EXERCISE 9.1

1. Convert the following to logarithmic form:
(i) $5^2 = 25$
(ii) $a^5 = 64$
(iii) $7^x = 100$
(iv) $9^o = 1$
(v) $6^1 = 6$
(vi) $3^{-2} = 1/9$
(vii) $10^{-2} = 0.01$
(viii) $(81)^{3/4} = 27$
Solution:
(i) $5^2 = 25$
Let us apply log, we get
$\text{Log}_5 25 = 2$

(ii) $a^5 = 64$
Let us apply log, we get
$\text{Log}_a 64 = 5$

(iii) $7^x = 100$
Let us apply log, we get
$\text{Log}_7 100 = x$

(iv) $9^o = 1$
Let us apply log, we get
$\text{Log}_9 1 = 0$

(v) $6^1 = 6$
Let us apply log, we get
$\text{Log}_6 6 = 1$

(vi) $3^{-2} = 1/9$
Let us apply log, we get
$\text{Log}_3 1/9 = -2$

(vii) $10^{-2} = 0.01$
Let us apply log, we get
$\text{Log}_{10} 0.01 = -2$

(viii) $(81)^{3/4} = 27$
Let us apply log, we get
$\text{Log}_{81} 27 = ¾$

2. Convert the following into exponential form:
(i) $\log_2 32 = 5$
(ii) $\log_3 81=4$
(iii) $\log_3 1/3 = -1$
(iv) $\log_3 4 = 2/3$
(v) $\log_8 32 = 5/3$
(vi) $\log_{10} (0.001) = -3$
(vii) $\log_2 0.25 = -2$
(viii) $\log_a (1/a) = -1$
Solution:
(i) $\log_2 32 = 5$
The exponential form of the expression is
$2^5 = 32$

(ii) $\log_3 81=4$
The exponential form of the expression is
$3^4 = 81$

(iii) $\log_3 1/3 = -1$
The exponential form of the expression is
$3^{-1} = 1/3$

(iv) $\log_3 4 = 2/3$
The exponential form of the expression is
$(8)^{2/3} = 4$

(v) $\log_8 32 = 5/3$
The exponential form of the expression is
$(8)^{5/3} = 32$

(vi) $\log_{10} (0.001) = -3$
The exponential form of the expression is
$10^{-3} = 0.001$

(vii) $\log_2 0.25 = -2$

The exponential form of the expression is
$2^{-2} = 0.25$

(viii) $\log_a (1/a) = -1$
The exponential form of the expression is
$a^{-1} = 1/a$

3. By converting to exponential form, find the values of:
(i) $\log_2 16$
(ii) $\log_5 125$
(iii) $\log_4 8$
(iv) $\log_9 27$
(v) $\log_{10} (.01)$
(vi) $\log_7 1/7$
(vii) $\log_{.5} 256$
(viii) $\log_2 0.25$
Solution:
(i) $\log_2 16$
Let us consider $\log_2 16 = x$
So,
$(2)^x = 16$
$= 2 \times 2 \times 2 \times 2$
$2^x = 2^4$
By comparing the powers,
$x = 4$

(ii) $\log_5 125$
Let us consider $\log_5 125 = x$
So,
$(5)^x = 125$
$= 5 \times 5 \times 5$
$5^x = 5^3$
By comparing the powers,
$x = 3$

(iii) $\log_4 8$
Let us consider $\log_4 8 = x$
So,
$(4)^x = 8$

$(2 \times 2)^x = 2 \times 2 \times 2$
$2^{2x} = 2^3$
By comparing the powers,
$2x = 3$
$x = 3/2$

(iv) $\log_9 27$
Let us consider $\log_9 27 = x$
So,
$(9)^x = 27$
$(3 \times 3)^x = 3 \times 3 \times 3$
$3^{2x} = 3^3$
By comparing the powers,
$2x = 3$
$x = 3/2$

(v) $\log_{10} (.01)$
Let us consider $\log_{10} (.01) = x$
So,
$(10)^x = 1/100$
$= 1/10 \times 1/10$
$10^x = 1/(10)^2$
$10^x = 10^{-2}$
By comparing the powers,
$x = -2$

(vi) $\log_7 1/7$
Let us consider $\log_7 1/7 = x$
So,
$(7)^x = 1/7$
$7^x = 7^{-1}$
By comparing the powers,
$x = -1$

(vii) $\log_{.5} 256$
Let us consider $\log_{.5} 256 = x$
So,
$(.5)^x = 256$
$(5/10)^x = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

$(1/2)^x = 2^8$
$(2)^{-x} = 2^8$
By comparing the powers,
-x = 8
x = -8

(viii) $\log_2 0.25$
Let us consider $\log_2 0.25 = x$
So,
$(2)^x = 0.25$
= 25/100
$2^x = 1/4$
$2^x = (2)^{-2}$
By comparing the powers,
x = -2

4. Solve the following equations for x:
(i) $\log_3 x = 2$
(ii) $\log_x 25 = 2$
(iii) $\log_{10} x = -2$
(iv) $\log_4 x = ½$
(v) $\log_x 11 = 2.5$
(vi) $\log_x ¼ = -1$
(vii) $\log_{81} x = 3/2$
(viii) $\log_9 x = 2.5$
(ix) $\log_4 x = -1.5$
Solution:
(i) $\log_3 x = 2$
Let us simplify the expression,
$(3)^2 = x$
x = 9

(ii) $\log_x 25 = 2$
Let us simplify the expression,
$(x)^2 = 25$
= 5 × 5
$x^2 = 5^2$
Since the powers are same,
So,

$x = 5$

(iii) $\log_{10} x = -2$
Let us simplify the expression,
$(10)^{-2} = x$
$x = 1/(10)^2$
$= 1/100$
$x = 0.01$

(iv) $\log_4 x = ½$
Let us simplify the expression,
$(4)^{1/2} = x$
$x = (2 \times 2)^{1/2}$
$= (2)^{2 \times 1/2}$
$x = 2$

(v) $\log_x 11 = 2.5$
Let us simplify the expression,
$(x)^1 = 11$
$x = 11$

(vi) $\log_x ¼ = -1$
Let us simplify the expression,
$(x)^{-1} = ¼$
$x^{-1} = 4^{-1}$
Since the powers are same,
So,
$x = 4$

(vii) $\log_{81} x = 3/2$
Let us simplify the expression,
$(81)^{3/2} = x$
$x = 81^{3/2}$
$= (3^4)^{3/2}$
$= 3^{4 \times 3/2}$
$= 3^6$
$= 3 \times 3 \times 3 \times 3 \times 3 \times 3$
$= 729$
$x = 729$

(viii) $\log_9 x = 2.5$
$\log_9 x = 5/2$
Let us simplify the expression,
$(9)^{5/2} = x$
$x = (3^2)^{5/2}$
$= 3^{2 \times 5/2}$
$= 3^5$
$= 3 \times 3 \times 3 \times 3 \times 3$
$= 234$
$x = 234$

(ix) $\log_4 x = -1.5$
$\log_4 x = -3/2$
Let us simplify the expression,
$(4)^{-3/2} = x$
$x = (2^2)^{-3/2}$
$= 2^{2 \times -3/2}$
$= 2^{-3}$
$= 1/2^3$
$= 1/(2 \times 2 \times 2)$
$= 1/8$
$x = 1/8$

5. Given $\log_{10} a = b$, express 10^{2b-3} in terms of a.
Solution:
Given:
$\log_{10} a = b$
$(10)^b = a$
Now,
$10^{2b-3} = (10)^{2b} / (10)^3$
$= (10^b)^2 / (10 \times 10 \times 10)$
Substitute the value of $(10)^b = a$, we get
$= a^2/1000$

6. Given $\log_{10} x = a$, $\log_{10} y = b$ and $\log_{10} z = c$,
(i) write down 10^{2a-3} in terms of x.
(ii) write down 10^{3b-1} in terms of y.
(iii) if $\log_{10} P = 2a + b/2 - 3c$, express P in terms of x, y and z.
Solution:

Given:
$\log_{10} x = a$
⇨ $(10)^a = x$
$\log_{10} y = b$
⇨ $(10)^b = y$
$\log_{10} z = c$
⇨ $(10)^c = z$

(i) Write down 10^{2a-3} in terms of x.
$10^{2a-3} = (10)^{2a} / (10)^3$
$= (10^a)^2 / (10 \times 10 \times 10)$
Substitute the value of $(10)^a = x$, we get
$= x^2/1000$

(ii) Write down 10^{3b-1} in terms of y.
$10^{3b-1} = (10)^{3b} / (10)^1$
$= (10^b)^3 / (10)$
Substitute the value of $(10)^b = y$, we get
$= y^3/10$

(iii) If $\log_{10} P = 2a + b/2 – 3c$, express P in terms of x, y and z.
we know that,
$(10)^a = x$
$(10)^b = y$
$(10)^c = z$
By substituting the values
$\log_{10} P = 2a + b/2 – 3c$
$= 2 \log_{10} x + ½ \log_{10} y – 3 \log_{10} z$
$= \log_{10} x^2 + \log_{10} y^{1/2} - \log_{10} z^3$
$= \log_{10} (x^2 + y^{1/2}) - \log_{10} z^3$
$= \log_{10} [(x^2\sqrt{y})/z^3]$
$P = (x^2\sqrt{y})/z^3$

7. If $\log_{10}x = a$ and $\log_{10}y = b$, find the value of xy.
Solution:
Given:
$\log_{10}x = a$
$(10)^a = x$

$\log_{10} y = b$
$(10)^b = y$
Then,
$xy = (10)^a \times (10)^b$
$= (10)^{a+b}$

8. Given $\log_{10} a = m$ and $\log_{10} b = n$, express a^3/b^2 in terms of m and n.
Solution:
Given:
$\log_{10} a = m$
$(10)^m = a$

$\log_{10} b = n$
$(10)^n = b$
So,
$a^3/b^2 = (10^m)^3 / (10^n)^2$
$= 10^{3m} / 10^{2n}$
$= 10^{3m - 2n}$

9. Given $\log_{10} a = 2a$ and $\log_{10} y = –b/2$
(i) write 10^a in terms of x.
(ii) write 10^{2b+1} in terms of y.
(iii) if $\log_{10} P = 3a - 2b$, express P in terms of x and y.
Solution:
Given:
$\log_{10} a = 2a$
$(10)^{2a} = a$

$\log_{10} y = –b/2$
$(10)^{-b/2} = y$
(i) Write 10^a in terms of x.
$10^a = (10^{2a})^{1/2}$
$= (x)^{1/2}$
$= \sqrt{x}$

(ii) Write 10^{2b+1} in terms of y.
$10^{2b+1} = 10^{2b} \times 10^1$
$= 10^{4(b/2)} \times 10^1$
$= (10^{b/2})^4 \times 10^1$

$= y^4 \times 10^1$
$= 10y^4$

(iii) If $\log_{10}P = 3a - 2b$, express P in terms of x and y.
$\log_{10}P = 3a - 2b$
Substitute the values,
$\log_{10}P = 3/2\ (2a) – 4(b/2)$
$= 3/2\ (\log_{10} x) – 4\ (\log_{10} y)$
$= (\log_{10} x)^{3/2} - (\log_{10} y)^4$
$= \log_{10} [(x^{3/2}) / y^4]$
$P = (x^{3/2}) / y^4$

10. If $\log_2 y = x$ and $\log_3 z = x$, find 72^x in terms of y and z.
Solution:
Given:
$\log_2 y = x$
$2^x = y$

$\log_3 z = x$
$3^x = z$
So,
$72^x = (2 \times 2 \times 2 \times 3 \times 3)^x$
$= (2^3 \times 3^2)^x$
$= 2^{3x} \times 3^{2x}$
$= (2^x)^3 \times (3^x)^2$
$= y^3 \times z^2$
$= y^3z^2$

11. If $\log_2 x = a$ and $\log_5 y = a$, write 100^{2a-1} in terms of x and y.
Solution:
Given:
$\log_2 x = a$
$2^a = x$

$\log_5 y = a$
$5^a = y$
So,
$100^{2a-1} = (2 \times 2 \times 5 \times 5)^{2a-1}$
$= (2^2 \times 5^2)^{2a-1}$

$= 2^{4a-2} \times 5^{a-2}$
$= (2^{4a})/2^2 \times (5^{4a})/5^2$
$= [(2^a)^4 \times (5^a)^4] / (4\times25)$
$= (x^4y^4)/100$

EXERCISE 9.2

1. Simplify the following:
(i) log a^3 – log a^2
(ii) log a^3 ÷ log a^2
(iii) log 4/log 2
(iv) (log 8 log 9)/log 27
(v) log 27/log √3
(vi) (log 9 – log 3)/log 27
Solution:
(i) log a^3 – log a^2
By using Quotient law,
log a^3 – log a^2 = log (a^3/a^2)
= log a

(ii) log a^3 ÷ log a^2
By using power law,
log a^3 ÷ log a^2 = 3log a ÷ 2 log a
= 3log a / 2log a
= 3/2

(iii) log 4/log 2
Let us simplify the expression,
log 4/log 2 = log(2×2)/log 2
By using power law,
= 2 log 2/log 2
= 2

(iv) (log 8 log 9)/log 27
Let us simplify the expression,
(log 8 log 9)/log 27 = (log 2^3. log 3^2)/log 3^3
By using power law,
= [(3 log 2).(2 log 3)]/(3 log 3)
= [(log 2).2] / 1
= 2 log 2
= log 2^2
= log 4

(v) log 27/log √3

Let us simplify the expression,

log 27/log √3 = log(3×3×3)/log$(3)^{1/2}$

= log 3^3/log $3^{1/2}$

By using power law

= 3log 3/((1/2)log 3)

= (3×2)/1 (log 3/log 3)

= (6) (1)

= 6

(vi) (log 9 – log 3)/log 27

Let us simplify the expression,

(log 9 – log 3)/log 27 = [log(3×3) – log 3] / log(3×3×3)

= [log 3^2 – log 3] / log 3^3

By using power law

= [2 log 3 – log 3]/3log 3

= log 3/3log 3

= 1/3

2. Evaluate the following:

(i) log (10 ÷ ∛10)

(ii) 2 + ½ log(10^{-3})

(iii) 2 log 5 + log 8 – ½ log 4

(iv) 2 log 10^3 + 3 log 10^{-2} - 1/3 log 5^{-3} + ½ log 4

(v) 2 log 2 + log 5 – ½ log 36 – log 1/30

(vi) 2 log 5 + log 3 + 3 log 2 – ½ log 36 – 2 log 10

(vii) log 2 + 16 log 16/15 + 12 log 25/24 + 7 log 81/80

(viii) 2 $\log_{10}$ 5 + $\log_{10}$ 8 – ½ $\log_{10}$ 4

Solution:

(i) log (10 ÷ ∛10)

Let us simplify the expression,

log (10 ÷ ∛10) = log (10 ÷ $10^{1/3}$)

= log ($10^{1 – 1/3}$)

= log ($10^{2/3}$)

= 2/3 log 10

= 2/3 (1)

= 2/3

(ii) 2 + ½ log(10^{-3})

Let us simplify the expression,

2 + ½ $\log(10^{-3})$ = 2 + ½ × (-3) log 10
= 2 – 3/2 log 10
= 2 – 3/2 (1)
= 2 – 3/2
= (4-3)/2
= ½

(iii) 2 log 5 + log 8 – ½ log 4
Let us simplify the expression,
2 log 5 + log 8 – ½ log 4 = $\log 5^2$ + log 8 – ½ $\log 2^2$
= log 25 + log 8 – ½ 2log 2
= log 25 + log 8 – log 2
= log (25×8)/2
= log (25×4)
= log 100
= $\log 10^2$
= 2 log 10
= 2 (1)
= 2

(iv) 2 $\log 10^3$ + 3 $\log 10^{-2}$ - 1/3 $\log 5^{-3}$ + ½ log 4
Let us simplify the expression,
2 $\log 10^3$ + 3 $\log 10^{-2}$ - 1/3 $\log 5^{-3}$ + ½ log 4 = 2×3 log 10 + 3(-2)log 10 – 1/3 (-3) log 5 + ½ $\log 2^2$
= 6 log 10 – 6 log 10 + log 5 + ½ 2 log 2
= 6 log 10 – 6 log 10 + log 5 + log 2
= 0 + log 5 + log 2
= log (5×2)
= log 10
= 1

(v) 2 log 2 + log 5 – ½ log 36 – log 1/30
Let us simplify the expression,
2 log 2 + log 5 – ½ log 36 – log 1/30 = $\log 2^2$ + log 5 – ½ $\log 6^2$ – log (1/30)
= log 4 + log 5 – log 6 – log 1/30
= log 4 + log 5 – log 6 – (log 1 – log 30)
= log 4 + log 5 – log 6 – log 1 + log 30
= log 4 + log 5 + log 30 – (log 6 + log 1)
= log (4×5×30) – log (6×1)

= log (4×5×30)/(6×1)
= log (4×5×5)
= log 100
= log 10^2
= 2 log 10
= 2 (1)
= 2

(vi) 2 log 5 + log 3 + 3 log 2 – ½ log 36 – 2 log 10
Let us simplify the expression,
2log 5 + log 3 + 3log 2 – ½ log 36 – 2log 10 **=** log 5^2 + log 3 + log 2^3 – ½ log 6^2 – log 10^2
= log 25 + log 3 + log 8 – log 6 – log 100
= log (25×3×8) – log(6×100)
= log (25×3×8)/ (6×100)
= log (1×3×8) / (6×4)
= log 24/24
= log 1
= 0

(vii) log 2 + 16 log 16/15 + 12 log 25/24 + 7 log 81/80
Let us simplify the expression,
log 2 + 16 log 16/15 + 12 log 25/24 + 7 log 81/80 = log 2 + 16(log 16 – log 15) + 12(log 25 – log 24) + 7(log 81 – log 80)
= log 2 + 16 (log 2^4 – log (3×5)) + 12 (log 5^2 – log (3×2×2×2)) + 7 (log (3×3×3×3) – log (2^4 × 5))
= log 2 + 16(4log 2 – (log 3 + log 5)) + 12 (2log 5 – log (3×2^3)) + 7 (log 3^4 – (log 2^4 + log 5))
= log 2 + 16 (4log 2 – log 3 – log 5) + 12 (2log 5 – (log 3 + 3log 2)) + 7 (4log 3 – 4log 2 – log 5)
= log 2 + 64log 2 – 16log 3 – 16log 5 + 24log 5 – 12log 3 – 36log 2 + 28log 3 – 28log 2 – 7log 5
= (log 2 + 64log 2 – 36log 2 – 28log 2) + (-16log 3 – 12log 3 + 28log 3) + (-16log 5 + 24log 5 – 7log 5)
= (65log 2 – 64log 2) + (-28log 3 + 28log 3) + (-23log 5 + 24log 5)
= log 2 + 0 + log 5
= log (2×5)
= log 10
= 1

(viii) $2 \log_{10} 5 + \log_{10} 8 – ½ \log_{10} 4$
Let us simplify the expression,
$2 \log_{10} 5 + \log_{10} 8 – ½ \log_{10} 4 = \log_{10} 5^2 + \log_{10} 8 – \log_{10} 4^{1/2}$
$= \log_{10} 25 + \log_{10} 8 – \log_{10} (2)^{2\times 1/2}$
$= \log_{10} 25 + \log_{10} 8 – \log_{10} 2$
$= \log_{10} [(25\times8)/2]$
$= \log_{10} (25\times4)$
$= \log_{10} 100$
$= \log_{10} 10^2$
$= 2 \log_{10} 10$
$= 2 (1)$
$= 2$

3. Express each of the following as a single logarithm:
(i) 2 log 3 – ½ log 16 + log 12
(ii) $2 \log_{10} 5 – \log_{10} 2 + 3 \log_{10} 4 + 1$
(iii) ½ log 36 + 2 log 8 – log 1.5
(iv) ½ log 25 – 2 log 3 + 1
(v) ½ log 9 + 2 log 3 – log 6 + log 2 - 2
Solution:
(i) 2 log 3 – ½ log 16 + log 12
Let us simplify the expression into single logarithm,
$2 \log 3 – ½ \log 16 + \log 12 = 2 \log 3 – ½ \log 4^2 + \log 12$
$= 2 \log 3 – \log 4 + \log 12$
$= \log 3^2 – \log 4 + \log 12$
$= \log 9 - \log 4 + \log 12$
$= \log (9\times12)/4$
$= \log (9\times3)$
$= \log 27$

(ii) $2 \log_{10} 5 – \log_{10} 2 + 3 \log_{10} 4 + 1$
Let us simplify the expression into single logarithm,
$2 \log_{10} 5 – \log_{10} 2 + 3 \log_{10} 4 + 1 = \log_{10} 5^2 – \log_{10} 2 + \log_{10} 4^3 + \log_{10} 10$
$= \log_{10} 25 – \log_{10} 2 + \log_{10} 64 + \log_{10} 10$
$= \log_{10} (25\times64\times10) – \log_{10} 2$
$= \log_{10} (16000) – \log_{10} 2$
$= \log_{10} (16000/2)$
$= \log_{10} 8000$

(iii) ½ log 36 + 2 log 8 – log 1.5

Let us simplify the expression into single logarithm,

½ log 36 + 2 log 8 – log 1.5 = log $36^{1/2}$ + log 8^2 – log 1.5
= log $(6)^{2\times1/2}$ + log 64 – log 1.5
= log 6 + log 64 – log (15/10)
= log 6 + log 64 – (log 15 – log 10)
= log (6×64) – log 15 + log 10
= log (6×64×10) – log 15
= log [(6×64×10)/15]
= log (4×64)
= log 256

(iv) ½ log 25 – 2 log 3 + 1

Let us simplify the expression into single logarithm,

½ log 25 – 2 log 3 + 1 = log $25^{1/2}$ – log 3^2 + log 10
= log $(5)^{2\times1/2}$ – log 9 + log 10
= log 5 – log 9 + log 10
= log (5×10) – log 9
= log ((5×10)/9)
= log 50/9

(v) ½ log 9 + 2 log 3 – log 6 + log 2 - 2

Let us simplify the expression into single logarithm,

½ log 9 + 2 log 3 – log 6 + log 2 – 2 = log $9^{1/2}$ + log 3^2 – log 6 + log 2 – log 100
= log $3^{2\times1/2}$ + log 9 – log 6 + log 2 – log 100
= log 3 + log 9 – log 6 + log 2 – log 100
= log [(3×9×2)/(6×100)]
= log 9/100

4. Prove the following:

(i) $\log_{10} 4 \div \log_{10} 2 = \log_3 9$

(ii) $\log_{10} 25 + \log_{10} 4 = \log_5 25$

Solution:

(i) $\log_{10} 4 \div \log_{10} 2 = \log_3 9$

Let us consider LHS, $\log_{10} 4 \div \log_{10} 2$

$\log_{10} 4 \div \log_{10} 2 = \log_{10} 2^2 \div \log_{10} 2$
$= 2 \log_{10} 2 \div \log_{10} 2$
$= 2 \log_{10} 2 / \log_{10} 2$
= 2 (1)

$= 2$

Now let us consider RHS,

$\log_3 9 = \log_3 3^2$

$= 2 \log_3 3$

$= 2(1)$

$= 2$

∴ LHS = RHS

Hence proved.

(ii) $\log_{10} 25 + \log_{10} 4 = \log_5 25$

Let us consider LHS, $\log_{10} 25 + \log_{10} 4$

$\log_{10} 25 + \log_{10} 4 = \log_{10} (25\times4)$

$= \log_{10} 100$

$= \log_{10} 10^2$

$= 2 \log_{10} 10$

$= 2(1)$

$= 2$

Now, let us consider RHS,

$\log_5 25 = \log_5 5^2$

$= 2 \log_5 5$

$= 2 (1)$

$= 2$

∴ LHS = RHS

Hence proved.

5. If $x = (100)^a$, $y = (10000)^b$ and $z = (10)^c$, express $\log [(10\sqrt{y})/x^2z^3]$ in terms of a, b, c.

Solution:

Given:

$x = (100)^a = (10^2)^a = 10^{2a}$

$y = (10000)^b = (10^4)^b = 10^{4b}$

$z = (10)^c$

It is given that, $\log [(10\sqrt{y})/x^2z^3]$

$\log [(10\sqrt{y})/x^2z^3] = (\log 10 + \log \sqrt{y}) – (\log x^2 + \log z^3)$

$= (1 + \log(y)^{1/2}) – (\log x^2 + \log z^3)$ [we know that, log 10 = 1]

$= (1 + ½ \log y) – (2 \log x + 3 \log z)$

Now substitute the values of x, y, z, we get

$= (1 + ½ \log 10^{4b}) – (2 \log 10^{2a} + 3 \log 10^c)$

$= (1 + ½\, 4b \log 10) – (2\times2a \log 10 + 3\times c \log 10)$

$= (1 + ½\, 4b) – (2\times2a + 3c)$ [Since, log 10 = 1]

$= (1 + 2b) – (4a + 3c)$
$= 1 + 2b – 4a – 3c$

6. If a = $\log_{10} x$, find the following in terms of a :
(i) x
(ii) $\log_{10} \sqrt[5]{x^2}$
(iii) $\log_{10} 5x$
Solution:
Given:
$a = \log_{10} x$
(i) x
$10^a = x$
$\therefore x = 10^a$

(ii) $\log_{10} \sqrt[5]{x^2}$
$\log_{10} \sqrt[5]{x^2} = \log_{10} (x^2)^{1/5}$
$= \log_{10} (x)^{2/5}$
$= 2/5 \log_{10} x$
$= 2/5 (a)$
$= 2a/5$

(iii) $\log_{10} 5x$
$x = (10)^a$
$= \log_{10} 5x$
$= \log_{10} 5(10)^a$
$= \log_{10} 5 + \log_{10} 10$
$= \log_{10} 5 + a(1)$
$= a + \log_{10} 5$

7. If a =log 2/3, b = log 3/5 and c = 2 log √(5/2). Find the value of
(i) a + b + c
(ii) 5^{a+b+c}
Solution:
Given:
a =log 2/3
b = log 3/5
c = 2 log √(5/2)
(i) a + b + c
Let us substitute the given values, we get

a + b + c = log 2/3 + log 3/5 + 2 log √(5/2)
= (log 2 – log 3) + (log 3 – log 5) + 2 log $(5/2)^{1/2}$
= log 2 – log 3 + log 3 – log 5 + 2 × ½ (log 5 – log 2)
= log 2 – log 3 + log 3 – log 5 + log 5 – log 2
= 0

(ii) 5^{a+b+c}
$5^{a+b+c} = 5^0$
= 1

8. If x = log 3/5, y = log 5/4 and z = 2 log √3/2, find the value of
(i) x + y – z
(ii) 3^{x+y-z}
Solution:
Given:
x = log 3/5 = log 3 – log 5
y = log 5/4 = log 5 – log 4
z = 2 log √3/2 = log $(\sqrt{3}/2)^2$ = log ¾ = log 3 – log 4
(i) x + y – z
Let us substitute the given values, we get
x + y – z = log 3 – log 5 + log 5 – log 4 – (log 3 – log 4)
= log 3 – log 5 + log 5 – log 4 – log 3 + log 4
= 0

(ii) 3^{x+y-z}
$3^{x+y-z} = 3^0$
= 1

9. If x = $\log_{10}$ 12, y = $\log_4$ 2 × $\log_{10}$ 9 and z = $\log_{10}$ 0.4, find the values of
(i) x – y – z
(ii) 7^{x-y-z}
Solution:
Given:
x = $\log_{10}$ 12
y = $\log_4$ 2 × $\log_{10}$ 9
z = $\log_{10}$ 0.4
(i) x – y – z
Let us substitute the given values, we get
x – y – z = $\log_{10}$ 12 - $\log_4$ 2 × $\log_{10}$ 9 - $\log_{10}$ 0.4

$= \log_{10} (3\times4) – \log_4 4^{1/2} \times \log_{10} 3^2 – \log_{10} 4/10$
$= \log_{10} 3 + \log_{10} 4 – ½ \log_4 4 \times 2 \log_{10} 3 – (\log_{10} 4 – \log_{10} 10)$
$= \log_{10} 3 + \log_{10} 4 – ½ \times 1 \times 2 \log_{10} 3 – \log_{10} 4 + 1$
$= \log_{10} 3 + \log_{10} 4 - \log_{10} 3 – \log_{10} 4 + 1$
$= 1$

(ii) 7^{x-y-z}
$7^{x-y-z} = 7^1$
$= 7$

10. If log V + log 3 = log π + log 4 + 3 log r, find V in terms of other quantities.
Solution:
Given:
log V + log 3 = log π + log 4 + 3 log r
Let us simplify the given expression to find V,
$\log (V \times 3) = \log (\pi \times 4 \times r^3)$
$\log 3V = \log 4\pi r^3$
$3V = 4\pi r^3$
$V = 4\pi r^3/3$

11. Given 3 (log 5 – log 3) – (log 5 - 2 log 6) = 2 – log n, find n.
Solution:
Given:
3 (log 5 – log 3) – (log 5 - 2 log 6) = 2 – log n
Let us simplify the given expression to find n,
3 log 5 – 3 log 3 – log 5 + 2 log 6 = 2 – log n
2 log 5 – 3 log 3 + 2 log 6 = 2 (1) – log n
$\log 5^2 – \log 3^3 + \log 6^2 = 2 \log 10 – \log n$ [Since, 1 = log 10]
$\log 25 – \log 27 + \log 36 – \log 10^2 = - \log n$
log n = - log 25 + log 27 – log 36 + log 100
= (log 100 + log 27) – (log 25 + log 36)
= log (100×27) – log (25×36)
= log (100×27)/(25×36)
log n = log 3
n = 3

12. Given that $\log_{10} y + 2 \log_{10} x = 2$, express y in terms of x.
Solution:
Given:

$\log_{10} y + 2 \log_{10} x = 2$
Let us simplify the given expression,
$\log_{10} y + \log_{10} x^2 = 2(1)$
$\log_{10} y + \log_{10} x^2 = 2 \log_{10} 10$
$\log_{10} (y \times x^2) = \log_{10} 10^2$
$yx^2 = 100$
$y = 100/x^2$

13. Express $\log_{10} 2 + 1$ in the form $\log_{10} x$.
Solution:
Given:
$\log_{10} 2 + 1$
Let us simplify the given expression,
$\log_{10} 2 + 1 = \log_{10} 2 + \log_{10} 10$ [Since, $1 = \log_{10} 10$]
$= \log_{10} (2 \times 10)$
$= \log_{10} 20$

14. If $a^2 = \log_{10} x$, $b^2 = \log_{10} y$ and $a^2/2 – b^2/3 = \log_{10} z$. Express z in terms of x and y.
Solution:
Given:
$a^2 = \log_{10} x$
$b^2 = \log_{10} y$
$a^2/2 – b^2/3 = \log_{10} z$
Let us substitute the given values in the expression, we get
$\log_{10} x/2 - \log_{10} y/3 = \log_{10} z$
$\log_{10} x^{1/2} – \log_{10} y^{1/3} = \log_{10} z$
$\log_{10} \sqrt{x} – \log_{10} \sqrt[3]{y} = \log_{10} z$
$\log_{10} \sqrt{x}/\sqrt[3]{y} = \log_{10} z$
$\sqrt{x}/\sqrt[3]{y} = z$
$z = \sqrt{x}/\sqrt[3]{y}$

15. Given that log m = x + y and log n = x - y, express the value of log m^2n in terms of x and y.
Solution:
Given:
$\log m = x + y$
$\log n = x – y$
$\log m^2n$
Let us simplify the given expression,

$\log m^2n = \log m^2 + \log n$
$= 2 \log m + \log n$
By substituting the given values, we get
$= 2 (x + y) + (x - y)$
$= 2x + 2y + x – y$
$= 3x + y$

16. Given that log x = m + n and log y = m – n, express the value of log ($10x/y^2$) in terms of m and n.
Solution:
Given:
$\log x = m + n$
$\log y = m – n$
$\log (10x/y^2)$
Let us simplify the given expression,
$\log (10x/y^2) = \log 10x – \log y^2$
$= \log 10 + \log x – 2 \log y$
$= 1 + \log x – 2 \log y$
$= 1 + (m + n) – 2(m - n)$
$= 1 + m + n – 2m + 2n$
$= 1 – m + 3n$

17. If log x/2 = log y/3, find the value of y^4/x^6.
Solution:
Given:
$\log x/2 = \log y/3$
Let us simplify the given expression,
By cross multiplying, we get
$3 \log x = 2 \log y$
$\log x^3 = \log y^2$
so, $x^3 = y^2$
now square on both sides, we get
$(x^3)^2 = (y^2)^2$
$x^6 = y^4$
$y^4/x^6 = 1$

18. Solve for x:
(i) log x + log 5 = 2 log 3
(ii) $\log_3 x – \log_3 2 = 1$

(iii) x = log 125/log 25
(iv) (log 8/log 2) × (log 3/log√3) = 2 log x
Solution:
(i) log x + log 5 = 2 log 3
Let us solve for x,
Log x = 2 log 3 – log 5
= log 3^2 – log 5
= log 9 – log 5
= log (9/5)
∴ x = 9/5

(ii) $\log_3$ x – $\log_3$ 2 = 1
Let us solve for x,
$\log_3$ x = 1 + $\log_3$ 2
= $\log_3$ 3 + $\log_3$ 2 [Since, 1 can be written as $\log_3$ 3 = 1]
= $\log_3$ (3×2)
= $\log_3$ 6
∴ x = 6

(iii) x = log 125/log 25
x = log 5^3/log5^2
= 3 log 5/ 2 log 5
= 3/2 [Since, log 5/log 5 = 1]
∴ x = 3/2

(iv) (log 8/log 2) × (log 3/log√3) = 2 log x
(log 2^3/log 2) × (log 3/log$3^{1/2}$) = 2 log x
(3log 2/log 2) × (log 3/½ log 3) = 2 log x
3 × 1/(½) = 2 log x
3 × 2 = 2 log x
6 = 2 log x
log x = 6/2
log x = 3
x = $(10)^3$
= 1000
∴ x = 1000

19. Given 2 $\log_{10}$ x + 1= $\log_{10}$ 250, find
(i) x

(ii) $\log_{10} 2x$
Solution:
Given:
$2 \log_{10} x + 1 = \log_{10} 250$
(i) let us simplify the above expression,
$\log_{10} x^2 + \log_{10} 10 = \log_{10} 250$ [Since, 1 can be written as $\log_{10} 10$]
$\log_{10} (x^2 \times 10) = \log_{10} 250$
$(x^2 \times 10) = 250$
$x^2 = 250/10$
$x^2 = 25$
$x = \sqrt{25}$
$= 5$
$\therefore x = 5$

(ii) $\log_{10} 2x$
We know that, x = 5
So, $\log_{10} 2x = \log_{10} 2\times5$
$= \log_{10} 10$
$= 1$

20. If log x/log 5 = log y^2/log 2 = log 9/log (1/3), find x and y.
Solution:
Given:
$\log x/\log 5 = \log y^2/\log 2 = \log 9/\log (1/3)$
let us consider,
$\log x/\log 5 = \log 9/\log (1/3)$
$\log x = (\log 9\times\log 5)/\log (1/3)$
$= (\log 3^2 \times \log 5) / (\log 1 – \log 3)$
$= (2 \log 3 \times \log 5) / (-\log 3)$ [$\log 1 = 0$]
$= -2 \times \log 5$
$= \log 5^{-2}$
$x = 5^{-2}$
$= 1/5^2$
$= 1/25$
Now,
$\log y^2/\log 2 = \log 9/\log (1/3)$
$\log y^2 = (\log 9\times\log 2)/\log (1/3)$
$= (\log 3^2 \times \log 2) / (\log 1 – \log 3)$
$= (2 \log 3 \times \log 2) / (-\log 3)$ [$\log 1 = 0$]

$= -2 \times \log 2$

$= \log 2^{-2}$

$y^2 = 2^{-2}$

$= 1/2^2$

$= 1/4$

$= \sqrt{(1/4)}$

$= 1/2$

21. Prove the following:

(i) $3^{\log 4} = 4^{\log 3}$

(ii) $27^{\log 2} = 8^{\log 3}$

Solution:

(i) $3^{\log 4} = 4^{\log 3}$

Let us take log on both sides,

If $\log 3^{\log 4} = \log 4^{\log 3}$

$\log 4 \,.\, \log 3 = \log 3 \,.\, \log 4$

$\log 2^2 \,.\, \log 3 = \log 3 \,.\, \log 2^2$

$2 \log 2 \,.\, \log 3 = \log 3 \,.\, 2 \log 2$

Which is true.

Hence proved.

(ii) $27^{\log 2} = 8^{\log 3}$

Let us take log on both sides,

If $\log 27^{\log 2} = \log 8^{\log 3}$

$\log 2 \,.\, \log 27 = \log 3 \,.\, \log 8$

$\log 2 \,.\, \log 3^3 = \log 3 \,.\, \log 2^3$

$\log 2 \,.\, 3 \log 3 = \log 3 \,.\, 3 \log 2$

$3 \log 2 \,.\, \log 3 = 3 \log 2 \,.\, \log 3$

Which is true.

Hence proved.

22. Solve the following equations:

(i) $\log (2x + 3) = \log 7$

(ii) $\log (x + 1) + \log (x - 1) = \log 24$

(iii) $\log (10x + 5) - \log (x - 4) = 2$

(iv) $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$

(v) $\log (4y - 3) = \log (2y + 1) - \log 3$

(vi) $\log_{10} (x + 2) + \log_{10} (x - 2) = \log_{10} 3 + 3 \log_{10} 4$

(vii) $\log (3x + 2) + \log (3x - 2) = 5 \log 2$

Solution:

(i) log (2x + 3) = log 7
Let us simplify the expression,
2x + 3 = 7
2x = 7 – 3
2x = 4
x = 4/2
= 2

(ii) log (x +1) + log (x – 1) = log 24
Let us simplify the expression,
log [(x +1) (x – 1)] = log 24
log (x^2 - 1) = log 24
(x^2 - 1) = 24
x^2 = 24 + 1
= 25
x = √25
= 5

(iii) log (10x + 5) – log (x – 4) = 2
Let us simplify the expression,
log (10x + 5) / (x – 4) = 2 log 10
log (10x + 5) / (x – 4) = log 10^2
(10x + 5) / (x – 4) = 100
10x + 5 = 100 (x - 4)
10x + 5 = 100x – 400
5 + 400 = 100x – 10x
90x = 405
x = 405/90
= 81/18
= 9/2
= 4.5

(iv) $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$
Let us simplify the expression,
$\log_{10} [5\times (5x + 1)] = \log_{10} (x + 5) + \log_{10} 10$
$\log_{10} [5\times (5x + 1)] = \log_{10} [(x + 5) \times 10]$
[5× (5x + 1)] = [(x + 5) × 10]
25x + 5 = 10x + 50

25x – 10x = 50 – 5
15x = 45
x = 45/15
= 3

(v) log (4y – 3) = log (2y + 1) – log 3
Let us simplify the expression,
log (4y – 3) = log (2y + 1) / 3
(4y – 3) = (2y + 1) / 3
By cross multiplying, we get
3(4y - 3) = 2y + 1
12y – 9 = 2y + 1
12y – 2y = 9 + 1
10y = 10
y = 10/10
= 1

(vi) $\log_{10}(x + 2) + \log_{10}(x – 2) = \log_{10}3 + 3 \log_{10} 4$
Let us simplify the expression,
$\log_{10} [(x + 2) \times (x - 2)] = \log_{10} 3 + \log_{10} 4^3$
$\log_{10} [(x + 2) \times (x - 2)] = \log_{10} (3\times4^3)$
$[(x + 2) \times (x - 2)] = (3\times4^3)$
$(x^2 - 4) = (3\times4\times4\times4)$
$(x^2 - 4) = 192$
$x^2 = 192 + 4$
= 196
x = √196
= 14

(vii) log (3x + 2) + log (3x – 2) = 5 log 2
Let us simplify the expression,
$\log (3x + 2) + \log (3x – 2) = \log 2^5$
log [(3x + 2) × (3x - 2)] = log 32
$\log (9x^2 - 4) = \log 32$
$(9x^2 - 4) = 32$
$9x^2 = 32 + 4$
$9x^2 = 36$
$x^2 = 36/9$
$x^2 = 4$

$x = \sqrt{4}$
$= 2$

23. Solve for x:
$\log_3 (x + 1) - 1 = 3 + \log_3 (x - 1)$
Solution:
Given:
$\log_3 (x + 1) - 1 = 3 + \log_3 (x - 1)$
Let us simplify the expression,
$\log_3 (x + 1) - \log_3 (x - 1) = 3 + 1$
$\log_3 (x + 1) / (x - 1) = 4 \log_3 3$ [Since, $\log_3 3 = 1$]
$\log_3 (x + 1) / (x - 1) = \log_3 3^4$
$(x + 1) / (x - 1) = 3^4$
By cross multiplying, we get
$(x + 1) = 81 (x - 1)$
$x + 1 = 81x - 81$
$81x - x = 1 + 81$
$80x = 82$
$x = 82/80$
$= 41/40$
$= 1\ 1/40$

24. Solve for x:
$5^{\log x} + 3^{\log x} = 3^{\log x+1} - 5^{\log x - 1}$
Solution:
Given:
$5^{\log x} + 3^{\log x} = 3^{\log x+1} - 5^{\log x - 1}$
Let us simplify the expression,
$5^{\log x} + 3^{\log x} = 3^{\log x} . 3^1 - 5^{\log x} . 5^{-1}$
$5^{\log x} + 3^{\log x} = 3.3^{\log x} - 1/5 . 5^{\log x}$
$5^{\log x} + 1/5 . 5^{\log x} = 3.3^{\log x} - 3^{\log x}$
$(1 + 1/5) 5^{\log x} = (3 - 1) 3^{\log x}$
$(6/5) 5^{\log x} = 2(3^{\log x})$
$5^{\log x} / 3^{\log x} = (2\times5)/6$
$(5/3)^{\log x} = 10/6$
$(5/3)^{\log x} = 5/3$
$(5/3)^{\log x} = (5/3)^1$
So, by comparing the powers
$\log x = 1$

log x = log 10
x = 10

25. If log (x-y)/2 = ½ (log x + log y), prove that $x^2 + y^2 = 6xy$
Solution:
Given:
log (x-y)/2 = ½ (log x + log y)
Let us simplify,
log (x-y)/2 = ½ (log x×y)
log (x-y)/2 = ½ log xy
log (x-y)/2 = log $(xy)^{1/2}$
(x-y)/2 = $(xy)^{1/2}$
By squaring on both sides, we get
$[(x-y)/2]^2 = [(xy)^{1/2}]^2$
$(x - y)^2/4 = xy$
By cross multiplying, we get
$(x - y)^2 = 4xy$
$x^2 + y^2 – 2xy = 4xy$
$x^2 + y^2 = 4xy + 2xy$
$x^2 + y^2 = 6xy$
Hence proved.

26. If $x^2 + y^2 = 23xy$, Prove that log (x + y)/5 = ½ (log x + log y)
Solution:
Given:
$x^2 + y^2 = 23xy$
So, the above equation can be written as
$x^2 + y^2 = 25xy – 2xy$
$x^2 + y^2 + 2xy = 25xy$
$(x + y)^2 = 25xy$
$(x + y)^2 / 25 = xy$
Now by taking log on both sides, we get
log $[(x + y)^2 / 25]$ = log xy
log $[(x + y)/5]^2$ = log xy
2 log (x+y)/5 = log x + log y
log (x+y)/5 = ½ log x + log y
Hence proved.

27. If p = $\log_{10}$ 20 and q = $\log_{10}$ 25, find the value of x if 2 $\log_{10}$ (x + 1) = 2p – q

Solution:
Given:
$p = \log_{10} 20$
$q = \log_{10} 25$
Then,
$2 \log_{10} (x + 1) = 2p – q$
Now substitute the values of p and q, we get
$2 \log_{10} (x + 1) = 2 \log_{10} 20 - \log_{10} 25$
$= 2 \log_{10} 20 - \log_{10} 5^2$
$= 2 \log_{10} 20 – 2 \log_{10} 5$
$2 \log_{10} (x + 1) = 2 (\log_{10} 20 – \log_{10} 5)$
$\log_{10} (x + 1) = (\log_{10} 20 – \log_{10} 5)$
$= \log_{10} (20/5)$
$\log_{10} (x + 1) = \log_{10} 4$
$(x + 1) = 4$
$x = 4 - 1$
$= 3$

28. Show that:
(i) $1/\log_2 42 + 1/\log_3 42 + 1/\log_7 42 = 1$
(ii) $1/\log_8 36 + 1/\log_9 36 + 1/\log_{18} 36 = 2$

Solution:
(i) $1/\log_2 42 + 1/\log_3 42 + 1/\log_7 42 = 1$
Let us consider LHS:
$1/\log_2 42 + 1/\log_3 42 + 1/\log_7 42$
By using the formula, $\log_n m = \log_m / \log_n$
$1/\log_2 42 + 1/\log_3 42 + 1/\log_7 42 = 1/(\log 42/\log_2) + 1/(\log 42/\log_3) + 1/(\log 42/\log_7)$
$= \log_2/\log 42 + \log_3/\log 42 + \log_7/\log 42$
$= (\log_2 + \log_3 + \log_7)/\log 42$
$= (\log 2×3×7)/\log 42$
$= \log 42 / \log 42$
$= \log 42/\log 42$
$= 1$
= RHS

(ii) $1/\log_8 36 + 1/\log_9 36 + 1/\log_{18} 36 = 2$
Let us consider LHS:
$1/\log_8 36 + 1/\log_9 36 + 1/\log_{18} 36$
By using the formula, $\log_n m = \log_m / \log_n$

$1/\log_8 36 + 1/\log_9 36 + 1/\log_{18} 36 = 1/(\log 36/\log_8) + 1/(\log 36/\log_9) + 1/(\log 36/\log_{18})$
$= \log_8/\log 36 + \log_9/\log 36 + \log_{18}/\log 36$
$= (\log_8 + \log_9 + \log_{18})/\log 36$
$= (\log 8\times9\times18)/\log 36$
$= \log 36^2/\log 36$
$= 2 \log 36/\log 36$
$= 2$
$=$ RHS

29. Prove the following identities:
(i) $1/\log_a abc + 1/\log_b abc + 1/\log_c abc = 1$
(ii) $\log_b a. \log_c b. \log_d c = \log_d a$
Solution:
(i) $1/\log_a abc + 1/\log_b abc + 1/\log_c abc = 1$
Let us consider LHS:
$1/\log_a abc + 1/\log_b abc + 1/\log_c abc$
By using the formula, $\log_n m = \log_m / \log_n$
$1/\log_a abc + 1/\log_b abc + 1/\log_c abc = 1/(\log abc/\log_a) + 1/(\log abc/\log_b) + 1/(\log abc/\log_c)$
$= \log_a/ \log abc + \log_b/ \log abc + \log_c/ \log abc$
$= (\log_a + \log_b + \log_c)/ \log abc$
$= (\log a\times b\times c)/ \log abc$
$= \log abc/\log abc$
$= 1$
$=$ RHS

(ii) $\log_b a. \log_c b. \log_d c = \log_d a$
Let us consider LHS:
$\log_b a. \log_c b. \log_d c = (\log a/ \log b) \times (\log b/ \log c) \times (\log c/ \log d)$
$= \log a/\log d$
$= \log_d a$
$=$ RHS

30. Given that $\log_a x = 1/ \alpha, \log_b x = 1/\beta, \log_c x = 1/\gamma$, find $\log_{abc} x$.
Solution:
It is given that:
$\log_a x = 1/ \alpha, \log_b x = 1/\beta, \log_c x = 1/\gamma$
So,
$\log_a x = 1/ \alpha \Rightarrow \log x/\log_a = 1/ \alpha \Rightarrow \log_a = \alpha \log x$
$\log_b x = 1/\beta \Rightarrow \log x/\log_b = 1/ \beta \Rightarrow \log_b = \beta \log x$

$\log_c x = 1/\gamma \Rightarrow \log x/\log_c = 1/\gamma \Rightarrow \log_c = \gamma \log x$
Now,
$\log_{abc} x = \log x/\log abc$
$= \log x/(\log a + \log b + \log c)$
$= \log x/(\alpha \log x + \beta \log x + \gamma \log x)$
$= \log x/\log x(\alpha + \beta + \gamma)$
$= 1/(\alpha + \beta + \gamma)$

31. Solve for x:
(i) $\log_3 x + \log_9 x + \log_{81} x = 7/4$
(ii) $\log_2 x + \log_8 x + \log_{32} x = 23/15$
Solution:
(i) $\log_3 x + \log_9 x + \log_{81} x = 7/4$
let us simplify the expression,
$1/\log_x 3 + 1/\log_x 9 + 1/\log_x 81 = 7/4$
$1/\log_x 3^1 + 1/\log_x 3^2 + 1/\log_x 3^4 = 7/4$
$1/\log_x 3 + 1/2\log_x 3 + 1/4\log_x 3 = 7/4$
$1/\log_x 3 [1 + ½ + ¼] = 7/4$
$1/\log_x 3 [(4+2+1)/4] = 7/4$
$\log_3 x [7/4] = 7/4$
$\log_3 x = (7/4) \times (4/7)$
$\log_3 x = 1$
$\log_3 x = \log_3 3$ [Since, $1= \log_a a$]
On comparing, we get
$x = 3$

(ii) $\log_2 x + \log_8 x + \log_{32} x = 23/15$
let us simplify the expression,
$1/\log_x 2 + 1/\log_x 8 + 1/\log_x 32 = 23/15$
$1/\log_x 2^1 + 1/\log_x 2^3 + 1/\log_x 2^5 = 23/15$
$1/\log_x 2 + 1/3\log_x 2 + 1/5\log_x 2 = 23/15$
$1/\log_x 2 [1 + 1/3 + 1/5] = 23/15$
$\log_2 x [(15 + 5 + 3)/15] = 23/15$
$\log_2 x [23/15] = 23/15$
$\log_2 x = (23/15) \times (15/23)$
$\log_2 x = 1$
$\log_2 x = \log_2 2$ [Since, $1= \log_a a$]
On comparing, we get
$x = 2$