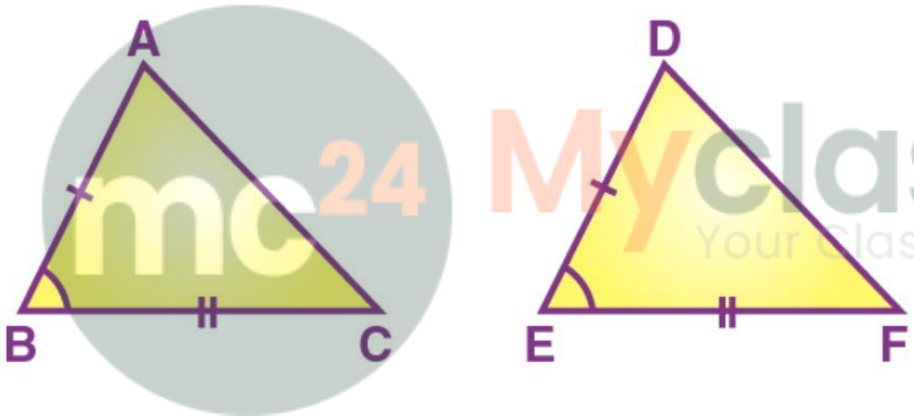
EXERCISE 9A

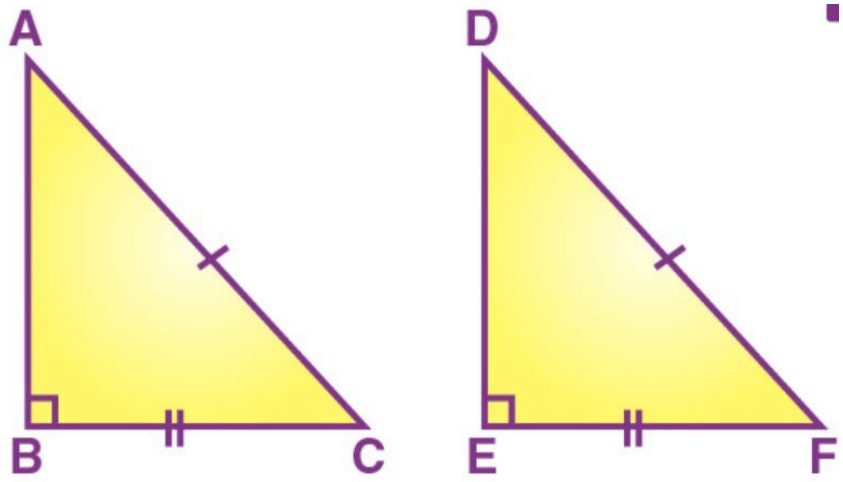
1. Which of the following pairs of triangles are congruent? In each case, state the condition of congruency:
(a) In Δ ABC and Δ DEF, AB = DE, BC = EF and ∠B = ∠E.
(b) In Δ ABC and Δ DEF, ∠B = ∠E = 90°; AC = DF and BC = EF.
(c) In Δ ABC and Δ QRP, AB = QR, ∠B = ∠R and ∠C = ∠P.
(d) In Δ ABC and Δ PQR, AB = PQ, AC = PR and BC = QR.
(e) In Δ ABC and Δ PQR, BC = QR, ∠A = 90°, ∠C = ∠R = 40° and ∠Q=50°.

Solution:
(a) In Δ ABC and Δ DEF
AB = DE (data)
BC = EF and ∠B = ∠E (given)



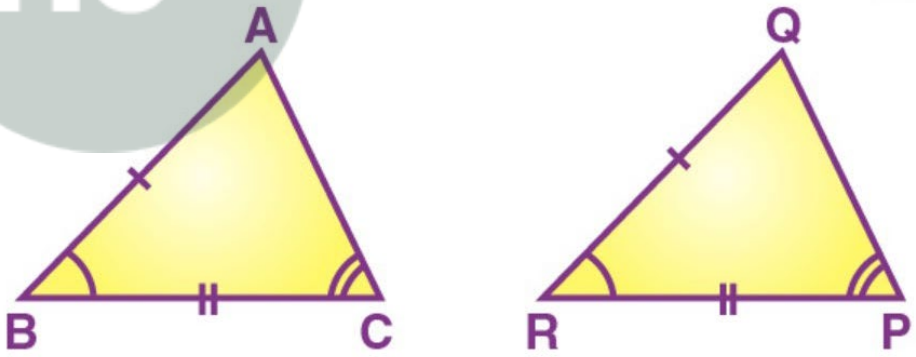
By SAS criteria of congruency and given data we can conclude that,
Δ ABC and Δ DEF are congruent to each other.
Therefore, Δ ABC ≅ Δ DEF.

(b) Given in Δ ABC and Δ DEF,
∠B = ∠E = 90°;
AC = DF
That is hypotenuse AC = hypotenuse DF
and BC = EF



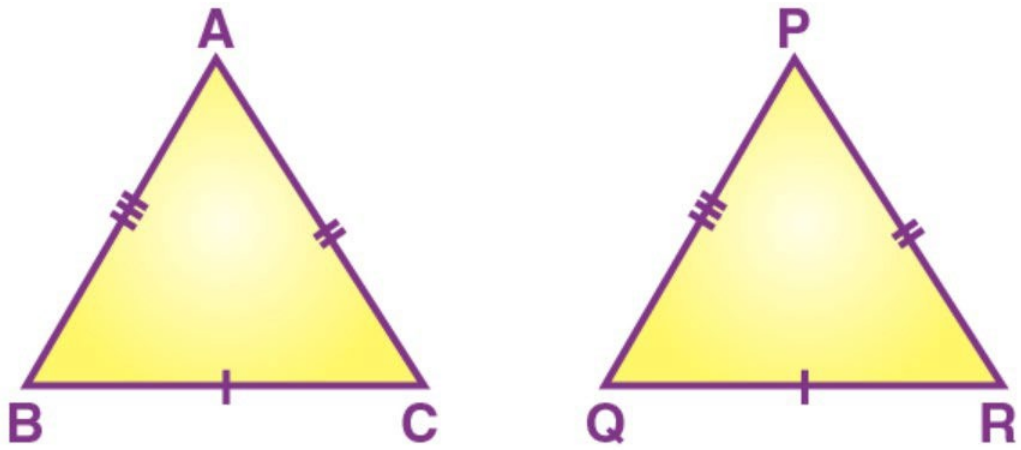
By right angle hypotenuse side postulate of congruency,
The given triangles Δ ABC and Δ DEF are congruent to each other.
Therefore, Δ ABC ≅ Δ DEF.

(c) In Δ ABC and Δ QRP,
Data: AB = QR,
∠B = ∠R and
∠C = ∠P.



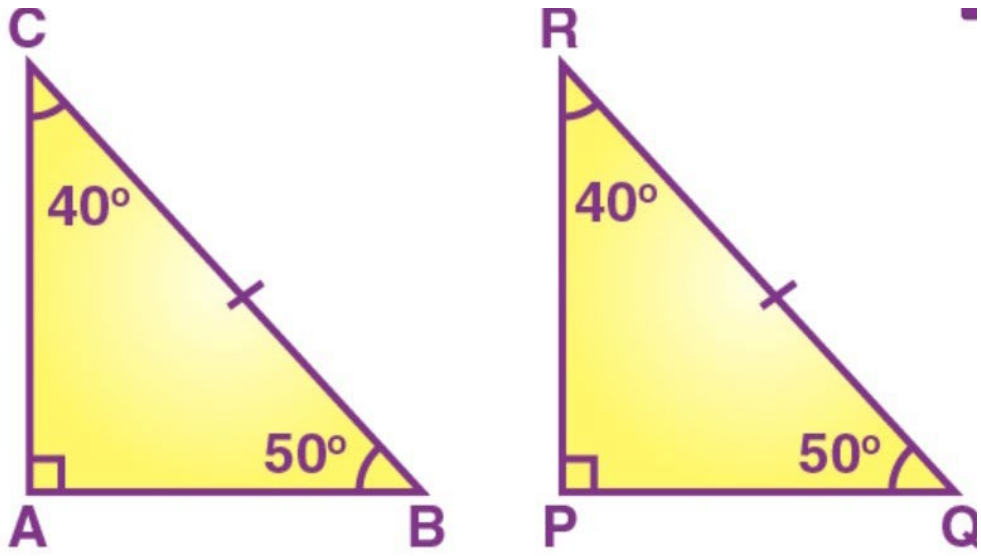
By using SAS postulate and given data we can conclude that
The given triangles Δ ABC and Δ QRP are congruent to each other.
Therefore, Δ ABC ≅ Δ QRP.

(d) In In Δ ABC and Δ PQR,
Data: AB = PQ, AC = PR and BC = QR.



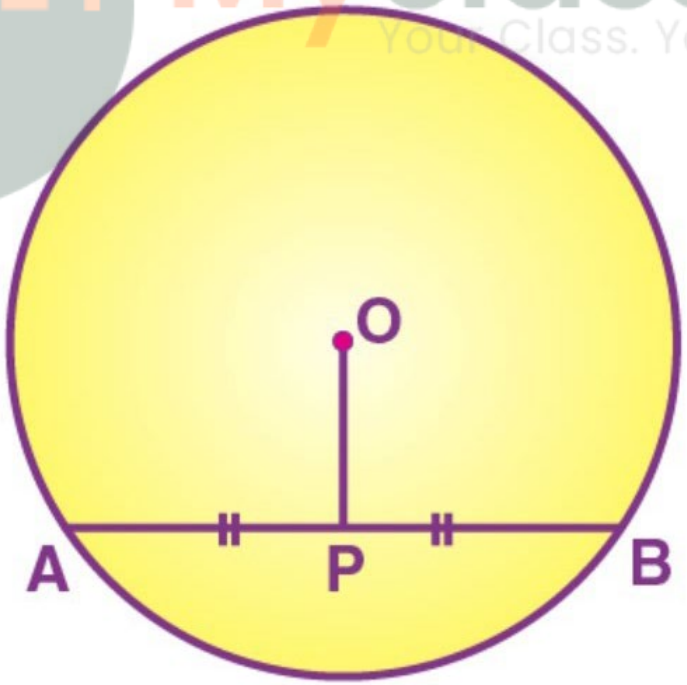
By using SSS postulate of congruency and given data we can conclude that
The given triangles Δ ABC and Δ PQR are congruent to each other.
Therefore, Δ ABC ≅ Δ PQR.

(e) In Δ ABC and Δ PQR,
Data: BC = QR, ∠A = 90°,
∠C = ∠R = 40° and ∠Q=50°
But we know that sum of angle of triangle = 180°
Therefore, ∠P + ∠Q + ∠R = 180°
∠P + 50° + 40° = 180°
∠P + 90° = 180°
∠P = 180° – 90°
∠P = 90°
In Δ ABC and Δ PQR,
∠A = ∠P
∠C = ∠R
BC = QR



By ASA postulate of congruency,
The given triangles Δ ABC and Δ PQR are congruent to each other.
Therefore, Δ ABC ≅ Δ PQR.

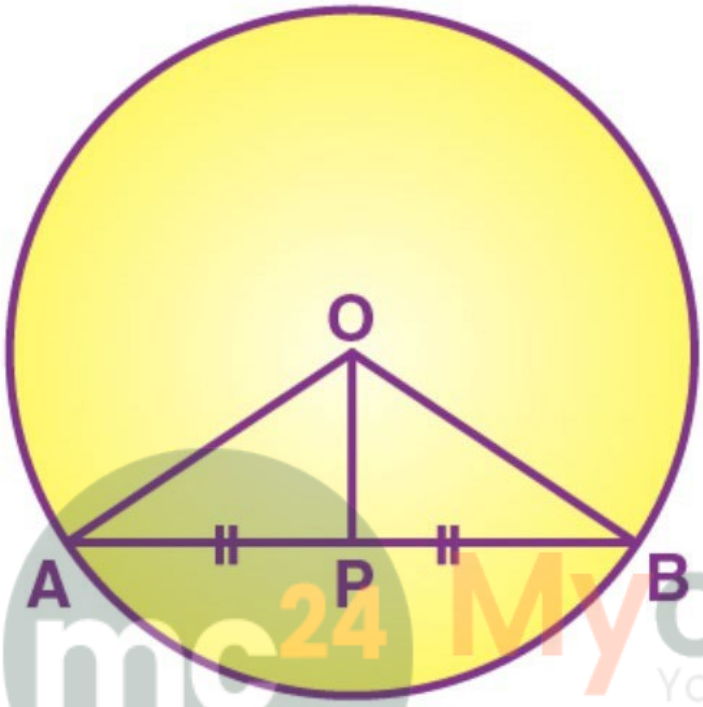
2. The given figure shows a circle with centre O. P is mid-point of chord AB.



Show that OP is perpendicular to AB.

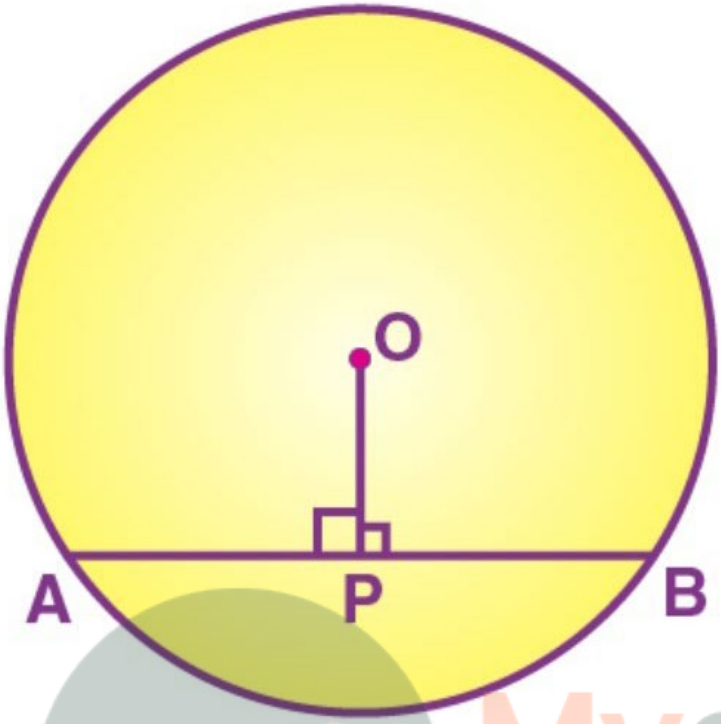
Solution:

Data: in the given figure O centre of circle.
P is mid-point of chord AB. AB is a chord
P is a point on AB such that AP = PB
Now we have to prove that OP ⊥ AB



Construction: Join OA and OB
Proof: In Δ OAP and Δ OBP
OA = OB (because radii of common circle)
OP = OP (common)
AP = PB (data)
By SSS postulate of congruent triangles
The given triangles Δ OAP and Δ OBP are congruent to each other.
Therefore, Δ OAP ≅ Δ OBP.
The corresponding parts of the congruent triangles are congruent.
∠OPA = ∠OPB (by Corresponding parts of Congruent triangles)
But ∠OPA + ∠OPB = 180^o (linear pair)
∠OPA = ∠OPB = 90^o
Hence OP ⊥ AB

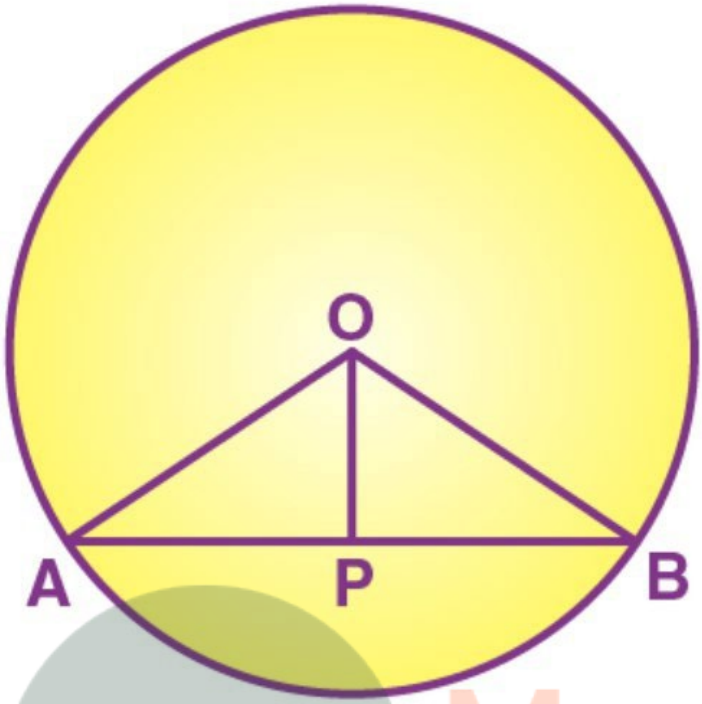
3. The following figure shows a circle with centre O.



If OP is perpendicular to AB, prove that AP=BP.

Solution:

Given: In the figure, O is the centre of the circle,
And AB is chord. P is a midpoint on AB such that AP = PB
We need to prove that AP = BP,

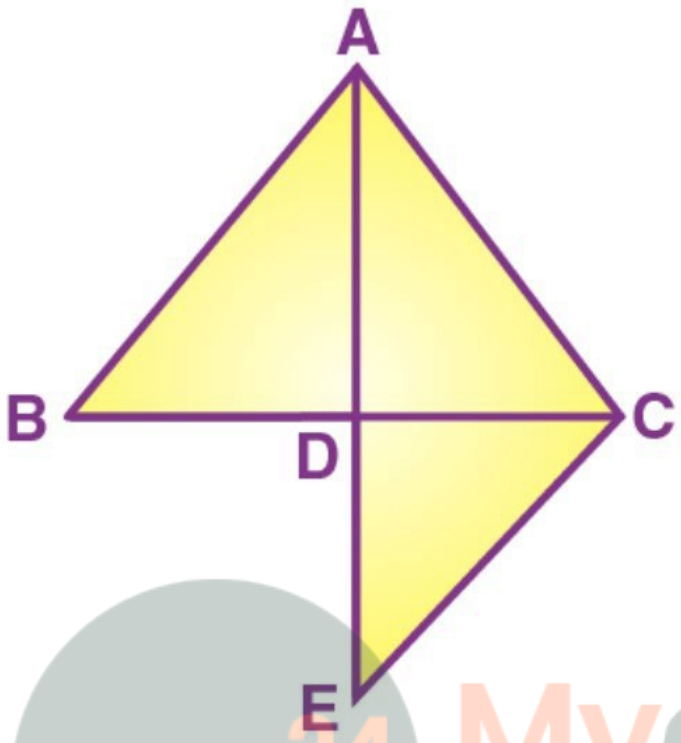


Construction: Join OA and OB
Proof: In right angle triangles Δ OAP and Δ OBP
Hypotenuse OA = Hypotenuse OB (because radii of common circle)
Side OP = OP (common)
AP = PB (data)
By SSS postulate of congruent triangles
The given triangles Δ OAP and Δ OBP are congruent to each other.
Therefore, Δ OAP ≅ Δ OBP.
The corresponding parts of the congruent triangles are congruent.
AP = BP (by Corresponding parts of Congruent triangles)
Hence the proof.

4. In a triangle ABC, D is mid-point of BC; AD is produced up to E so that DE = AD. Prove that: (i) Δ ABD and Δ ECD are congruent.
(ii) AB=EC
(iii) AB is parallel to EC.

Solution:
Given Δ ABC in which D is the mid-point of BC
AD is produced to E so that DE = AD
We need to prove that
(i) Δ ABD ≅ Δ ECD
(ii) AB = EC

(iii) AB ∥ EC



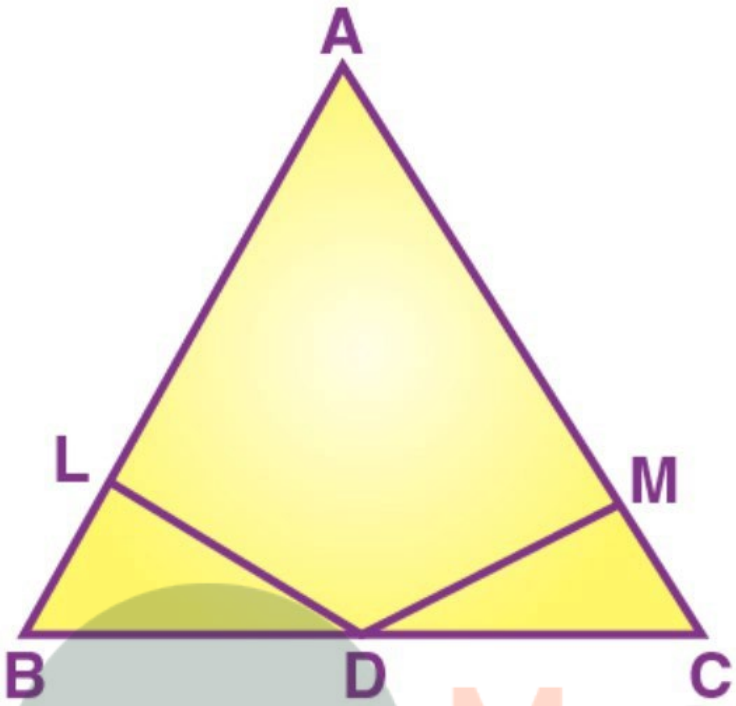
(i) In Δ ABD and Δ ECD
BD = DC (D is the midpoint of BC)
∠ADB = ∠CDE (vertically opposite angles)
AD = DE (Given)
By SAS postulate of congruency of triangles, we have
Δ ABD ≅ Δ ECD

(ii) The corresponding parts of congruent triangles are congruent
Therefore, AB = EC (corresponding parts of congruent triangles)

(iii) Also, we have ∠DAB = ∠DEC (corresponding parts of congruent triangles)
AB ∥ EC [∠DAB = ∠DEC are alternate angles]

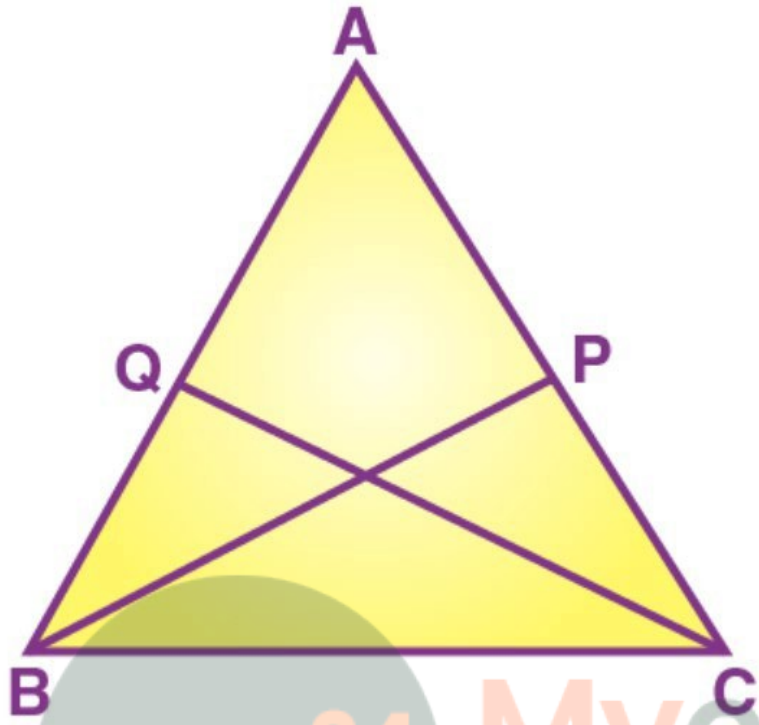
5. A triangle ABC has ∠B = ∠C. Prove that:
(i) The perpendiculars from the mid-point of BC to AB and AC are equal.
(ii) The perpendiculars form B and C to the opposite sides are equal.

Solution:
(i) Given Δ ABC in which ∠B = ∠C.
DL is perpendicular from D to AB
DM is the perpendicular from D to AC.



We need to prove that
DL = DM
Proof:
In Δ ABC and Δ DMC (DL perpendicular to AB and DM perpendicular to AC)
∠DLB = ∠DMC = 90°
∠B = ∠C (Given)
BD = DC (D is midpoint of BC)
By AAS postulate of congruent triangles
Δ DLB ≅ Δ DMC
The corresponding parts of the congruent triangles are congruent
Therefore DL = DM

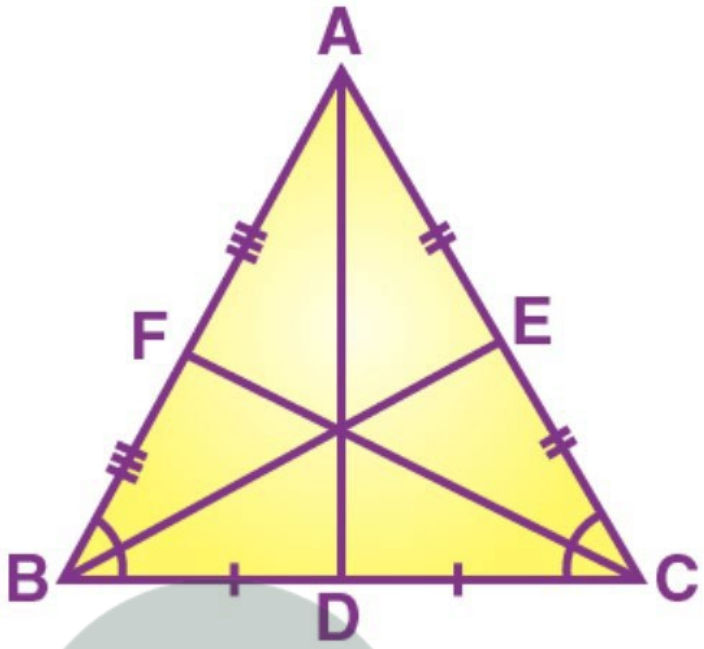
(ii) Given Δ ABC in which ∠B = ∠C.
BP is perpendicular from D to AC
CQ is the perpendicular from C to AB.



We need to prove that
BP = CQ
Proof:
In Δ BPC and Δ CQB (BP perpendicular to AC and CQ perpendicular to AB)
∠BPC = ∠CQB = 90°
∠B = ∠C (Given)
BC = BC (common)
By AAS postulate of congruent triangles
Δ BPC ≅ Δ CQB
The corresponding parts of the congruent triangles are congruent
Therefore BP = CQ

6. The perpendicular bisector of the sides of a triangle AB meet at I. Prove that: IA = IB = IC

Solution:
Given triangle ABC in which AD is the perpendicular bisector of BC
BE is the perpendicular bisector of CA
CF is the perpendicular bisector of AB
AD, BE and CF meet at I

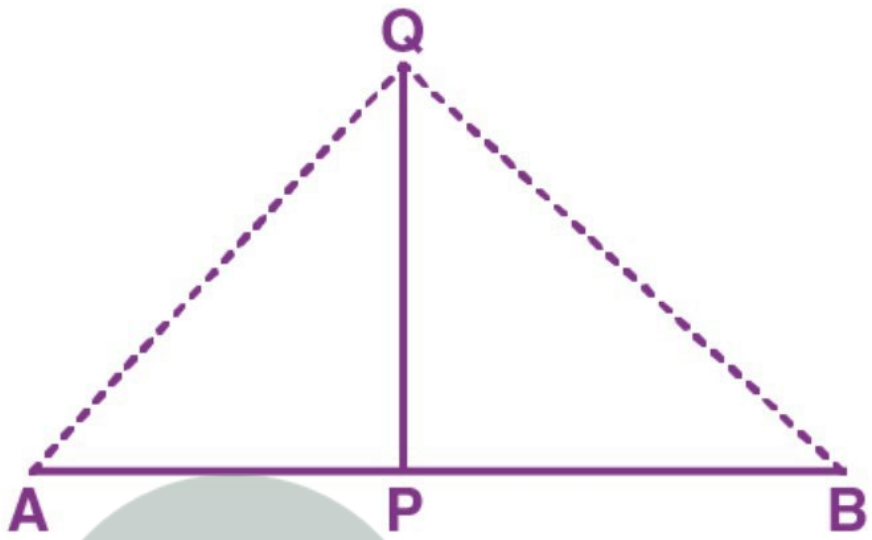


We need to prove that
IA = IB = IC
Proof:
In Δ BID and Δ CID
BD = DC (given)
∠BDI = ∠CDI = 90° (AD is perpendicular bisector of BC)
BC = BC (common)
By SAS postulate of congruent triangles
Δ BID ≅ Δ CID
The corresponding parts of the congruent triangles are congruent
Therefore IB = IC

Similarly, In Δ CIE and Δ AIE
CE = AE (given)
∠CEI = ∠AEI = 90° (AD is perpendicular bisector of BC)
IE = IE (common)
By SAS postulate of congruent triangles
Δ CIE ≅ Δ AIE
The corresponding parts of the congruent triangles are congruent
Therefore IC = IA
Thus, IA = IB = IC

7. A line segment AB is bisected at point P and through point P another line segment PQ, which is perpendicular to AB, is drawn. Show that: QA = QB.

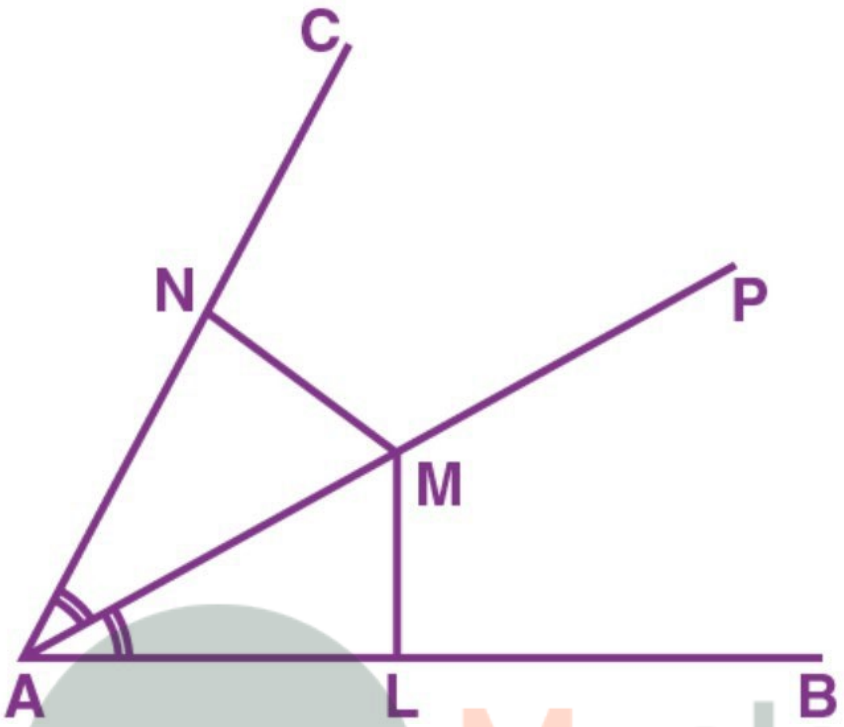
Solution:
Given triangle ABC in which AB is bisected at P
PQ is the perpendicular to AB



We need to prove that
QA = QB
Proof:
In Δ APQ and Δ BPQ
AP = PB (P is the midpoint of AB)
∠APQ = ∠BPQ = 90^o (PQ is perpendicular to AB)
BC = BC (common)
By SAS postulate of congruent triangles
Δ APQ ≅ Δ BPQ
The corresponding parts of the congruent triangles are congruent
Therefore QA = QB

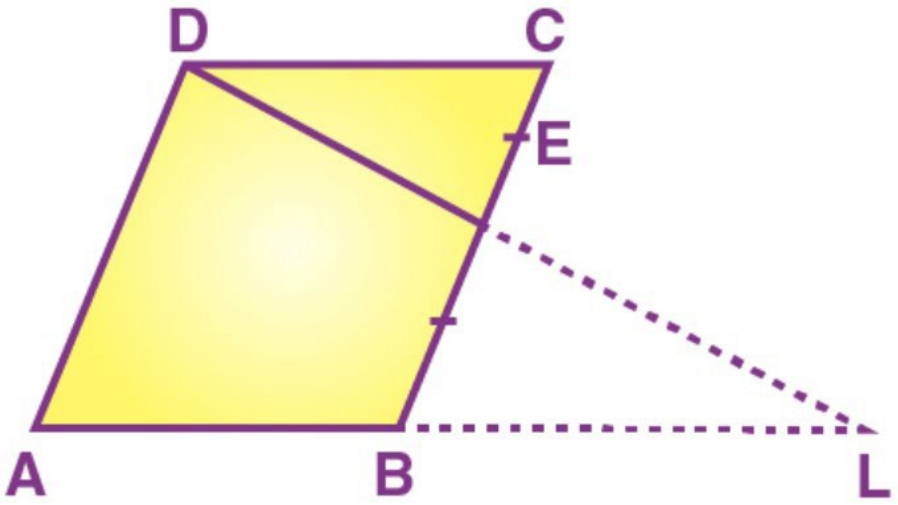
8. If AP bisects angle BAC and M is any point on AP, prove that the perpendiculars drawn from M to AB and AC are equal.

Solution:
From M, draw ML such that ML is perpendicular to AB and MN is perpendicular to AC



In Δ ALM and Δ ANM
∠LAM = ∠MAN (AP is the bisector of ∠BAC)
∠ALM = ∠ANM = 90^o (ML is perpendicular to AB and MN is perpendicular to AC)
AM = AM (common)
By AAS postulate of congruent triangles
Δ ALM ≅ Δ ANM
The corresponding parts of the congruent triangles are congruent
Therefore ML = MN
Hence the proof.

9. From the given diagram, in which ABCD is a parallelogram, ABL is al line segment and E is mid-point of BC.
Prove that:
(i) Δ DCE ≅ Δ LBE
(ii) AB = BL.
(iii) AL = 2DC



Solution:

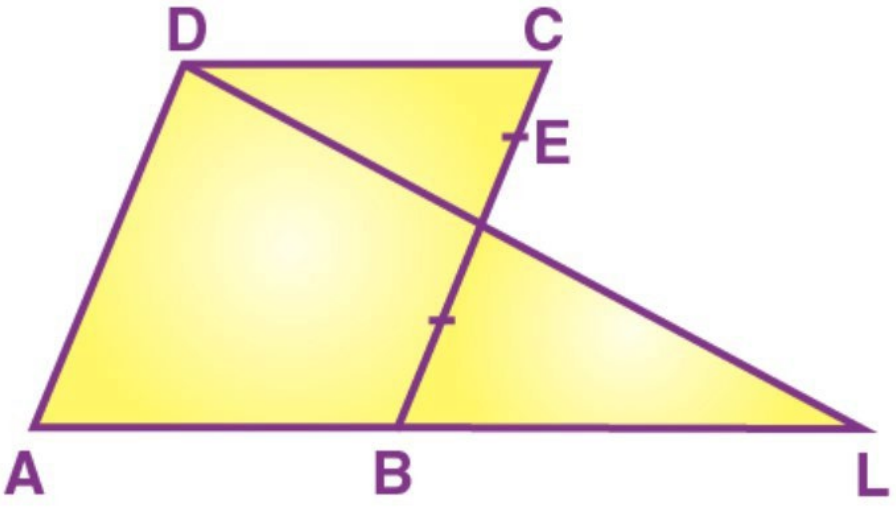
Given ABCD is a parallelogram in which E is the midpoint of BC

We need to prove that

(i) Δ DCE ≅ Δ LBE

(ii) AB = BL.

(iii) AL = 2DC



(i) In Δ DCE and Δ LBE

∠DCE = ∠LBE (DC parallel to AB, alternate angles)

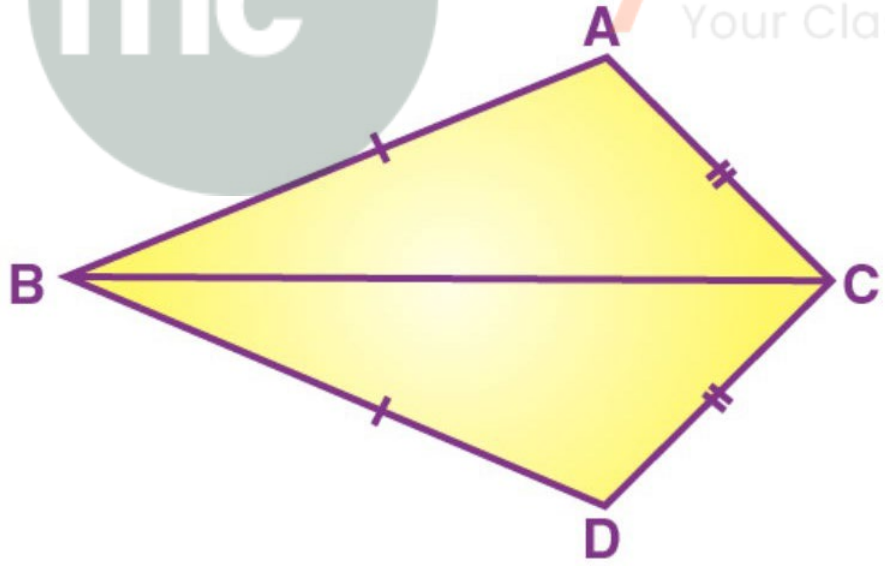
CE = EB (E is the midpoint of BC)

∠DCE = ∠LBE (vertically opposite angles)
By ASA postulate of congruent triangles
Δ DCE ≅ Δ LBE
The corresponding parts of the congruent triangles are congruent
Therefore DC = LB…. (i)

(ii) DC = AB…… (ii)
From (i) and (ii) AB = BL…… (iii)

(iii) AL = AB + BL…… (iv)
From (iii) and (iv) AL = AB + AB
AL = 2AB
AL = 2DC from (ii)

10. In the given figure, AB = DB and Ac = DC.
If ∠ABD = 58°,
∠ DBC = (2x - 4)°,
∠ACB = y + 15° and
∠DCB = 63°; find the values of x and y.



Solution:
Given: In the given figure, AB = DB and Ac = DC.
If ∠ABD = 58°,
∠ DBC = (2x - 4)°,
∠ACB = y + 15° and
∠DCB = 63°;

We need to find the values of x and y.
In Δ ABC and Δ DBC
AB = DB (given)
AC = DC (given)
BC = BC (common)
By SSS postulate of congruent triangles
Δ ABC ≅ Δ DBC
The corresponding parts of the congruent triangles are congruent
Therefore
∠ACB = ∠DCB
$y^o + 15^o = 63^o$
$y^o = 63^o - 15^o$
$y^o = 48^o$
∠ACB = ∠DCB (corresponding parts of the congruent triangles)
But ∠DCB = $(2x - 4)^o$
We have ∠ACB + ∠DCB = ∠ABD
$(2x - 4)^o + (2x - 4)^o = 58^o$
$4x - 8^o = 58^o$
$4x = 58^o + 8^o$
$4x = 66^o$
$x = 66^o/4$
$x = 16.5^o$
Thus, the values of x and y are
$x = 16.5^o$ and $y = 48^o$