

# NCERT Solutions for Class-XII Maths

## Chapter-5.7

### NCERT Math Class 12

Find the second order derivatives of the functions given in Exercises 1 to 10.

1.  $x^2 + 3x + 2$

1. Let  $y = x^2 + 3x + 2$ , therefore,

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + 3x + 2) = 2x + 3 \Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}(2x + 3) = 2$$

2.  $x^{20}$

2. Let us take  $y = x^{20}$

Now,

$$\frac{dy}{dx} = \frac{d(x^{20})}{dx} = 20x^{19}$$

Therefore,

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{d(20x^{19})}{dx} = 20 \frac{d(x^{19})}{dx} \\ &= 20 \times 19 \times x^{18} \\ &= 380 x^{18} \end{aligned}$$

3.  $x \cdot \cos x$

3. Let  $y = x \cdot \cos x$ , therefore,

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx}(x \cdot \cos x) = x \cdot \frac{d}{dx} \cos x + \cos x \cdot \frac{d}{dx} x = -x \sin x + \cos x \\ \Rightarrow \frac{d^2y}{dx^2} &= \frac{d}{dx}(-x \sin x + \cos x) = -\left(x \frac{d}{dx} \sin x + \sin x \frac{d}{dx} x\right) - \sin x \\ &= -x \cos x - \sin x - \sin x = -(x \cos x + 2 \sin x) \end{aligned}$$

4.  $\log x$

4. Let us take  $y = \log x$

Now,

$$\frac{dy}{dx} = \frac{d(\log x)}{dx} = \frac{1}{x}$$

Therefore,

$$\frac{d^2y}{dx^2} = \frac{d\left(\frac{1}{x}\right)}{dx} = \left(-\frac{1}{x^2}\right)$$

5.  $x^3 \log x$

5. Let  $y = x^3 \log x$ , therefore,

$$\frac{dy}{dx} = \frac{d}{dx}(x^3 \log x) = x^3 \cdot \frac{d}{dx} \log x + \log x \cdot \frac{d}{dx} x^3 = x^3 \cdot \frac{1}{x} + \log x \cdot 3x^2 = x^2 + 3x^2 \log x$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}(x^2 + 3x^2 \log x) = 2x + 3\left(x^2 \frac{d}{dx} \log x + \log x \frac{d}{dx} x^2\right)$$

$$= 2x + 3\left(x^2 \cdot \frac{1}{x} + \log x \cdot 2x\right) = 2x + 3x + 6x \log x = 5x + 6x \log x = x(5 + 6 \log x)$$

6.  $e^x \sin 5x$

6. Let us take  $y = e^x \sin 5x$

Now,

$$\frac{dy}{dx} = \frac{d(e^x \sin 5x)}{dx}$$

$$= \sin 5x \cdot \frac{d(e^x)}{dx} + e^x \cdot \frac{d(\sin 5x)}{dx}$$

$$= \sin 5x \cdot e^x + e^x \cdot \cos 5x \cdot \frac{d(5x)}{dx}$$

$$= e^x \sin 5x + e^x \cos 5x \cdot 5$$

$$= e^x (\sin 5x + 5 \cos 5x)$$

$$\frac{d^2y}{dx^2} = \frac{d[e^x (\sin 5x + 5 \cos 5x)]}{dx}$$

$$= (\sin 5x + 5 \cos 5x) \cdot \frac{d(e^x)}{dx} + e^x \cdot \frac{d(\sin 5x + 5 \cos 5x)}{dx}$$

$$= (\sin 5x + 5 \cos 5x) e^x + e^x \left[ \cos 5x \cdot \frac{d(5x)}{dx} + 5(-\sin 5x) \cdot \frac{d(5x)}{dx} \right]$$

$$= e^x (\sin 5x + 5 \cos 5x) + e^x (5 \cos 5x - 25 \sin 5x)$$

$$= e^x (10 \cos 5x - 24 \sin 5x)$$

$$= 2e^x (5 \cos 5x - 12 \sin 5x)$$

7.  $e^{6x} \cos 3x$

7. Let  $y = e^{6x} \cos 3x$ , therefore,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx}(e^{6x} \cos 3x) = e^{6x} \cdot \frac{d}{dx} \cos 3x + \cos 3x \cdot \frac{d}{dx} e^{6x} \\ &= e^{6x} \cdot (-\sin 3x) \cdot 3 + \cos 3x \cdot e^{6x} \cdot 6 = 3e^{6x} (-\sin 3x + 2\cos 3x)\end{aligned}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx} [3e^{6x} (-\sin 3x + 2\cos 3x)]$$

$$= 3e^{6x} \cdot \frac{d}{dx} (-\sin 3x + 2\cos 3x) + (-\sin 3x + 2\cos 3x) \cdot \frac{d}{dx} 3e^{6x}$$

$$= 3e^{6x} \cdot (-3\cos 3x - 6\sin 3x) + (-\sin 3x + 2\cos 3x) \cdot 18e^{6x}$$

$$= e^{6x} (-9\cos 3x - 18\sin 3x - 18\sin 3x + 36\cos 3x)$$

$$= e^{6x} (27\cos 3x - 36\sin 3x)$$

$$= 9e^{6x} (3\cos 3x - 4\sin 3x)$$

8.  $\tan^{-1} x$

8. Let us take  $y = \tan^{-1} x$  Now,

$$\frac{dy}{dx} = \frac{d(\tan^{-1})}{dx} = \frac{1}{1+x^2}$$

$$\frac{d^2y}{dx^2} = \frac{d\left[\frac{1}{1+x^2}\right]}{dx}$$

$$= \frac{d(+x^2)^{-1}}{dx} = (-1) \cdot (1+x^2) \cdot \frac{d(1+x^2)}{dx}$$

$$= \frac{1}{(1+x^2)^2} \times 2x = \frac{-2x}{(1+x^2)^2}$$

9.  $\log(\log x)$

9. Let  $y = \log(\log x)$ , therefore,

$$\frac{dy}{dx} = \frac{d}{dx}(\log(\log x)) = \frac{1}{\log x} \cdot \frac{1}{x} = \frac{1}{x \log x}$$

$$\begin{aligned}\Rightarrow \frac{d^2y}{dx^2} &= \frac{d}{dx} \left( \frac{1}{x \log x} \right) = \frac{(x \log x) \frac{d}{dx} 1 - 1 \cdot \frac{d}{dx} (x \log x)}{(x \log x)^2} \\ &= \frac{0 - \left( x \cdot \frac{1}{x} + \log x \right)}{(x \log x)^2} = -\frac{1 + \log x}{(x \log x)^2}\end{aligned}$$

10.  $\sin(\log x)$

10. Let us take  $y = \sin(\log x)$

Now,

$$\begin{aligned}\frac{dy}{dx} &= \frac{d[\sin(\log x)]}{dx} \\ &= \cos(\log x) \cdot \frac{d(\log x)}{dx} \\ &= \frac{\cos(\log x)}{x}\end{aligned}$$

Then,

$$\begin{aligned}\frac{d^2y}{dx^2} &= \frac{d\left(\frac{\cos(\log x)}{x}\right)}{dx} \\ &= \frac{x \cdot \frac{d[\cos(\log x)]}{dx} - \cos(\log x) \cdot \frac{d(x)}{dx}}{x^2} \\ &= \frac{x \cdot \left[ -\sin(\log x) \cdot \frac{d(\log x)}{dx} \right] - \cos(\log x) \cdot 1}{x^2} \\ &= \frac{-x \sin(\log x) \cdot \frac{1}{x} - \cos(\log x)}{x^2} \\ &= \frac{-[\sin(\log x) + \cos(\log x)]}{x^2}\end{aligned}$$

11. If  $y = 5 \cos x - 3 \sin x$ , prove that  $\frac{d^2y}{dx^2} + y = 0$

11. Given that:  $y = 5 \cos x - 3 \sin x$ , therefore,

$$\frac{dy}{dx} = \frac{d}{dx} (5 \cos x - 3 \sin x) = -5 \sin x - 3 \cos x$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}(-5\sin x - 3\cos x) = -5\cos x + 3\sin x = -(5\cos x - 3\sin x) = -y$$

$$\Rightarrow \frac{d^2y}{dx^2} + y = 0$$

12. If  $y = \cos^{-1} x$ , Find  $\frac{d^2y}{dx^2}$  in terms of  $y$  alone.

12. It is given that  $y = \cos^{-1} x$

Now,

$$\frac{dy}{dx} = \frac{d(\cos^{-1})}{dx} = \frac{-1}{\sqrt{1-x^2}} = -(1-x^2)^{-\frac{1}{2}}$$

Therefore,

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{d\left(-(1-x^2)^{-\frac{1}{2}}\right)}{dx} \\ &= -\left(-\frac{1}{2}\right) \cdot (1-x^2)^{-\frac{3}{2}} \cdot \frac{d(1-x^2)}{dx} \end{aligned}$$

$$= \frac{1}{2\sqrt{(1-x^2)^3}} \times (-2x)$$

$$\frac{d^2y}{dx^2} = \frac{-x}{\sqrt{(1-x^2)^3}} \dots \dots \dots (1)$$

Now it is given that  $y = \cos^{-1} x$

$$\Rightarrow x = \cos y$$

Now putting the value of  $x$  in equation (1), we get

$$\frac{d^2y}{dx^2} = \frac{-\cos y}{\sqrt{(1-\cos^2 y)^3}}$$

$$= \frac{-\cos y}{\sqrt{(\sin^2 y)^3}}$$

$$= \frac{-\cos y}{(\sin y)^3} = \frac{-\cos y}{\sin y} \cdot \frac{1}{\sin^2 y}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\cot y \cdot \operatorname{cosec}^2 y$$



13. If  $y = 3 \cos(\log x) + 4 \sin(\log x)$ , show that  $x^2 y_2 + xy_1 + y = 0$

13. Given that:  $y = 3\cos(\log x) + 4\sin(\log x)$ , therefore,

$$\frac{dy}{dx} = \frac{d}{dx}(3\cos(\log x) + 4\sin(\log x)) = -3\sin(\log x) \cdot \frac{1}{x} + 4\cos(\log x) \cdot \frac{1}{x}$$

$$\Rightarrow x \frac{dy}{dx} = -3\sin(\log x) + 4\cos(\log x)$$

$$\Rightarrow x \frac{d^2 y}{dx^2} + \frac{dy}{dx} \cdot \frac{d}{dx} x = \frac{d}{dx} [-3\sin(\log x) + 4\cos(\log x)]$$

$$= -3\cos(\log x) \cdot \frac{1}{x} - 4\sin(\log x) \cdot \frac{1}{x} = -\frac{1}{x} [3\cos(\log x) + 4\sin(\log x)] = -\frac{1}{x} \cdot y$$

$$\Rightarrow x \frac{d^2 y}{dx^2} + \frac{dy}{dx} = -\frac{1}{x} y \Rightarrow x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} = -y \Rightarrow x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = 0$$

$$\Rightarrow x^2 y_2 + xy_1 + y = 0$$

14. If  $y = Ae^{mx} + Be^{nx}$ , show that  $\frac{d^2 y}{dx^2} - (m+n) \frac{dy}{dx} + mny = 0$

14. According to given equation, we have,

$$y = Ae^{mx} + Be^{nx}$$

$$\text{Then, } \frac{dy}{dx} = \frac{d(Ae^{mx} + Be^{nx})}{dx}$$

$$= A \cdot \frac{d(e^{mx})}{dx} + B \cdot \frac{d(e^{nx})}{dx}$$

$$= A \cdot e^{mx} \frac{d(mx)}{dx} + B \cdot e^{nx} \frac{d(nx)}{dx}$$

$$= Ame^{mx} + Bne^{nx}$$

Now, on again differentiating we get,

$$\frac{d^2 y}{dx^2} = \frac{d(Ame^{mx} + Bne^{nx})}{dx}$$

$$= Am \cdot \frac{d(e^{mx})}{dx} + Bn \cdot \frac{d(e^{nx})}{dx}$$

$$= Am \cdot e^{mx} \frac{d(mx)}{dx} + Bn \cdot e^{nx} \frac{d(nx)}{dx}$$

$$= Am^2 e^{mx} + Bn^2 e^{nx}$$

$$\therefore \frac{d^2 y}{dx^2} - (m+n) \frac{dy}{dx} + mny$$

$$= Am^2 e^{mx} + Bn^2 e^{nx} - (m+n)(Ame^{mx} + Bne^{nx}) + mn(Ae^{mx} + Be^{nx})$$

$$= Am^2 e^{mx} + Bn^2 e^{nx} - Am^2 e^{mx} - Bmne^{nx} - Amne^{mx} - Bn^2 e^{nx} + Amne^{mx} + Bmne^{nx}$$



$$= 0$$

$$\Rightarrow \frac{d^2y}{dx^2} - (m+n) \frac{dy}{dx} + mny = 0$$

**Hence Proved**

15. If  $y = 500e^{7x} + 600e^{-7x}$ , show that  $\frac{d^2y}{dx^2} = 49y$

15. Given that:  $y = 500e^{7x} + 600e^{-7x}$ , therefore,

$$\frac{dy}{dx} = \frac{d}{dx}(500e^{7x} + 600e^{-7x}) = 500e^{7x} \cdot 7 + 600e^{-7x} \cdot (-7) = 7(500e^{7x} - 600e^{-7x})$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx} 7(500e^{7x} - 600e^{-7x}) = 7[500e^{7x} \cdot 7 + 600e^{-7x} \cdot (-7)]$$

$$= 49(500e^{7x} - 600e^{-7x}) = 49y$$

$$\Rightarrow \frac{d^2y}{dx^2} = 49y$$

16. If  $e^y(x+1) = 1$ , show that  $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

16. It is given that

$$e^y(x+1) = 1$$

$$\Rightarrow e^y = \frac{1}{x+1}$$

Now, taking logarithm on both the sides we get,

$$y = \log \frac{1}{x+1}$$

On differentiating both sides we get,

$$\begin{aligned} \frac{dy}{dx} &= (x+1) \frac{d\left(\frac{1}{x+1}\right)}{dx} \\ &= (x+1) \cdot \frac{-1}{(x+1)^2} = \frac{-1}{x+1} \end{aligned}$$

Again, on differentiating we get,

$$\begin{aligned} \therefore \frac{d^2y}{dx^2} &= -\frac{d\left(\frac{1}{x+1}\right)}{dx} \\ &= -\left(\frac{-1}{(x+1)^2}\right) = \frac{1}{(x+1)^2} \end{aligned}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{(x+1)^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$$

**Hence Proved**

17. If  $y = (\tan^{-1} x)^2$ , show that  $(x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1 = 2$

17. Given that:  $y = (\tan^{-1} x)^2$ , therefore,

$$\frac{dy}{dx} = \frac{d}{dx} \left[ (\tan^{-1} x)^2 \right] = 2 \tan^{-1} x \cdot \frac{1}{1+x^2} = \frac{2 \tan^{-1} x}{1+x^2}$$

$$\Rightarrow (1+x^2) \frac{dy}{dx} = 2 \tan^{-1} x$$

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{d}{dx} (1+x^2) = \frac{d}{dx} (2 \tan^{-1} x)$$

$$\Rightarrow (1+x^2) \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot 2x = \frac{2}{1+x^2}$$

$$\Rightarrow (1+x^2)^2 \frac{d^2y}{dx^2} + 2x(1+x^2) \frac{dy}{dx} = 2$$

$$\Rightarrow (x^2 + 1)^2 y_2 + 2x(x^2 + 1) y_1 = 2$$

**myclass24**  
Your Class. Your Pace.