

EXERCISE 4.6

1. Find the principal values of each of the following:

- (i) $\cot^{-1}(-\sqrt{3})$
- (ii) $\cot^{-1}(\sqrt{3})$
- (iii) $\cot^{-1}(-1/\sqrt{3})$
- (iv) $\cot^{-1}(\tan 3\pi/4)$

Solution:

(i) Given $\cot^{-1}(-\sqrt{3})$

Let $y = \cot^{-1}(-\sqrt{3})$

$$-\cot(\pi/6) = \sqrt{3}$$

$$= \cot(\pi - \pi/6)$$

$$= \cot(5\pi/6)$$

The range of principal value of \cot^{-1} is $(0, \pi)$ and $\cot(5\pi/6) = -\sqrt{3}$

Thus, the principal value of $\cot^{-1}(-\sqrt{3})$ is $5\pi/6$

(ii) Given $\cot^{-1}(\sqrt{3})$

Let $y = \cot^{-1}(\sqrt{3})$

$$\cot(\pi/6) = \sqrt{3}$$

The range of principal value of \cot^{-1} is $(0, \pi)$ and

Thus, the principal value of $\cot^{-1}(\sqrt{3})$ is $\pi/6$

(iii) Given $\cot^{-1}(-1/\sqrt{3})$

Let $y = \cot^{-1}(-1/\sqrt{3})$

$$\cot y = (-1/\sqrt{3})$$

$$-\cot(\pi/3) = 1/\sqrt{3}$$

$$= \cot(\pi - \pi/3)$$

$$= \cot(2\pi/3)$$

The range of principal value of $\cot^{-1}(0, \pi)$ and $\cot(2\pi/3) = -1/\sqrt{3}$

Therefore the principal value of $\cot^{-1}(-1/\sqrt{3})$ is $2\pi/3$

(iv) Given $\cot^{-1}(\tan 3\pi/4)$

But we know that $\tan 3\pi/4 = -1$

By substituting this value in $\cot^{-1}(\tan 3\pi/4)$ we get

$$\cot^{-1}(-1)$$

Now, let $y = \cot^{-1}(-1)$

$$\cot y = (-1)$$

$$- \cot (\pi/4) = 1$$

$$= \cot (\pi - \pi/4)$$

$$= \cot (3\pi/4)$$

The range of principal value of $\cot^{-1}(0, \pi)$ and $\cot (3\pi/4) = -1$

Therefore the principal value of $\cot^{-1}(\tan 3\pi/4)$ is $3\pi/4$



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