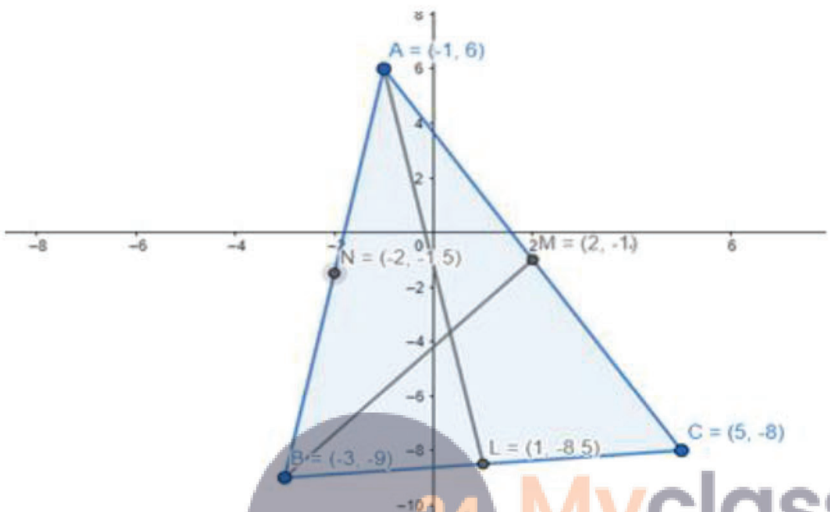
$$\text{coordinates of M} = \left(\frac{-1+5}{2}, \frac{6+(-8)}{2}\right) \Rightarrow (2,-1)$$

$$\text{coordinates of N} = \left(\frac{-1+(-3)}{2}, \frac{6+(-9)}{2}\right) \Rightarrow \left(-2, \frac{-3}{2}\right)$$



Now equation of medians AL, BM and CN using two point form

For median AL,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 6 = \frac{\frac{-17}{2} - 6}{1-(-1)}(x-(-1))$$

$$y - 6 = \frac{\frac{-17-12}{2}}{2}(x+1) \Rightarrow y - 6 = \frac{-29}{4}(x+1)$$

4(y - 6) = - 29(x + 1)

4y - 24 + 29x + 29 = 0

29x + y + 5 = 0

For median BM,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - (-9) = \frac{-1 - (-9)}{2 - (-3)}(x - (-3))$$

$$y + 9 = \frac{8}{5}(x + 3) \Rightarrow 5(y + 9) = 8(x + 3)$$

5y + 45 = 8x + 24

8x - 5y + 24 - 45 = 0

8x - 5y - 21 = 0

For median CN,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - (-8) = \frac{\frac{-3}{2} - (-8)}{-2 - 5}(x - 5)$$

$$y + 8 = \frac{\frac{-3 + 16}{2}}{2}(x - 5) \Rightarrow y + 8 = \frac{13}{4}(x - 5)$$

4(y + 8) = 13(x - 5)

4y + 32 = 13x - 65

13x - 4y - 65 - 32 = 0

13x - 4y - 97 = 0

So, the required line of equations for medians are for AL: 29x + y + 5 = 0

For BM: 8x - 5y - 21 = 0

For CN: 13x - 4y - 97 = 0

Q. 20. Find the equation of the perpendicular bisector of the line segment whose end points are A(10, 4) and B(- 4, 9).

Answer : <u>Perpendicular bisector:</u> A perpendicular bisector is a line segment which is perpendicular to the given line segment and passes through its mid - point (or we can say bisects the line segment).

Now to find the equation of perpendicular bisector first, we will find mid - point of the given line using mid - point formula (call it midpoint as M),

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{coordinates of } M = \left(\frac{10 + (-4)}{2}, \frac{4 + 9}{2}\right) \Rightarrow \left(3, \frac{13}{2}\right)$$

Now we will calculate the slope of the given line and since lines are perpendicular, so the slope of two is related as m1.m2 = - 1.

$$\text{Slope of AB}: \ m_1 = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{9 - 4}{-4 - 10} = -\frac{5}{14}$$

Now the slope of perpendicular bisector is

$$m_1.m_2 = -1 \Rightarrow -\frac{5}{14}.m_2 = -1$$

$$m_2 = \frac{14}{5}$$

Now equation of perpendicular bisector using two point form,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - \frac{13}{2} = \frac{14}{5}(x - 3) \Rightarrow 5(2y - 13) = 28(x - 3)$$

10y - 65 = 28x - 84

28x - 10y - 84 + 65 = 0

28x - 10y - 19 = 0

So, required equation of perpendicular bisector 28x - 10y - 19 = 0.

Q. 21. Find the equations of the altitudes of a ΔABC, whose vertices are A(2, - 2), B(1, 1) and C(- 1, 0).

Answer : **Altitude:** A line drawn from the vertex that meets the opposite side at right angles. It determines the height of the triangle.

In triangle ABC, let the altitudes from vertices A, B and C are AL, BM and CN on sides BC,AC and AB respectively.

Now we will find slope of sides and using the relation between the slopes of perpendicular lines i.e. m1.m2 = - 1 we will find the slopes of altitudes.

$$\text{Slope of BC}: m_1 = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{0 - 1}{-1 - 1} = \frac{-1}{-2}$$

$$m_1 = \frac{1}{2}$$

$$\text{Slope of AC}: m_2 = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{0 - (-2)}{-1 - 2} = -\frac{2}{3}$$

$$\text{Slope of AB}: m_3 = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{1 - (-2)}{1 - 2} = -3$$

$$\text{Slope of AL}: m_1.m_1' = -1 \Rightarrow \frac{1}{2}.m_1' = -1$$

$$m_1' = -2$$

$$\text{Slope of BM}: m_2.m_2' = -1 \Rightarrow \frac{-2}{3}.m_2' = -1$$

$$m_2' = \frac{3}{2}$$

$$\text{Slope of CN}: m_3.m_3' = -1 \Rightarrow -3.m_3' = -1$$

$$m_3' = \frac{1}{3}$$

Now equation of altitudes using two point form

For altitude AL,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

y - (- 2) = - 2(x - 2)

y + 2 + 2x - 4 = 0

2x + y - 2 = 0

For altitude BM,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

y - 1 = - 1(x - 1)

y - 1 + x - 1 = 0

x + y - 2 = 0

For altitude CN,

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 0 = \frac{1}{3}(x - (-1))$$

3y = x + 1

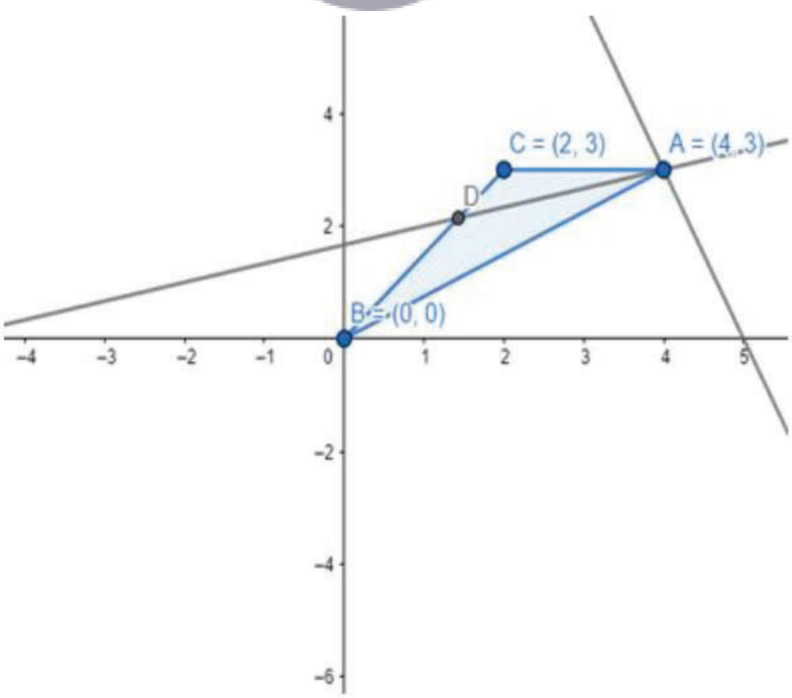
x - 3y + 1 = 0

So, the required equations of altitudes are for AL: 2x + y - 2 = 0

For BM: x + y - 2 = 0
For CN: x - 3y + 1 = 0

Q. 22. If A(4, 3), B(0, 0) and C(2, 3) are the vertices of a ΔABC, find the equation of the bisector of ∠A.

Answer : <u>Construction</u>: Draw a line from vertex A intersecting side BC of the triangle at D (as there is one bisector for exterior angle also but it is the default that we have to find interior angle bisector).



As the angle between the sides AB and angle bisector AD and side AC and angle bisector AD is equal.

$$\angle A = 2\theta \Rightarrow \angle BAD = \angle CAD = \theta$$

Then using the angle between two lines, if the slope of AD be m and slope of AB

$$\text{Slope of AB} = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{3-0}{4-0} = \frac{3}{4}$$

Putting the values in the equation

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1.m_2} \quad \text{.................(1)}$$

$$\Rightarrow \frac{\frac{3}{4} - m}{1 + m.\frac{3}{4}} = \frac{\frac{3-4m}{4}}{\frac{4+3m}{4}}$$

$$\tan\theta = \frac{3-4m}{4+3m} \quad \text{......................(2)}$$

Again for side AC slope

$$\text{Slope of AC} = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{3-3}{2-4} = 0$$

Putting in equation (1)

$$\tan\theta = \frac{m_2 - m_1}{1 + m_1.m_2} \Rightarrow \frac{m-0}{1+0.m} = m \quad \text{.....................(3)}$$

From equation (2) and (3),we have

$$m = \frac{3-4m}{4+3m} \Rightarrow 4m + 3m^2 + 4m - 3 = 0$$

$$3m^2 + 8m - 3 = 0$$

$$\text{From equation we have two values of m} - 3, \frac{1}{3}$$

tanθ = - 3 as tanx is negative in II and IV quadrant means it is obtuse angle either way(exterior here)we require interior angle so will consider the positive value of m.

$$m = \tan\theta = \frac{1}{3}$$

As we obtained the slope of angle bisector which passes through A vertex so using slope intercept form first calculate the value of the intercept

y = mx + c ……………….(4)

$$3 = \frac{1}{3}(4) + c \Rightarrow c = 3 - \frac{4}{3} \Rightarrow c = \frac{9-4}{3} \Rightarrow \frac{5}{3}$$

Putting the value of c in equation (4),we have

$$y = \frac{1}{3}x + \frac{5}{3} \Rightarrow x - 3y + 5 = 0$$

So, the required equation of angle bisector is x - 3y + 5 = 0.

Q. 23. the midpoints of the sides BC, CA and AB of a ΔABC are D(2, 1), B(- 5, 7) and P(- 5, - 5) respectively. Find the equations of the sides of ΔABC.

Answer : Let us consider the coordinates of vertices of triangle A, B, C be (a, b), (c, d) and (e, f). Now using mid - point formula

$$(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$\text{For side BC (midpoint D)}: (2,1) = \frac{c+e}{2}, \frac{d+f}{2}$$

$$\text{For side AC(midpoint E)}: (-5,7) = \frac{a+e}{2}, \frac{b+f}{2}$$

$$\text{For side AB(midpoint F)}: (-5,-5) = \frac{a+c}{2}, \frac{b+d}{2}$$

Now from above equations, we have

c + e = 4, d + f = 2 (i)

a + e = - 10, b + f = 14 (ii)

a + c = - 10, b + d = - 10 (iii)

From subtract (i) from (ii),we get

a - c = - 14, b - d = 12 (iv)

Adding (iii) and (iv)

$$2a = -24 \Rightarrow a = -12, 2b = 2 \Rightarrow b = 1$$

Putting values of a, b in equation (iii)

$$c = -10 - (-12) \Rightarrow c = 2,\ d = -10 - 1 \Rightarrow d = -11$$

Again putting values in (i)

$$e = 4 - 2 \Rightarrow e = 2,\ f = 2 - (-11) \Rightarrow f = 13$$

So coordinates of A (- 12,1), B(2, - 11) and C(2,13).

Using two point form of the equation

Equation of side AB:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 1 = \frac{-11-1}{2-(-12)}(x-(-12)) \Rightarrow y - 1 = \frac{-12}{14}(x+12)$$

14(y - 1) = - 12(x + 12)

14y - 14 + 12x + 144 = 0

12x + 14y + 130 = 0

6x + 7y + 65 = 0

Equation of side BC:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - (-11) = \frac{13-(-11)}{2-2}(x-2) \Rightarrow y + 11 = \frac{24}{0}(x-2)$$

y = - 11(slope is not defined i.e. line is vertical)

Equation of side CA:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 13 = \frac{1-13}{-12-2}(x-2) \Rightarrow y - 13 = \frac{-12}{-14}(x-2)$$

14(y - 13) = 12(x - 2)

12x - 24 - 14y + 182 = 0

12x - 14y + 158 = 0

6x - 7y + 79 = 0

So, the required equations of sides for AB: 6x + 7y + 65 = 0

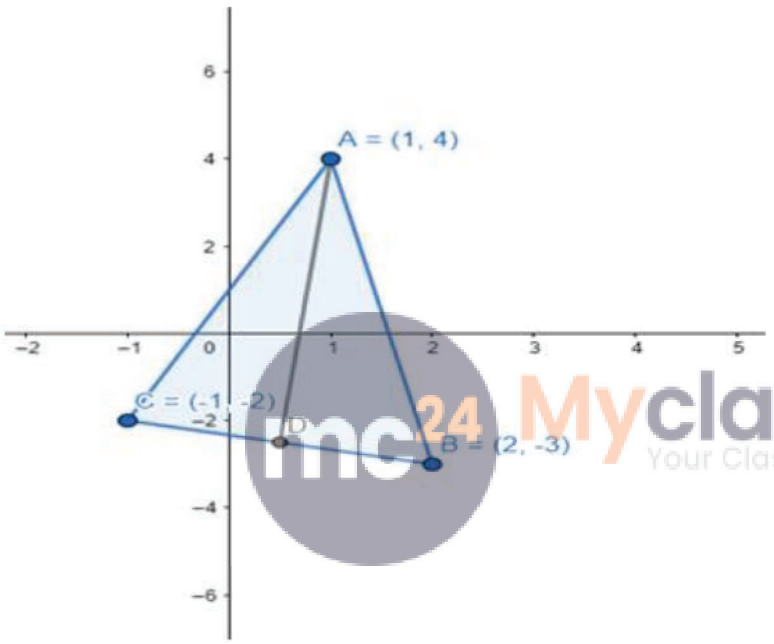
For BC: y = - 11

For CA: 6x - 7y + 79 = 0

Q. 24. If A(1, 4), B(2, - 3) and C(- 1, - 2) are the vertices of a ΔABC, find the equation of

(i) the median through A
(ii) the altitude through A
(iii) the perpendicular bisector of BC

Answer : <u>Construction:</u> Draw a line segment from vertex A intersecting BC at the midpoint (D).



(i) Equation of median AD, we will find the midpoint of side BC

$$\text{For side BC(midpoint D)}: (x, y) = \frac{2+(-1)}{2}, \frac{-3+(-2)}{2}$$

$$(x, y) = \left(\frac{1}{2}, \frac{-5}{2}\right)$$

Now using two point form of the equation of the line, we have

Equation of side AD:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

$$y - 4 = \frac{\frac{-5}{2} - 4}{\frac{1}{2} - 1}(x - 1) \Rightarrow y - 4 = \frac{\frac{-5-8}{2}}{\frac{1-2}{2}}(x - 1)$$

$$y - 4 = \frac{-13}{-1}(x - 1) \Rightarrow y - 4 = 13x - 13$$

13x - y - 13 + 4 = 0

13x - y - 9 = 0

So, required equation of altitude is 3x - y - 9 = 0.

(ii) For the equation of altitude, we will need slope as we have a point through which line passes (A).

Now we will find the slope of side BC and using the relation between the slopes of perpendicular lines, i.e. $m_1 . m_2 = -1$ we will find the slopes of altitude.

$$\text{Slope of BC}: m_1 = \frac{y_2 - y_1}{x_2 - x_1} \Rightarrow \frac{-2 + (-3)}{-1 - 2} = \frac{-5}{-3}$$

$$m_1 = \frac{5}{3}$$

$$\text{Slope of AM}: m_1 . m_1' = -1 \Rightarrow \frac{5}{3} . m_1' = -1$$

$$m_1' = \frac{-3}{5}$$

Using slope intercept form, we will first calculate intercept,

y = mx + c ……………………(1)

$$4 = \frac{-3}{5}(1) + c \Rightarrow c = 4 + \frac{3}{5}$$

$$c = \frac{20+3}{5} \Rightarrow c = \frac{23}{5}$$

Putting in equation (1)

$$y = \frac{-3}{5}x + \frac{23}{5} \Rightarrow 3x + 5y - 23 = 0$$

So, required equation of altitude is 3x + 5y - 23 = 0.

(iii) We have a slope of perpendicular and a mid point from the previous solution

$$m_1' = \frac{-3}{5}, \quad \text{midpoint of BC(point D)}\,(x, y) = \left(\frac{1}{2}, \frac{-5}{2}\right)$$

Now for perpendicular bisector, it passes through the midpoint of BC, i.e. we have a slope of the equation and a point through which it passes so we can use the slope - intercept form and calculate intercept,

y = mx + c …………………(i)

$$\frac{-5}{2} = \frac{-3}{5}\left(\frac{1}{2}\right) + c \Rightarrow c = \frac{-5}{2} + \frac{3}{10}$$

$$c = \frac{-25+3}{10} \Rightarrow c = \frac{-22}{10}$$

$$c = \frac{-11}{5}$$

Putting in equation (i) value of c,

$$y = \frac{-3}{5}x + \frac{-11}{5} \Rightarrow 3x + y + 11 = 0$$

So, the required equation of perpendicular bisector is $3x + y + 11 = 0$.

Exercise 20D

Q. 1. Find the equation of the line whose

(i) slope = 3 and y - intercept = 5
(ii) slope = - 1 and y - intercept = 4
(iii) slope = $-\frac{2}{5}$ and y - intercept = - 3

Answer : (i) Formula to be used: $y = mx + c$ where m is the slope of the line and c is the y - intercept.

Here, m = 3 and c = 5.

Hence, $y = (3)x + (5)$

i.e. $y = 3x + 5$

(ii) Formula to be used: $y = mx + c$ where m is the slope of the line and c is the y - intercept.

Here, m = - 1 and c = 4.

Hence, $y = (-1)x + (4)$

i.e. $x + y = 4$

(iii) Formula to be used: $y = mx + c$ where m is the slope of the line and c is the y - intercept.

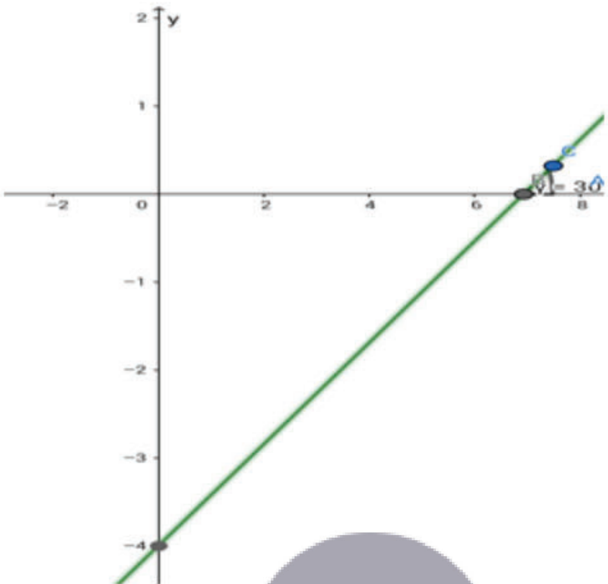
Here, $m = -\frac{2}{5}$ and c = - 3.

Hence, $y = (-\frac{2}{5})x + (-3)$

Or, $5y = -2x - 3$ i.e. $2x + 5y + 3 = 0$

Q. 2. Find the equation of the line which makes an angle of 30^0 with the positive direction of the x - axis and cuts off an intercept of 4 units with the negative direction of the y - axis.

Answer :



Given : The given line makes an angle of 30° with the x - axis. The y - intercept = - 4.

So, the slope of the line is m = tanθ = tan30° = $^1/_{\sqrt{3}}$.

Formula to be used: y = mx + c where m is the slope of the line and c is the y - intercept.

The equation of the line is y = $\frac{1}{\sqrt{3}}x - 4$

Or, $\sqrt{3}y = x - 4\sqrt{3}$ i.e. $x - \sqrt{3}y = 4\sqrt{3}$

Q. 3. Find the equation of the line whose inclination is $\frac{5\pi}{6}$ and which makes an intercept of 6 units on the negative direction of the y - axis.

Answer : Given:

$$\theta = \frac{5\pi}{6}$$

$$\therefore \text{slope, } m = \tan\theta = \tan\frac{5\pi}{6} = -\frac{1}{\sqrt{3}}$$

The y - intercept is 6 units.

Formula to be used: y = mx + c where m is the slope of the line and c is the y - intercept.

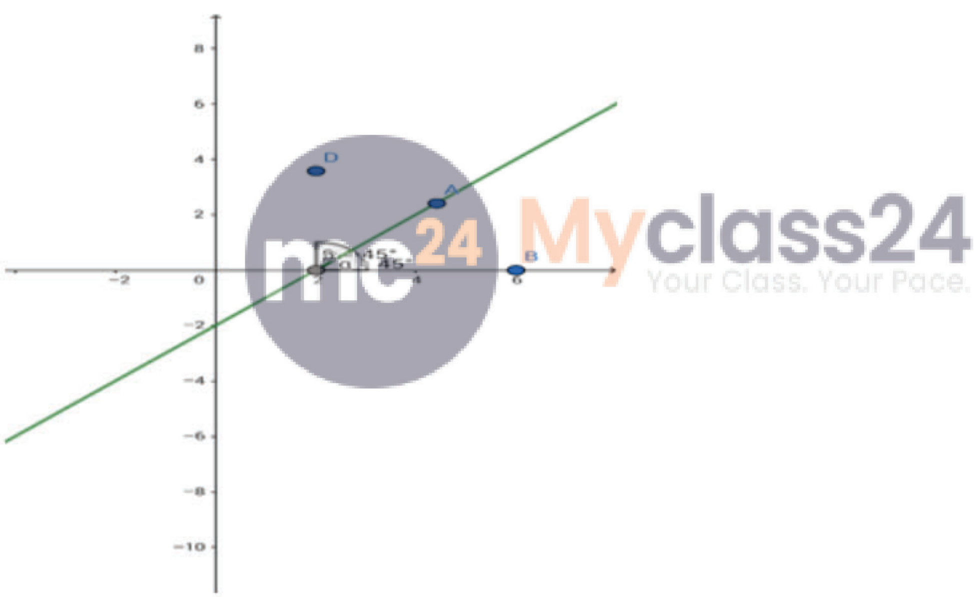
The equation of the line is

$$y = -\frac{1}{\sqrt{3}}x - 6$$

i.e. $\sqrt{3}y + x + 6\sqrt{3} = 0$

Q. 4. Find the equation of the line cutting off an intercept - 2 from the y - axis and equally inclined to the axes.

Answer :



Given: The line is equally inclined to both the axes.

The angle between the coordinate axes = 90°

If the inclination to both the axes is θ then θ+θ = 90°

i.e. θ = 45 θ°

∴ slope of the line, m = tan θ = tan45° = 1

The y - intercept = - 2 units

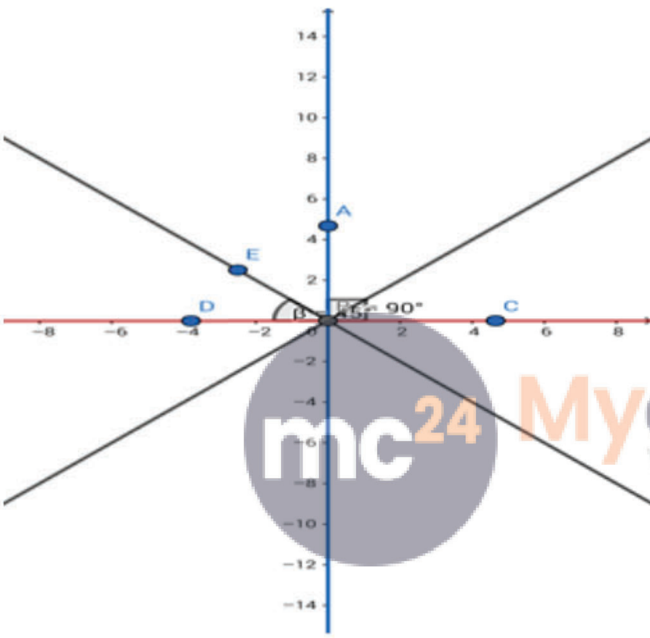
Formula to be used: y = mx + c where m is the slope of the line and c is the y - intercept.

The equation of the line is y = 1.x + (-2) = x - 2

i.e. x - y = 2

Q. 5. Find the equation of the bisectors of the angles between the coordinate axes.

Answer :



Given: The straight lines are x = 0 and y = 0.

Formula to be used: If θ is the angle between two straight lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ then the equation of their angle bisector is

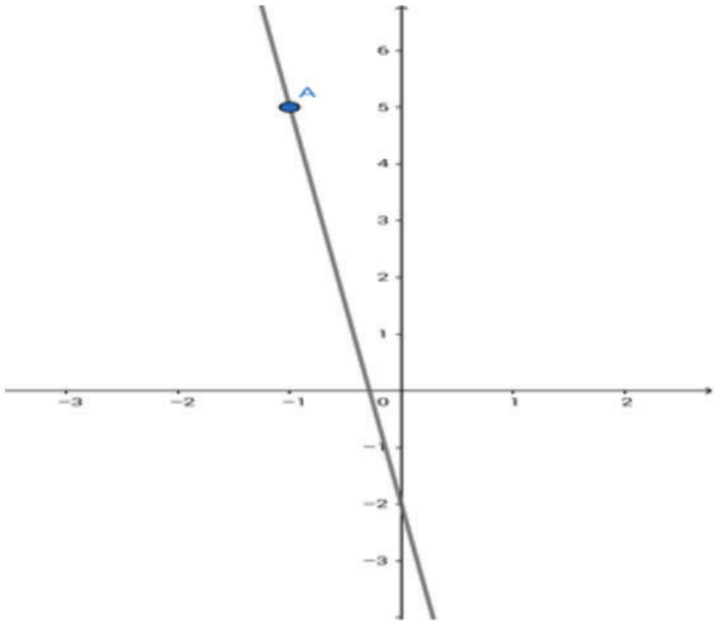
$$\left|\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}}\right| = \left|\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}\right|$$

∴the equation of the angle bisectors is $\left|\frac{x}{\sqrt{1^2}}\right| = \left|\frac{y}{\sqrt{1^2}}\right|$

i.e. x = ±y

Q. 6. Find the equation of the line through the point (- 1, 5) and making an intercept of - 2 on the y - axis.

Answer :



Given: The y - intercept = - 2.

The line passes through (- 1,5).

Formula to be used: y = mx + c where m is the slope of the line and c is the y - intercept.

The equation of the line is y = mx + (- 2) = mx - 2.

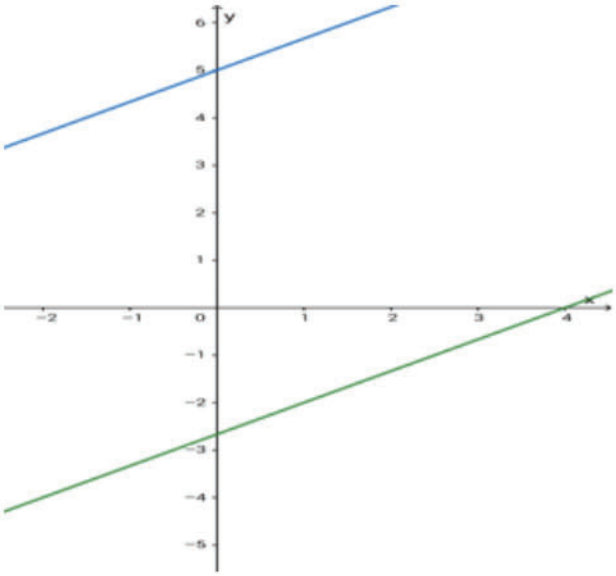
Now, this line passes through (- 1,5).

∴ 5 = m(- 1) - 2 = - m - 2 i.e. m = - (5 + 2) = - 7

∴ y = (- 7)x + (- 2) = - 7x - 2 i.e.7x + y + 2 = 0

Q. 7. Find the equation of the line which is parallel to the line 2x – 3y = 8 and whose y - intercept is 5 units.

Answer :



Given: The given line is 2x - 3y = 8. The line parallel to this line has a y - intercept of 5units.

Formula to be used: If ax + by = c is a straight line then the line parallel to the given line is of the form ax + by = d, where a,b,c,d are arbitrary real constants.

A line parallel to the given line has a slope of $\frac{2}{3}$ and is of the form 2x - 3y = k , where k is any arbitrary real constant.

Now, 2x - 3y = k

or, 3y = 2x - k

$$\text{or}, y = \left(\frac{2}{3}\right)x + \left(-\frac{k}{3}\right)$$

which is of the form y = mx + c , where c is the y - intercept.

$$\therefore c = -\frac{k}{3} = 5$$

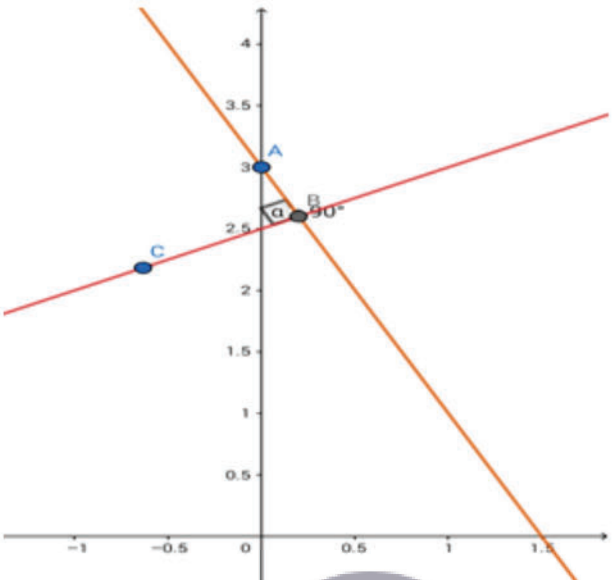
So, k = (- 3)x5 = - 15

Equation of the required line is 2x - 3y = - 15

i.e. 2x - 3y + 15 = 0

Q. 8. Find the equation of the line passing through the point (0, 3) and perpendicular to the line x – 2y + 5 = 0

Answer :



Given: The given line is x - 2y + 5 = 0. The line perpendicular to this given line passes through (0,3)

Formula to be used: The product of slopes of two perpendicular lines = - 1.

The slope of this line is 1/2 .

$\therefore$ the slope of the perpendicular line $= \frac{-1}{1/2} = -2.$

The equation of the line can be written in the form y = (- 2)x + c

(c is the y - intercept)

This line passes through (0,3) so the point will satisfy the equation of the line.

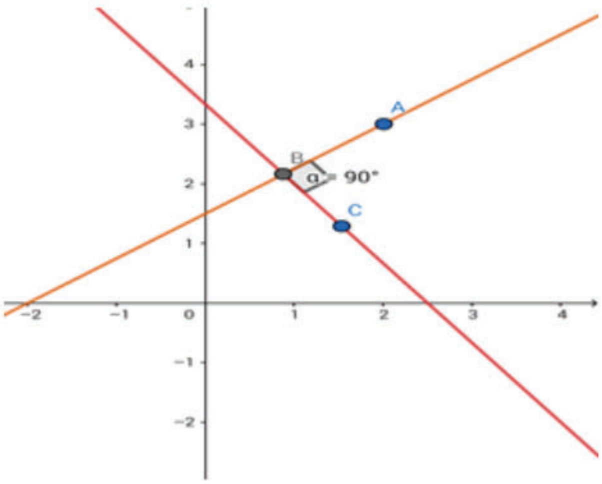
$\therefore$ 3 = (- 2)x0 + c i.e. c = 3

The required equation is y = - 2x + 3

i.e. 2x + y = 3

Q. 9. Find the equation of the line passing through the point (2, 3) and perpendicular to the line 4x + 3y = 10

Answer :



Given: The given line is 4x + 3y = 10. The line perpendicular to this line passes through (2,3).

Formula to be used: The product of slopes of two perpendicular lines = - 1

Slope of this line is $-{}^{4}/_{3}$.

∴the slope of the perpendicular line = $\frac{-1}{-{}^{4}/_{3}} = {}^{3}/_{4}$.

The equation of the line can be written in the form $y = \left(\frac{3}{4}\right)x + c$

(c is the y - intercept)

This line passes through (2,3), so the point will satisfy the equation of the line.

$\therefore 3 = \left(\frac{3}{4}\right)x2 + c \text{ i.e. } c = 3 - \frac{3}{2} = \frac{3}{2}$

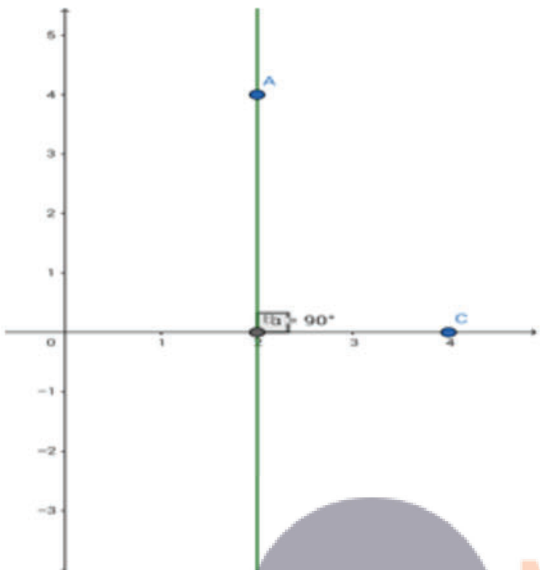
The required equation is

$y = \frac{3}{4}x + \frac{3}{2}$

or, 4y = 3x + 6 i.e. 3x - 4y + 6 = 0.

Q. 10. Find the equation of the line passing through the point (2, 4) and perpendicular to the x - axis.

Answer :



Given: The line is perpendicular to x - axis and passes through (2,4)

The equation of the line perpendicular to the x - axis (y = 0) can be represented as x = c, where c is a real constant.

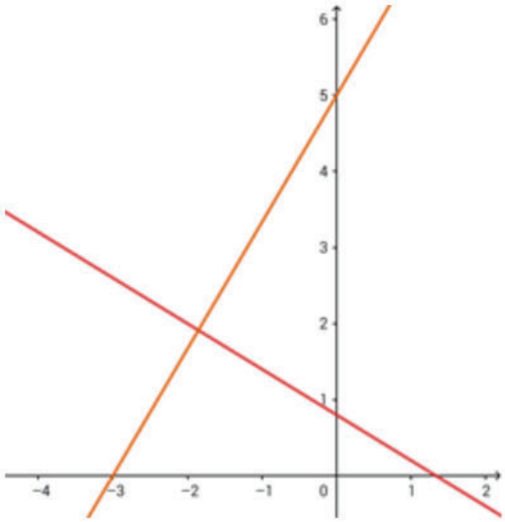
Now, this line passes through (2,4).

∴ c = 2

The required equation is x = 2

Q. 11. Find the equation of the line that has x - intercept - 3 and which is perpendicular to the line 3x + 5y = 4

Answer :



Given: The given line is 3x + 5y = 4. The perpendicular line has an x - intercept of - 3.

Formula to be used: The product of slopes of two perpendicular lines = - 1.

The slope of this line is $-{}^{3}/_{5}$.

∴ the slope of the perpendicular line =

$\frac{-1}{-{}^{3}/_{5}} = {}^{5}/_{3}.$

The equation of the line can be written in the form

$y = \left(\frac{5}{3}\right)x + c$

(c is the y - intercept)

This line intercepts the x - axis when y = 0.

So, the x - intercept:

$0 = \left(\frac{5}{3}\right)x + c \text{ i.e. } x = -\frac{3c}{5}$

Now, it is given that the x - intercept is - 3.

$\therefore -\frac{3c}{5} = -3 \text{ i.e. } c = 5$

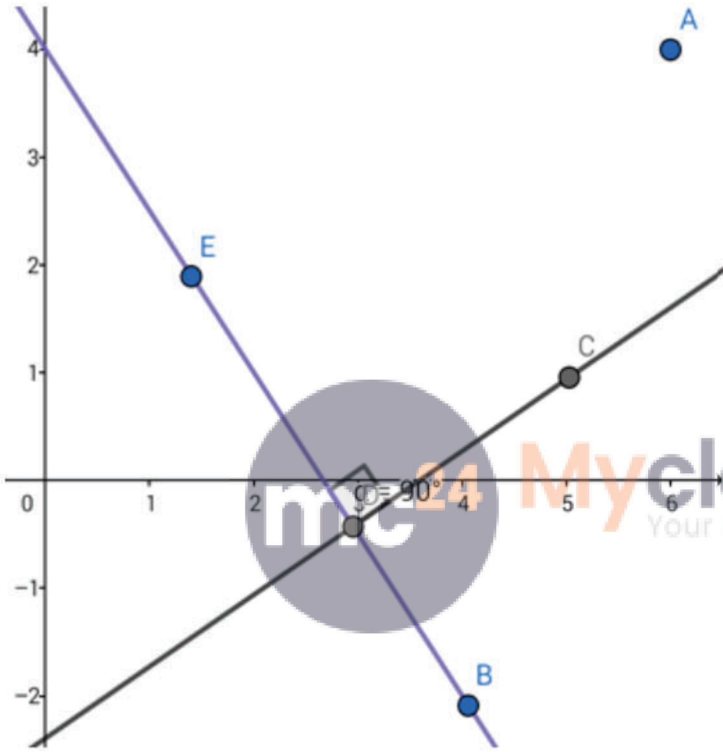
The required equation of the line is

$$y = \left(\frac{5}{3}\right)x + 5$$

i.e. 5x - 3y + 15 = 0

Q. 12. Find the equation of the line which is perpendicular to the line 3x + 2y = 8 and passes through the midpoint of the line joining the points (6, 4) and (4, - 2).

Answer :



Given: The given line is 3x + 2y = 8. The perpendicular line passes through the midpoint of (6,4) and (4, - 2).

Formulae to be used: The product of slopes of two perpendicular lines = - 1.

If (a,b) and (c,d) be two points, then their midpoint is given by

$$\left(\frac{a+c}{2}, \frac{b+d}{2}\right)$$

The slope of this line is $-{}^{3}/_{2}$.

∴ the slope of the perpendicular line =

$\frac{-1}{-3/2} = 2/3.$

The equation of the line can be written in the form

$$y = \left(\frac{2}{3}\right)x + c$$

(c is the y - intercept)

This line passes through the midpoint of (6,4) and (4, - 2).

The co - ordinates of the midpoint of the line joining the given points is

$$\left(\frac{6+4}{2}, \frac{4+(-2)}{2}\right) = (5,1)$$

(5,1) satisfies the equation

$$y = \left(\frac{2}{3}\right)x + c$$

$$\therefore 1 = \left(\frac{2}{3}\right)x5 + c \text{ or, } c = 1 - \frac{10}{3} = -\frac{7}{3}$$

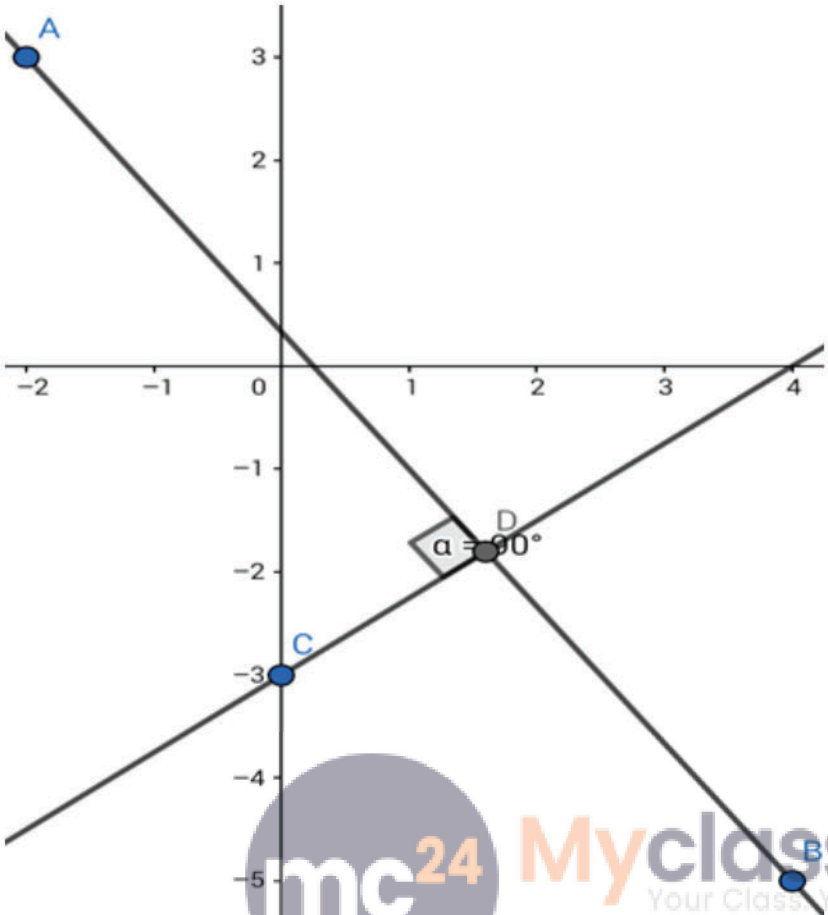
The required equation is

$$y = \left(\frac{2}{3}\right)x + \left(-\frac{7}{3}\right)$$

i.e. 2x - 3y = 7

Q. 13. Find the equation of the line whose y - intercept is - 3 and which is perpendicular to the line joining the points (- 2, 3) and (4, - 5).

Answer :



Given: The line perpendicular to the line passing through (- 2,3) and (4, - 5) has the y - intercept of - 3.

Formula to be used: If (a,b) and (c,d) are two points then the equation of the line passing through them is

$$\frac{y-d}{x-c}=\frac{d-b}{c-a}$$

Product of slopes of two perpendicular lines = - 1

The equation of the line joining points (- 2,3) and (4, - 5) is

$$\frac{y-(-5)}{x-4}=\frac{(-5)-3}{4-(-2)}$$

$$\text{or,}\frac{y\ +\ 5}{x-4}=\frac{-8}{6}=-\frac{4}{3}$$

or,3y + 15 = - 4x + 16 or, 4x + 3y = 1

Slope of this line is $-{}^{4}/_{3}$.

∴ the slope of the perpendicular line =

$$\frac{-1}{-{}^{4}/_{3}} = {}^{3}/_{4}.$$

The equation of the line can be written in the form

$$y = \left(\frac{3}{4}\right)x + c$$

(c is the y - intercept)

But, the y - intercept is - 3.

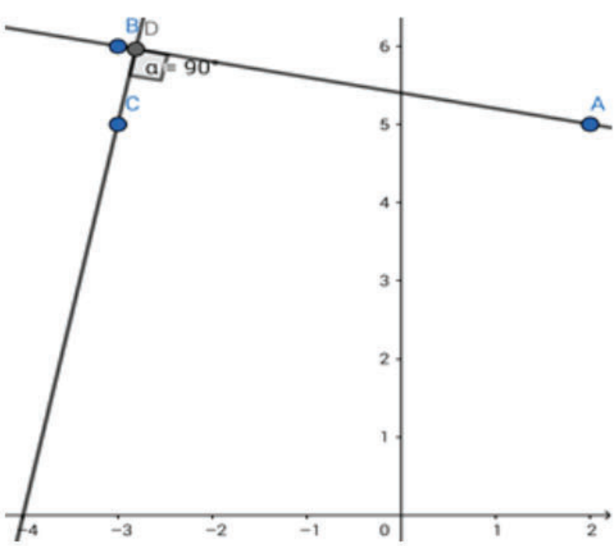
The required line is

$$y = \frac{3}{4}x + (-3)$$

i.e. 3x - 4y = 12

Q. 14. Find the equation of the line passing through (- 3, 5) and perpendicular to the line through the points (2, 5) and (- 3, 6).

Answer :



Given: The line perpendicular to the line passing through (2,5) and (- 3,6) passes through (- 3,5).

Formula to be used: If (a,b) and (c,d) are two points then the equation of the line passing through them is

$$\frac{y-b}{x-a} = \frac{b-d}{a-c}$$

Product of slopes of two perpendicular lines = - 1

The equation of the line joining points (2,5) and (- 3,6) is

$$\frac{y-5}{x-2} = \frac{5-6}{2-(-3)}$$

$$\text{or, } \frac{y-5}{x-2} = \frac{-1}{5}$$

Or, 5y - 25 = - x + 2

i.e. the given line is x + 5y = 27.

The slope of this line is $-{}^{1}/_{5}$.

∴ the slope of the perpendicular line =

$$\frac{-1}{-{}^{1}/_{5}} = 5.$$

The equation of the line can be written in the form y = 5x + c.

(c is the y - intercept)

This line passes through (- 3,5).

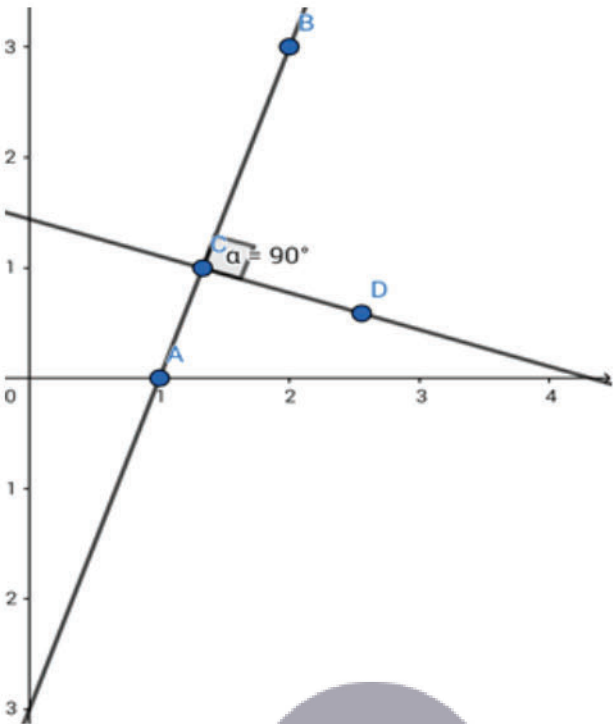
Hence, 5 = 5x(- 3) + c or, c = 20

The required equation of the line will be y = 5x + 20

i.e. 5x - y + 20 = 0

Q. 15. A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ratio 1 : 2. Find the equation of the line.

Answer :



Given: A line perpendicular to the line segment joining the points (1, 0) and (2, 3) divides it in the ratio 1 : 2.

Formula to be used: If (a,b) and (c,d) are two points then the equation of the line passing through them is

$$\frac{y-b}{x-a} = \frac{b-d}{a-c}$$

If (a_1,b_1) and (a_2,b_2) be two points , then the co - ordinates of the point dividing their join in the ratio a:b is given by

$$x - \text{co ordinate} = \left(\frac{a_1 X b + a_2 X a}{a + b}\right)$$

$$y - \text{co ordinate} = \left(\frac{b_1 X b + b_2 X a}{a + b}\right)$$

The equation of the line joining points (1,0) and (2,3) is

$$\frac{y-0}{x-1} = \frac{0-3}{1-2}$$

$$\text{or,} \frac{y}{x-1} = \frac{-3}{-1} = 3$$

or,

$$y = 3x - 3 \text{ or}, 3x - y = 3$$

i.e. the given line is 3x - y = 3.

Accordingly, the required co - ordinates of the point dividing the join of (1,0) and (2,3) in the ratio 1:2 are

$$\left(\left(\frac{1X2 + 2X1}{1 + 2}\right), \left(\frac{0X2 + 3X1}{1 + 2}\right)\right) = \left(\frac{4}{3}, 1\right)$$

The given line is 3x - y = 3.

The slope of this line is 3.

∴the slope of the perpendicular line = $\frac{-1}{3} = -\frac{1}{3}$.

The equation of the line can be written in the form $y = -\frac{1}{3}x + c$

(c is the y - intercept)

This line will pass through $\left(\frac{4}{3}, 1\right)$.

$$\therefore 1 = -\frac{1}{3}X\frac{4}{3} + c \text{ or}, c = 1 + \frac{4}{9} = \frac{13}{9}$$

The required equation is $y = -\frac{1}{3}x + \frac{13}{9}$

i.e. 3x + 9y = 13