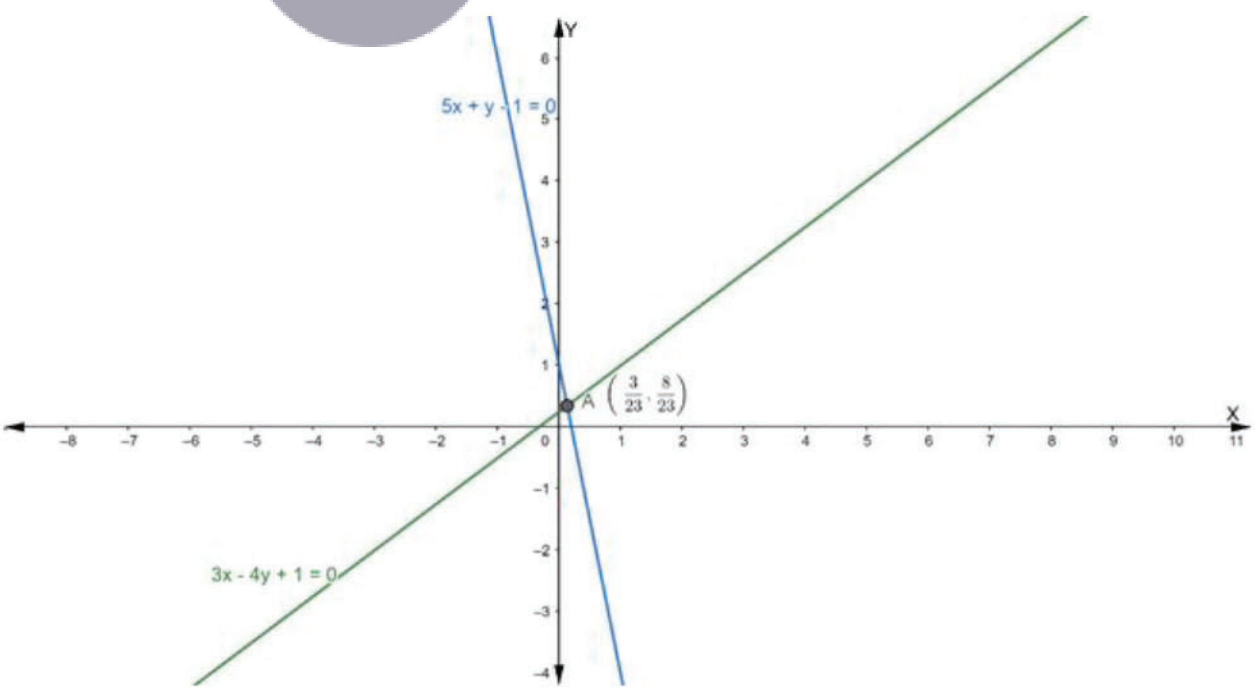
$$\Rightarrow y=1-\frac{15}{23}$$

$$\Rightarrow y=\frac{23-15}{23}$$

$$\Rightarrow y=\frac{8}{23}$$

Hence, the point of intersection $P(x_1, y_1)$ is

$$\left(\frac{3}{23},\frac{8}{23}\right)$$



Now, the equation of line in intercept form is:

$$\frac{x}{a}+\frac{y}{b}=1$$

where a and b are the intercepts on the axis.

Given that: a = b

$$\Rightarrow \frac{x}{a}+\frac{y}{a}=1$$

$$\Rightarrow \frac{x+y}{a}=1$$

⇒ x + y = a ...(i)

If eq. (i) passes through the point $\left(\frac{3}{23}, \frac{8}{23}\right)$, we get

$$\frac{3}{23}+\frac{8}{23}=a$$

$$\Rightarrow \frac{11}{23}=a$$

$$\Rightarrow a=\frac{11}{23}$$

Putting the value of ‘a’ in eq. (i), we get

$$x+y=\frac{11}{23}$$

⇒ 23x + 23y = 11

Hence, the required line is 23x + 23y = 11