

EXERCISE 4.3

1. Find the principal value of each of the following:

(i) $\tan^{-1} (1/\sqrt{3})$

(ii) $\tan^{-1} (-1/\sqrt{3})$

(iii) $\tan^{-1} (\cos (\pi/2))$

(iv) $\tan^{-1} (2 \cos (2\pi/3))$

Solution:

(i) Given $\tan^{-1} (1/\sqrt{3})$

We know that for any $x \in \mathbb{R}$, \tan^{-1} represents an angle in $(-\pi/2, \pi/2)$ whose tangent is x .

So, $\tan^{-1} (1/\sqrt{3}) =$ an angle in $(-\pi/2, \pi/2)$ whose tangent is $(1/\sqrt{3})$

But we know that the value is equal to $\pi/6$

Therefore $\tan^{-1} (1/\sqrt{3}) = \pi/6$

Hence the principal value of $\tan^{-1} (1/\sqrt{3}) = \pi/6$

(ii) Given $\tan^{-1} (-1/\sqrt{3})$

We know that for any $x \in \mathbb{R}$, \tan^{-1} represents an angle in $(-\pi/2, \pi/2)$ whose tangent is x .

So, $\tan^{-1} (-1/\sqrt{3}) =$ an angle in $(-\pi/2, \pi/2)$ whose tangent is $(1/\sqrt{3})$

But we know that the value is equal to $-\pi/6$

Therefore $\tan^{-1} (-1/\sqrt{3}) = -\pi/6$

Hence the principal value of $\tan^{-1} (-1/\sqrt{3}) = -\pi/6$

(iii) Given that $\tan^{-1} (\cos (\pi/2))$

But we know that $\cos (\pi/2) = 0$

We know that for any $x \in \mathbb{R}$, \tan^{-1} represents an angle in $(-\pi/2, \pi/2)$ whose tangent is x .

Therefore $\tan^{-1} (0) = 0$

Hence the principal value of $\tan^{-1} (\cos (\pi/2))$ is 0.

(iv) Given that $\tan^{-1} (2 \cos (2\pi/3))$

But we know that $\cos \pi/3 = 1/2$

So, $\cos (2\pi/3) = -1/2$

Therefore $\tan^{-1} (2 \cos (2\pi/3)) = \tan^{-1} (2 \times -1/2)$

$= \tan^{-1}(-1)$

$= -\pi/4$

Hence, the principal value of $\tan^{-1} (2 \cos (2\pi/3))$ is $-\pi/4$