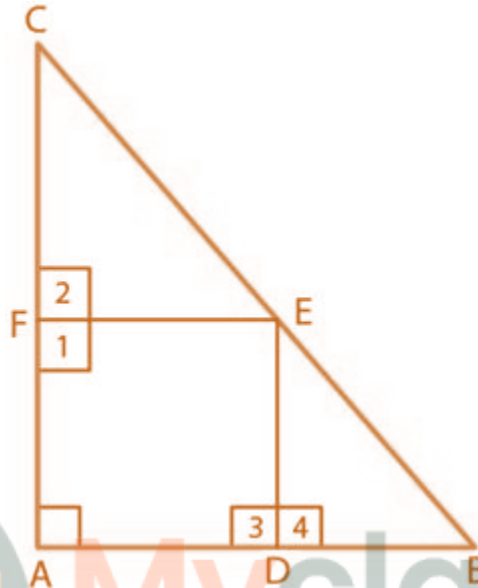


EXERCISE 8.4

A square is inscribed in an isosceles right triangle so that the square and the triangle have one angle common. Show that the vertex of the square opposite the vertex of the common angle bisects the hypotenuse.

Solution:



According to the question,
 $\triangle ABC$ with $\angle A = 90^\circ$ and
 Since, ABC is an isosceles triangle,

We get,
 $AB = AC \dots(i)$

Let $ADEF$ be the square inscribed in the isosceles triangle ABC .

Then, we have,
 $AD = AF = EF = DE \dots(ii)$

Subtracting equation (ii) from (i),

$$AB - AD = AC - AF$$

$$BD = CF$$

Now,

Considering $\triangle CFE$ and $\triangle EDB$,

$$BD = CF$$

$$DE = EF$$

$\angle CFE = \angle EDB = 90^\circ$ (Since, they are the side of a square)

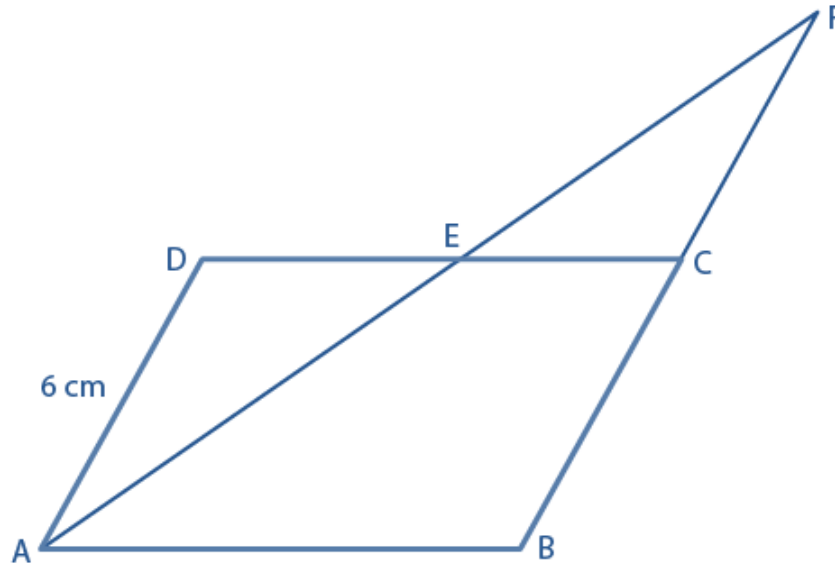
$\triangle CEF \sim \triangle BED$ (By SAS criteria)

Hence, $CE = BE$

Therefore, vertex E of the square bisects the hypotenuse BC .

1. In a parallelogram $ABCD$, $AB = 10$ cm and $AD = 6$ cm. The bisector of $\angle A$ meets DC in E . AE and BC produced meet at F . Find the length of CF .

Solution:



According to the question,

We have,

ABCD is a parallelogram

AB = 10 cm

AD = 6 cm.

The bisector of $\angle A$ meets DC at E.

AE and BC produced meet at F.

Since, AF bisects $\angle A$,

We get,

$\angle BAE = \angle EAD \dots (1)$

$\angle EAD = \angle EFB \dots (2)$ [Alternate angles]

From equations (1) and (2),

We get,

$\angle BAE = \angle EFB$

Since sides opposite to equal angles are equal,

We get,

BF = AB

Here, AB = 10 cm

So, BF = 10 cm

$\Rightarrow BC + CF = 10$ cm

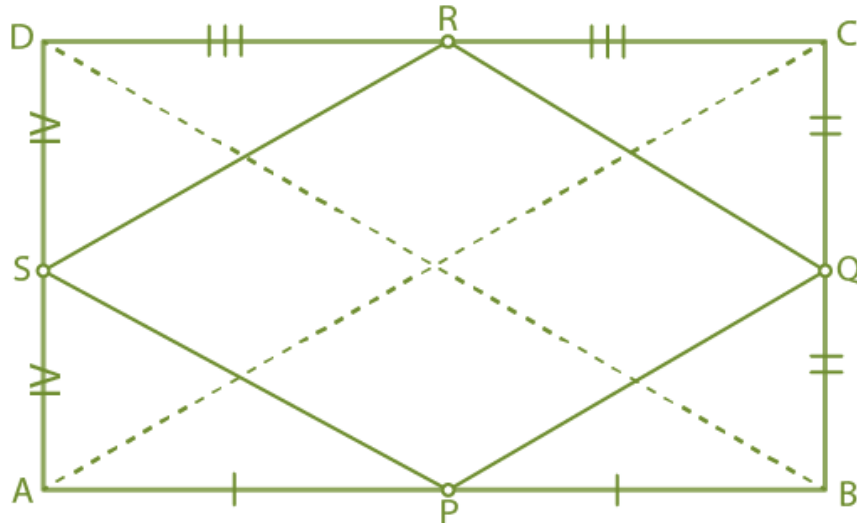
6 cm + CF = 10 cm [BC = AD = 6 cm, opposite sides of a parallelogram]

$\Rightarrow CF = 10 - 6$ cm = 4 cm

$\Rightarrow CF = 4$ cm

2. P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD in which AC = BD. Prove that PQRS is a rhombus.

Solution:



According to the question,

We have,

P is the mid-point of the sides AB

Q is the mid-point of the sides BC

R is the mid-point of the sides CD

S is the mid-point of the sides DA

Also, we know that,

$AC = BD$.

In $\triangle ADC$, by mid-point theorem,

$SR = \frac{1}{2} AC$

And, $SR \parallel AC$

In $\triangle ABC$, by mid-point theorem,

$PQ = \frac{1}{2} AC$

And, $PQ \parallel AC$

Hence, $SR = PQ = \frac{1}{2} AC$

Similarly,

In $\triangle BCD$, by mid-point theorem,

$RQ = \frac{1}{2} BD$

And, $RQ \parallel BD$

In $\triangle BAD$, by mid-point theorem,

$SP = \frac{1}{2} BD$

And, $SP \parallel BD$

So, we get,

$SP = RQ = \frac{1}{2} BD = \frac{1}{2} AC$

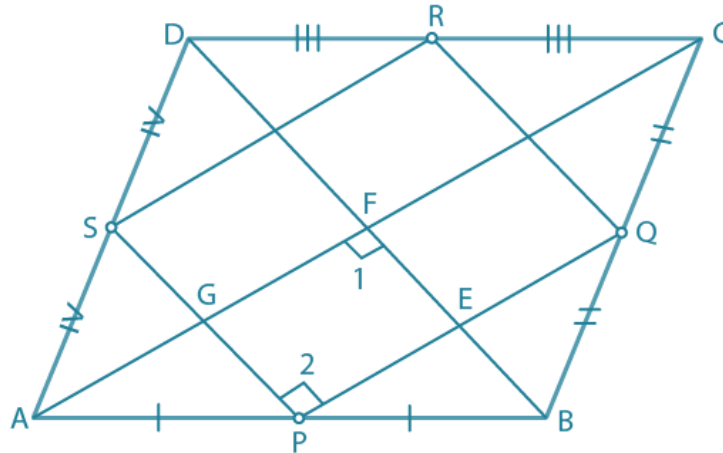
Then,

$SR = PQ = SP = RQ$

Hence, PQRS is a rhombus.

3. P, Q, R and S are respectively the mid-points of the sides AB, BC, CD and DA of a quadrilateral ABCD such that $AC \perp BD$. Prove that PQRS is a rectangle.

Solution:



According to the question,

We have,

P is the mid-point of the sides AB

Q is the mid-point of the sides BC

R is the mid-point of the sides CD

S is the mid-point of the sides DA

Also,

$AC \perp BD$

$\angle COD = \angle AOD = \angle AOB = \angle COB = 90^\circ$

In $\triangle ADC$, by mid-point theorem,

$SR = \frac{1}{2} AC$

And, $SR \parallel AC$

In $\triangle ABC$, by mid-point theorem,

$PQ = \frac{1}{2} AC$

And, $PQ \parallel AC$

So, we have,

$PQ \parallel SR$ and $SR = PQ = \frac{1}{2} AC$

Similarly,

$SP \parallel RQ$ and $SP = RQ = \frac{1}{2} BD$

Now, in quadrilateral EOFR,

$OE \parallel FR$ and $OF \parallel ER$

So, we get,

$\angle EOF = \angle ERF = 90^\circ$

Hence, PQRS is a rectangle.

4. P, Q, R and S are respectively the mid-points of sides AB, BC, CD and DA of quadrilateral ABCD in which $AC = BD$ and $AC \perp BD$. Prove that PQRS is a square.

Solution:

According to the question,

We have,

P is the mid-point of the sides AB

Q is the mid-point of the sides BC

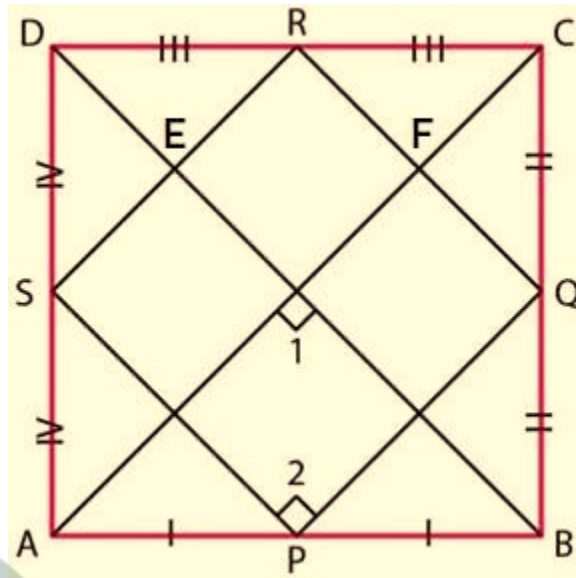
R is the mid-point of the sides CD

S is the mid-point of the sides DA

Also,

$AC \perp BD$

And $AC = BD$



In $\triangle ADC$, by mid-point theorem,

$SR = \frac{1}{2} AC$

And, $SR \parallel AC$

In $\triangle ABC$, by mid-point theorem,

$PQ = \frac{1}{2} AC$

And, $PQ \parallel AC$

So, we have,

$PQ \parallel SR$ and $PQ = SR = \frac{1}{2} AC$

Now, in $\triangle ABD$, by mid-point theorem,

$SP \parallel BD$ and $SP = \frac{1}{2} BD = \frac{1}{2} AC$

In $\triangle BCD$, by mid-point theorem,

$RQ \parallel BD$ and $RQ = \frac{1}{2} BD = \frac{1}{2} AC$

$SP = RQ = \frac{1}{2} AC$

$PQ = SR = SP = RQ$

Thus, we get that,

All four sides are equal.

Considering the quadrilateral EOPF,

$OE \parallel FR$, $OF \parallel ER$

$\angle EOF = \angle ERF = 90^\circ$ (Opposite angles of parallelogram)

$\angle QRS = 90^\circ$

Hence, PQRS is a square.

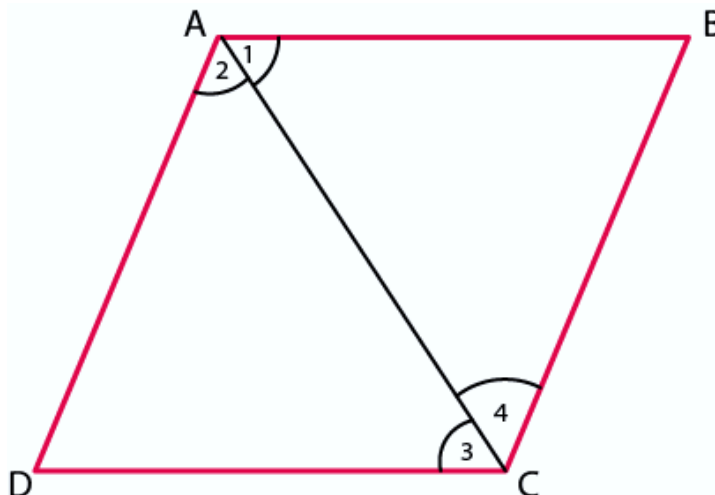
5. A diagonal of a parallelogram bisects one of its angles. Show that it is a rhombus.

Solution:

Let the parallelogram be = ABCD

Diagonal AC bisect $\angle A$.

$$\angle CAB = \angle CAD$$



Now,

$AB \parallel CD$ and AC is a transversal.

$$\angle CAB = \angle ACD$$

Again, $AD \parallel BC$ and AC is a transversal.

$$\angle DAC = \angle ACB$$

Now,

$$\angle A = \angle C$$

$$\frac{1}{2} \angle A = \frac{1}{2} \angle C$$

$$\angle DAC = \angle DCA$$

$$AD = CD$$

But, $AB = CD$ and $AD = BC$ (Opposite sides of parallelograms)

$$AB = BC = CD = AD$$

Thus, $ABCD$ is a rhombus.

6. P and Q are the mid-points of the opposite sides AB and CD of a parallelogram ABCD. AQ intersects DP at S and BQ intersects CP at R. Show that PRQS is a parallelogram.

Solution:

According to the question,

Q is the midpoint of AB

P is the midpoint of CD

Now,

$AB \parallel CD$,

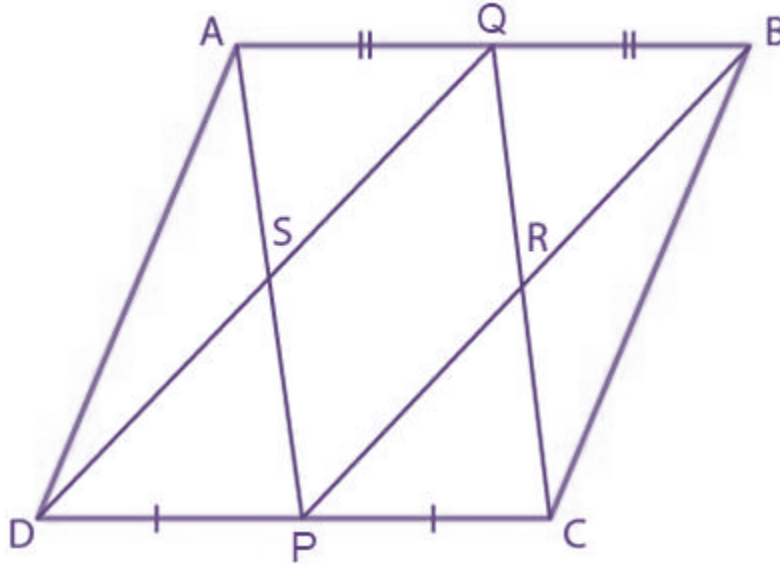
Also,

$AP \parallel QC$

And, $AB = DC$

$$\frac{1}{2} AB = \frac{1}{2} DC$$

$$AP = QC$$



Now,
 $AP \parallel QC$ and $AP = QC$
 $APCQ$ is a parallelogram.
 $AQ \parallel PC$ or $SQ \parallel PR$

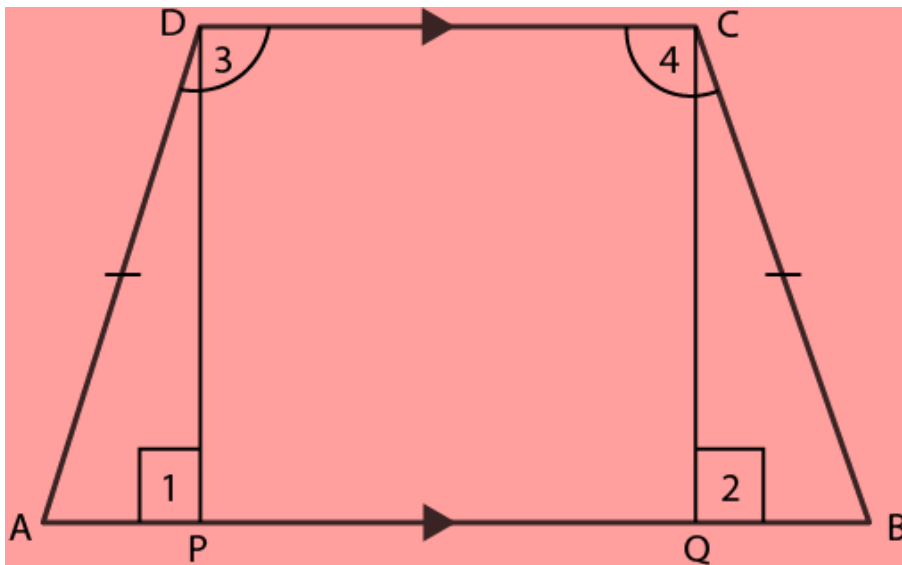
Again,
 $AB \parallel DC$ means $\frac{1}{2} AB = \frac{1}{2} DC$
 $BP = QD$

Now, $BP \parallel QD$ and $BP = QD$
 $BPDQ$ is a parallelogram
 So, $PD \parallel BQ$ or $PS \parallel QR$

Thus, $SQ \parallel RP$ and $PS \parallel QR$
 $PQRS$ is a parallelogram.

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7. $ABCD$ is a quadrilateral in which $AB \parallel DC$ and $AD = BC$. Prove that $\angle A = \angle B$ and $\angle C = \angle D$.
Solution:



According to the question,

We have,

Quadrilateral ABCD

$AB \parallel CD$ and $AD = BC$.

To prove: $\angle A = \angle B$ and $\angle C = \angle D$.

Construction: Draw $DP \perp AB$ and $CQ \perp AB$.

Proof: In $\triangle APD$ and $\triangle BQC$,

Since $\angle 1$ and $\angle 2$ are equal to 90°

$\angle 1 = \angle 2$

Distance between parallel line,

$AD = BC$ [Given]

By RHS criterion of congruence,

We have

$\triangle APD \cong \triangle BQC$ [CPCT]

$\angle A = \angle B$

Now, $DC \parallel AB$

Since, sum of consecutive interior angles is 180°

$\angle A + \angle 3 = 180 \dots(1)$

And,

$\angle B + \angle 4 = 180 \dots(2)$

From equations (1) and (2),

We get

$\angle A + \angle 3 = \angle B + \angle 4$

Since, $\angle A = \angle B$,

We have,

$\Rightarrow \angle 3 = \angle 4$

$\Rightarrow \angle C = \angle D$

Hence, proved.

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8. In Fig. 8.11, $AB \parallel DE$, $AB = DE$, $AC \parallel DF$ and $AC = DF$. Prove that $BC \parallel EF$ and $BC = EF$.

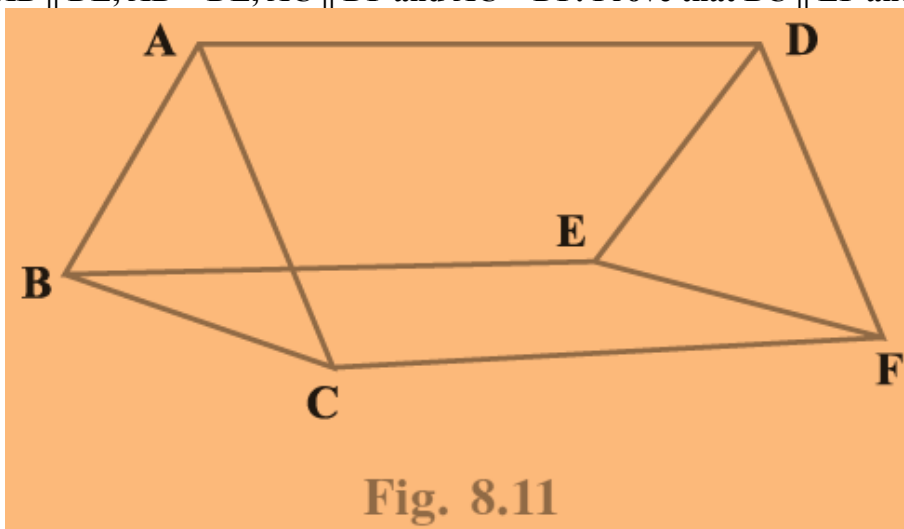
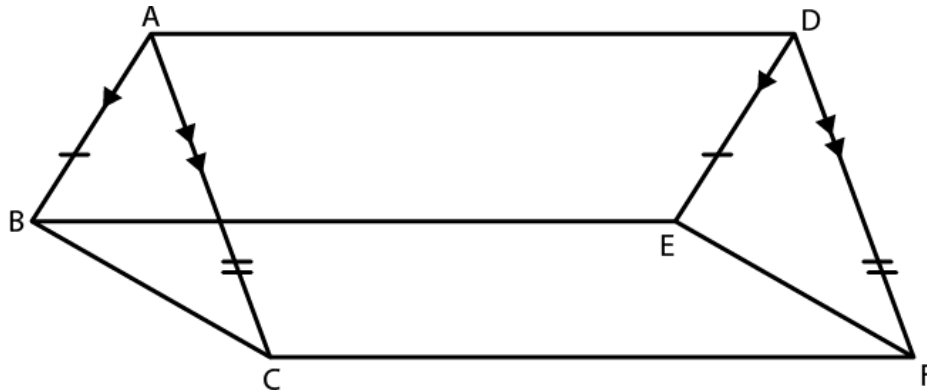


Fig. 8.11

Solution:



According to the question,

In quadrilateral ABED,

We have,

$AB \parallel DE$ and $AB = DE$

ABED is a parallelogram.

$AD \parallel BE$ and $AD = BE$

In quadrilateral ACFD,

We have,

$AC \parallel FD$ and $AC = FD$

ACFD is a parallelogram.

$AD \parallel CF$ and $AD = CF$

$AD = BE = CF$ and $CF \parallel BE$

In quadrilateral BCFE,

$BE = CF$ and $BE \parallel CF$.

BCFE is a parallelogram.

$BC = EF$ and $BC \parallel EF$

Hence proved.