Chapter 6

Chapter 6: Triangles

EXERCISE 6.4:

1. Let Δ ABC ~ Δ DEF and their areas be, respectively, 64 cm^2 and 121 cm^2. If EF = 15.4 cm, find BC

Solution: It is given that,

Δ ABC ~ Δ DEF

Therefore,

$\frac{ar(\Delta\ ABC)}{ar(\Delta\ DEF)} = (\frac{AB}{DE})^2 = (\frac{BC}{EF})^2 = (\frac{AC}{DF})^2$

Given:

EF = 15.4 cm

ar (ΔABC) = 64cm^2

ar (ΔDEF) = 121 cm^2

Hence,

$\frac{ar(\Delta\ ABC)}{ar(\Delta\ DEF)} = (\frac{BC}{EF})^2$

$(\frac{64}{121}) = (\frac{BC*BC}{15.4*15.4})$

$$\frac{BC}{15.4} = \frac{8}{11}$$

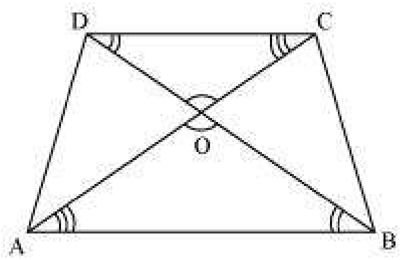
BC = $(8 * \frac{15.4}{11})$

BC = 8 * 1.4 = 11.2 cm

2. Diagonals of a trapezium ABCD with AB || DC intersect each other at the point O. If AB = 2 CD, find the ratio of the areas of triangles AOB and COD

Solution: Since AB || CD,

∴∠OAB = ∠OCD and ∠OBA = ∠ODC (Alternate interior angles)



In ΔAOB and ΔCOD,

∠AOB = ∠COD (Vertically opposite angles)

∠OAB = ∠OCD (Alternate interior angles)

∠OBA = ∠ODC (Alternate interior angles)

ΔAOB ~ΔCOD (By AAA similarity)

$$\frac{ar(\Delta\, AOB)}{ar(\Delta\, COD)} = \left(\frac{AB}{CD}\right) * \left(\frac{AB}{CD}\right)$$

Since, AB = 2 CD

Therefore,

$$\frac{ar(\Delta\, AOB)}{ar(\Delta\, COD)} = \frac{2 * CD * CD}{CD * CD}$$

= $\frac{4}{1} = 4:1$

3. In Fig. 6.44, ABC and DBC are two triangles on thesame base BC. If AD intersects BC at O, show that $\frac{ar(ABC)}{ar(DBC)} = \frac{AO}{DO}$.

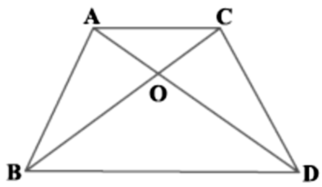


Fig. 6.44

Solution: Let us draw two perpendiculars AP and DM on line BC

Area of a triangle =$\frac{1}{2} * Base * Height$

Therefore,

$$\frac{ar(\Delta\, ABC)}{ar(\Delta\, DBC)} = \frac{\frac{1}{2} * BC * AP}{\frac{1}{2} * BC * DM}$$

= $\frac{AP}{DM}$

In ΔAPO and ΔDMO,

∠APO = ∠DMO (Each = 90°)

∠AOP = ∠DOM (Vertically opposite angles)

∴ΔAPO ~ ΔDMO (By AA similarity)

$$\frac{AP}{DM} = \frac{AO}{DO}$$
$$\frac{ar(\Delta\, ABC)}{ar(\Delta DBC)} = \frac{AO}{DO}$$

4. If the areas of two similar triangles are equal, prove that they are congruent

Solution: Let us consider two similar triangles as ΔABC ~ ΔPQR

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$ (i)

$$Given\ that,$$
$$ar(\Delta ABC) = ar(\Delta PQR)$$

1 = $\left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

AB = PQ

BC = QR

And,

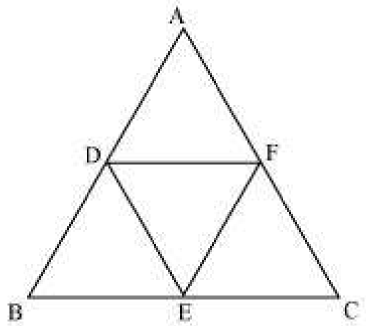
AC = PR

Hence,

$\Delta ABC \cong \Delta PQR$ (By SSS rule)

5. D, E and F are respectively the mid-points of sides AB, BC and CA of Δ ABC. Find theratio of the areas of Δ DEF and Δ ABC

Solution: D and E are the mid-points of ΔABC



DE parallel AC

And, DE = $\frac{1}{2}$ AC

In ΔBED and ΔBCD

$\angle BED = \angle BCA$ (Corresponding angles)

$\angle BDE = \angle BAC$ (Corresponding angles)

$\angle EBD = \angle CBA$ (Common angles)

Therefore,

$\Delta BED \sim \Delta BCA$ (AAA similarity)

$\frac{ar(\Delta BED)}{ar(\Delta BCA)} = \left(\frac{DE}{AC}\right)^2$

$\frac{ar(\Delta BED)}{ar(\Delta BCA)} = \frac{1}{4}$

ar(ΔBED) = $\frac{1}{4}$ ar(ΔBCA)

Similarly,

ar(ΔCFE) = $\frac{1}{4}$ ar(ΔCBA)

And,

ar(ΔADF) = $\frac{1}{4}$ ar(ΔABC)

Also,

ar(ΔDEF) = ar(ΔABC) – [ar(ΔBED) + ar(ΔCFE) + ar(ΔADF)]

ar(ΔDEF) = ar(ΔABC) – $\frac{3}{4}ar(\Delta ABC)$

= $\frac{1}{4}$ar(ΔABC)

$\frac{ar(\Delta DEF)}{ar(\Delta ABC)} = \frac{1}{4}$

6. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding medians

Solution: Let us assume two similar triangles as ΔABC ~ ΔPQR. Let AD and PS be the medians of these triangles

ΔABC ~ ΔPQR

$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$ (i)

∠A = ∠P, ∠B = ∠Q, ∠C = ∠R (ii)

Since AD and PS are medians,

BD = DC $=\frac{BC}{2}$

And, QS = SR $=\frac{QR}{2}$

Equation (i) becomes,

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AC}{PR}$ (iii)

In ΔABD and ΔPQS,

∠B = ∠Q [From (ii)]

And,

$\frac{AB}{PQ} = \frac{BD}{QS}$ [From (iii)]

ΔABD ~ ΔPQS (SAS similarity)

Therefore, it can be said that

$\frac{AB}{PQ} = \frac{BD}{QS} = \frac{AD}{PS}$(iv)

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{BC}{QR}\right)^2 = \left(\frac{AC}{PR}\right)^2$

From (i) and (iv), we get

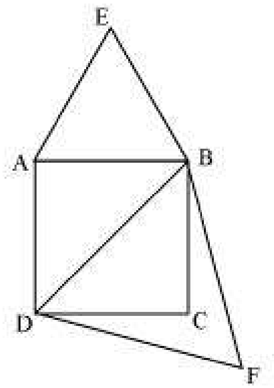
$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{AD}{PS}$$

And hence,

$\frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \left(\frac{AD}{PS}\right)^2$

7. Prove that the area of an equilateral triangle described on one side of a square is equalto half the area of the equilateral triangle described on one of its diagonals

Solution: Let ABCD be a square of side a



Hence, it's diagonal = $\sqrt{2}a$

Two desired equilateral triangles are formed as ΔABE and ΔDBF

Side of an equilateral triangle, ΔABE, described on one of its sides = *a*

Side of an equilateral triangle, ΔDBF, described on one of its diagonals = $\sqrt{2}a$

We know that equilateral triangles have all its angles as 60 º and all its sides of the same length Therefore, allequilateral triangles are similar to each other. Hence, the ratio between the areas of these triangles will be equal tothe square of the ratio between the sides of these triangles

$\frac{Area\ of\ (\Delta ABE)}{Area\ of\ (\Delta DBF)} = \left(\frac{a}{\sqrt{2}a}\right)^2$

$$= \frac{1}{2}$$

Tick the correct answer and justify:

8. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio ofthe areas of triangles ABC and BDE is

A. 2 : 1 B. 1 : 2

C. 4 : 1 D. 1 : 4

Solution: We know:

Equilateral triangles have all its angles as 60º and all its sides are of the same length. Therefore, all equilateral triangles are similar to each other.

Hence, the ratio between the areas of these triangles will be equal tothe square of the ratio between the sides of these triangles.

Let side of ΔABC = *x*

Therefore,

Side ofΔBDE = $\frac{x}{2}$

Therefore,

$\frac{Area(\Delta ABC)}{Area(\Delta BDE)} = \left(\frac{x}{\frac{x}{2}}\right)^2$

$$= \frac{4}{1}$$

Hence, the correct answer is C

9. Sides of two similar triangles are in the ratio 4: 9. Areas of these triangles are in the ratio

A. 2 : 3 B. 4 : 9

C. 81 : 16 D. 16 : 81

Solution: If two triangles are similar to each other, then the ratio of the areas of these triangles will be equal to the square of theratio of the corresponding sides of these triangles.

It is given that the sides are in the ratio 4:9

Therefore,

Ratio between areas of these triangles $=\left(\frac{4}{9}\right)^2$

$=\frac{16}{81}$

Hence, the correct answer is (D)