Chapter 4
EXERCISE 4.3

1. Find the roots of the following quadratic equations, if they exist, by the method of completing the square:

(i) $2x^2 - 7x + 3 = 0$

(ii) $2x^2 + x - 4 = 0$

(iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

(iv) $2x^2 + x + 4 = 0$

Solution:

i) $2x^2 - 7x + 3 = 0$

Dividing by coefficient of $x^2, we\ get,$

$= x^2 - \frac{7}{2}x + \frac{3}{2} = 0$

Adding and subtracting the square of $\left(\frac{b}{2}\right) = \frac{7}{4}$ $, we\ get,$

$= x^2 - 2 \times \frac{7}{4} \times x + \left(\frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2 + \frac{3}{2} = 0$

$= \left(x - \frac{7}{4}\right)^2 + \frac{3}{2} - \frac{49}{16} = 0$

$= \left(x - \frac{7}{4}\right)^2 + \frac{24-49}{16} = 0$

$= \left(x - \frac{7}{4}\right)^2 - \frac{25}{16} = 0$

$= \left(x - \frac{7}{4}\right)^2 = \frac{25}{16} = \left(\frac{5}{4}\right)^2$

So,

$= \left(x - \frac{7}{4}\right) = \pm\frac{5}{4}$

When, $x - \frac{7}{4} = -\frac{5}{4}$

$= x = -\frac{5}{4} + \frac{7}{4} = \frac{2}{4} = \frac{1}{2}$

When, $x - \frac{7}{4} = \frac{5}{4}$

$= x = \frac{5}{4} + \frac{7}{4} = \frac{12}{4} = 3$.

ii) **$2x^2 + x - 4 = 0$**

Dividing by coefficient of x^2

$= x^2 + \frac{x}{2} - 2 = 0$

Adding and subtracting the square of $\frac{b}{2} = \frac{1}{4}, we\ get$

$= x^2 + 2(x) \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 - 2 = 0$

$= \left(x + \frac{1}{4}\right)^2 + \left(\frac{1}{16} - 2\right) = 0$

$= \left(x + \frac{1}{4}\right)^2 + \frac{1-32}{16} = 0$

$= \left(x + \frac{1}{4}\right)^2 = \frac{31}{16}$

Taking square root , we get,

$= \left(x + \frac{1}{4}\right) = \pm\frac{\sqrt{31}}{4}$

= x = $\frac{-1\pm\sqrt{31}}{4}$

∴ x = $\frac{-1+\sqrt{31}}{4}$ *and* $\frac{-1-\sqrt{31}}{4}$

iii) $4x^2 + 4\sqrt{3}x + 3 = 0$

Compare the quadratic equation with $ax^2 + bx + c$

$= (2x)^2 + 2(2x) \times \sqrt{3} + \left(\sqrt{3}\right)^2 = 0$

$= \left(2x + \sqrt{3}\right)^2 = 0$

$= 2x = -\sqrt{3}$

$= x = \frac{-\sqrt{3}}{2}$

∴ Roots of equation are = x = $\left(-\frac{\sqrt{3}}{2}\right)$, $\left(-\frac{\sqrt{3}}{2}\right)$

iv) **$2x^2$ + x + 4**

Divide by coefficient of x^2, *we get*,

$= x^2 + \frac{x}{2} + 2 = 0$

Adding and subtracting the square of $\left(\frac{b}{2}\right)$, *we get*,

$= x^2 + 2x \times \frac{1}{4} + \left(\frac{1}{4}\right)^2 - \left(\frac{1}{4}\right)^2 + 2 = 0$

$= \left(x + \frac{1}{4}\right)^2 + \frac{-1+16}{16} = 0$

$= \left(x + \frac{1}{4}\right)^2 + \frac{15}{16} = 0$

$= \left(x + \frac{1}{4}\right)^2 = -\frac{15}{16}$

Square of a number cannot be negative, Hence roots of following equation do not exist.

2. Find the roots of the quadratic equations given in Q.1 above by applying the quadratic formula.

i) $2x^2 - 7x + 3 = 0$

According to the quadratic formula; $ax^2 + bx + c$

$= a = 2 \ , b = -7 \ , c = 3$

∴ The roots are = $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \frac{[-(-7)\pm\sqrt{49-4\times2\times3}]}{2\times2} = \frac{[7\pm\sqrt{49-24}]}{4} = \frac{(7+\sqrt{25})}{4} = \frac{(7\pm5)}{4}$

$= \frac{7+5}{4} = \frac{12}{4} = 3 \quad and \quad \frac{7-5}{4} = \frac{2}{4} = \frac{1}{2}$

∴ The roots are 3 and $\frac{1}{2}$

ii) **$2x^2 + x - 4 = 0$**

According to the quadratic formula; $ax^2 + bx + c$

= $a = 2\ , b = 1\ , c = -4$

∴ The roots are = $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

= $\frac{\left[-1 \pm \sqrt{1-4(2)(-4)}\right]}{2 \times 2}$ = $\frac{-1 \pm \sqrt{33}}{4}$

∴ The roots are = $\frac{-1+\sqrt{33}}{4}$ and $\frac{-1-\sqrt{33}}{4}$

iii) **$4x^2 + 4\sqrt{3}x + 3 = 0$**

According to the quadratic formula; $ax^2 + bx + c$

= $a = 4\ , b = 4\sqrt{3}\ , c = 3$

∴ The roots are = $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

= $\left(-4\sqrt{3} \pm \frac{\sqrt{16 \times 3 - (4 \times 4 \times 3)}}{2 \times 4}\right) = \frac{(-4\sqrt{3} \pm \sqrt{48-48})}{8} = \frac{-4\sqrt{3} \pm 0}{8} = \left(-\frac{\sqrt{3}}{2}\right), \left(-\frac{\sqrt{3}}{2}\right)$

∴ The roots are $\left(-\frac{\sqrt{3}}{2}\right)$ and $\left(-\frac{\sqrt{3}}{2}\right)$

iv) **$2x^2 + x + 4 = 0$**

According to the quadratic formula; $ax^2 + bx + c$

= $a = 2\ , b = 1\ , c = 4$

∴ The roots are = $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

= $\frac{\left[-1 \pm \sqrt{1-4(2)(4)}\right]}{2 \times 2} = \frac{-1 \pm \sqrt{1-32}}{4} = \frac{-1 \pm \sqrt{-31}}{4}$

This is not possible , Hence the roots do not exists.

3. Find the roots of the following equations:

(i) $x - \frac{1}{x} = 3, x \neq 0$

Solution:

= $x^2 - 1 = 3x$

= $x^2 - 3x - 1 = 0$

According to the quadratic formula; $ax^2 + bx + c$

= $a = 1\ , b = -3\ , c = -1$

So, the roots are,

= $\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{\left[-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}\right]}{2} = \frac{3 \pm \sqrt{13}}{2}$

∴ The roots are = $\frac{3+\sqrt{13}}{2}$ and $\frac{3-\sqrt{13}}{2}$

(ii) $\frac{1}{x+4} - \frac{1}{x-7} = \frac{11}{30}, x \neq -4, 7$

Solution:

$= \frac{[(x-7)-(x+4)]}{[(x+4)(x-7)]} = \frac{11}{30}$

$= -\frac{11}{[(x+4)(x-7)]} = \frac{11}{30}$

$= (x+4)(x-7) = -30$

$= x^2 - 3x - 28 = -30$

$= x^2 - 3x + 2 = 0$

According quadratic equation $ax^2 + bx + c$.

$= x(x-2) - 1(x-2) = 0$

$=(x-2)(x-1) = 0$

$= x = 1 \ \ and \ x = 2 \ are \ the \ roots \ of \ equation$

4. The sum of the reciprocals of Rehman's ages, (in years) 3 years ago and 5 years from now is $\frac{1}{3}$. Find his present age.

Solution:

Let the present age of Rehman be *x* years.

Three years ago, his age was (*x* – 3) years.

Five years hence, his age will be (*x* + 5) years.

It is given that the sum of the reciprocals of Rehman's ages 3 years

ago and 5 years from now is $\frac{1}{3}$.

$\therefore \frac{1}{x-3} + \frac{1}{x+5} = \frac{1}{3}$

$= \frac{x+5+x-3}{(x-3)(x+5)} = \frac{1}{3}$

$= \frac{2x+2}{(x-3)(x+5)} = \frac{1}{3}$

= 3(2x+2) = (x-3)(x+5)

= 6x+6 = x^2 +2x – 15

= x^2 – 4x -21 = 0

= x^2 – 7x +3x – 21 = 0

= x(x-7) +3(x-7) = 0

= x = 7, -3

However, age cannot be negative.

Therefore, Rehman's present age is 7 years.

5. **In a class test, the sum of Shefali's marks in Mathematics and English is 30. Had she got2 marks more in Mathematics and 3 marks less in English, the product of their marks would have been 210. Find her marks in the two subjects.**

Solution:

Let the marks in Maths be *x*.

Then, the marks in English will be 30 – *x*.

According to the given question,

(x+2)(30-x-3) = 210

(x+2)(27-x) = 210

= - x^2 +25x + 54 = 210

= x^2 – 25x +156 = 0

= x^2 – 12x – 13x +156 = 0

$= x(x-12) – 13 (x-12) = 0$

$=(x-12)(x-13) = 0$

$= x = 12, 13$

If the marks in Maths are 12, then marks in English will be 30 − 12 = 18

If the marks in Maths are 13, then marks in English will be 30 − 13 = 17

6. **The diagonal of a rectangular field is 60 metres more than the shorter side. If the longer side is 30 metres more than the shorter side, find the sides of the field.**

Solution:

Let the shorter side of the rectangle be x m.

Then, larger side of the rectangle = $(x + 30)$ m

Diagonal of the rectangle = $\sqrt{x^2 + (x + 30)^2}$

It is given that the diagonal of the rectangle is 60 m more than the shorter side.

$\therefore \sqrt{x^2 + (x + 30)^2} = x+60$

$= x^2 + (x+30)^2 = (x+60)^2$

$= x^2 + x^2 + 900+60x = x^2+3600+120x$

$= x^2 – 60x – 2700 = 0$

$= x(x – 90)+30(x – 90)$

$=(x - 90)(x+30) = 0$

$= x = 90, -30$

However, side cannot be negative. Therefore, the length of the shorter side will be

90 m.

Hence, length of the larger side will be (90 + 30) m = 120 m

7. **The difference of squares of two numbers is 180. The square of the smaller number is 8 times the larger number. Find the two numbers.**

Solution:

Let the larger and smaller number be *x* and *y* respectively.

According to the given question,

$= x^2 – y^2 = 180$ and $y^2 = 8x$

$= x^2 – 8x = 180$

$= x^2 – 8x – 180 = 0$

$= x^2 – 18x +10x – 180 = 0$

$= x(x – 18) +10(x – 18) = 0$

$= (x – 18)(x+10) = 0$

$= x = 18, -10$

The larger number cannot be negative as it makes square of smaller
number negative , which is not possible.

Therefore, the larger number will be 18 only.

X = 18

$\therefore y^2 = 8x = 8×18 = 144$

$= y = \pm\sqrt{144} = \pm 12$

∴ smaller number = ± 12

Therefore, the numbers are 18 and 12 or 18 and −12.

8. **A train travels 360 km at a uniform speed. If the speed had been 5 km/h more, it would have taken 1 hour less for the same journey. Find the speed of the train.**

Solution:

Let the speed of the train be *x* km/hr.

Time taken to cover 360 km = $\frac{360}{x}$ hr

According to the given question,

$(x+5)\left(\frac{360}{x} - 1\right) = 360$

= $(x+5)\left(\frac{360}{x} - 1\right) = 360$

= $360 - x + \frac{1800}{x} - 5 = 360$

= $x^2 + 5x - 1800 = 0$

= $x^2 + 45x - 40x - 1800 = 0$

= $x(x + 45) - 40(x+45) = 0$

= $(x+45)(x - 40) = 0$

= $x = -45, 40$

Hence, speed of train can't be negative ,

so, speed will be 40 km/hour.

9. Two water taps together can fill a tank in $9\frac{3}{8}$ hours. The tap of larger diameter takes 10hours less than the smaller one to fill the tank separately. Find the time in which each tap can separately fill the tank.

Solution:

Let the time taken by the smaller pipe to fill the tank be x hr.

Time taken by the larger pipe = $(x - 10)$ hr

Part of tank filled by smaller pipe in 1 hour = $\frac{1}{x}$

Part of tank filled by larger pipe in 1 hour = $\frac{1}{x-10}$

It is given that the tank can be filled in $9\frac{3}{8} = \frac{75}{8}$ hours by both the pipes together.

Therefore,

$$\frac{1}{x} + \frac{1}{x-10} = \frac{8}{75}$$

$$\frac{x - 10 + x}{x(x-10)} = \frac{8}{75}$$

$= \frac{2x-10}{x(x-10)} = \frac{8}{75}$

$= 75(2x-10) = 8x^2 - 80x$

$= 150x - 750 = 8x^2 - 80x$

$= 8x^2 - 230x + 750 = 0$

$= 8x^2 - 200x - 30x + 750 = 0$

$= 8x(x - 25) - 30(x - 25) = 0$

$=(x - 25)(8x - 30) = 0$

$= x = 25, \frac{30}{8}$

Time taken by the smaller pipe cannot be $\frac{30}{8} = 3.75$ hours.

As in this case, the time taken by the larger pipe will be negative, which is logically not possible.

Therefore, time taken individually by the smaller pipe and the larger pipe will be 25 and 25 − 10 =15 hours respectively.

10. An express train takes 1 hour less than a passenger train to travel 132 km between Mysore and Bangalore (without taking into consideration the time they stop at intermediate stations). If the average speed of the express train is 11km/h more than that of the passenger train, find the average speed of the two trains.

Solution:

Let the average speed of passenger train be *x* km/h.

Average speed of express train = (*x* + 11) km/h

It is given that the time taken by the express train to cover 132 km is 1 hour less than the passenger train to cover the same distance.

$$\therefore \frac{132}{x} - \frac{132}{x+11} = 1$$

$= 132\left[\frac{x+11-x}{x(x+11)}\right] = 1$

$= \frac{132\times 11}{x(x+11)} = 1$

= 132×11 = x(x+11)

= x^2 +11x – 1452 = 0

= x^2 + 44x – 33x – 1452 = 0

= x(x+44) – 33(x+44) = 0

= (x+44)(x – 33) = 0

= x = - 44 , 33

Speed cannot be negative.

Therefore, the speed of the passenger train will be 33 km/h and thus, the speed of the express train will be 33 + 11 = 44 km/h.

11. Sum of the areas of two squares is 468 m². If the difference of their perimeters is 24 m, find the sides of the two squares.

Solution:

Let the sides of the two squares be x m and y m.

Therefore, their perimeter will be 4x and 4y respectively and their areas will be x^2 and y^2 respectively.

It is given that $4x - 4y = 24$ or $x - y = 6$

$x = y + 6$

Also, $x^2 + y^2 = 468$

$= (6+y)^2 + y^2 = 468$

$= 36 + y^2 + 12y + y^2 = 468$

$= 2y^2 + 12y – 432 = 0$

$= y^2 + 6y – 216 = 0$

$= y^2 + 18y – 12y – 216 = 0$

$= y(y+18)(y – 12) = 0$

$= y = - 18$ or 12.

However, side of a square cannot be negative.

Hence, the sides of the squares are 12 m and (12 + 6) m = 18 m