$\left(\frac{1}{2k^{\frac{1}{3}}}\right)\left(-\frac{1}{k^{\frac{1}{3}}}\right) = -1$

$k^{\frac{2}{3}} = \frac{1}{2} \Rightarrow k^2 = \frac{1}{8} \Rightarrow 8k^2 = 1$

27. Question

Show that the curves $xy = a^2$ and $x^2 + y^2 = 2a^2$ touch each other.

Answer

If the two curve touch each other then the tangent at their intersecting point formed a angle of 0.

We have to find the intersecting point of these two curves.

$xy = a^2$ and $x^2 + y^2 = 2a^2$

$\Rightarrow x^2 + \left(\frac{a^2}{x}\right)^2 = 2a^2$

$\Rightarrow x^4 - 2a^2x^2 + a^4 = 0$

$\Rightarrow (x^2 - a^2) = 0$

$\Rightarrow x = +a$ and $-a$

At $x = a$, $y = a$

At $x = -a$, $y = -a$

$m_1 : \frac{dy}{dx} = \frac{-a^2}{x^2}$

m_1 at $(a, a) = -1$

m_1 at $(-a, -a) = -1$

$m_2 : 2x + 2y\frac{dy}{dx} = 0$

m_2 at $(a, a) = -1$

m_2 at $(-a, -a) = -1$

At (a, a)

$\tan\theta = \frac{m_1 - m_2}{1 + m_1m_2}$

$\tan\theta = \frac{(-1)-(-1)}{1+(-1)(-1)} = 0 \Rightarrow \theta = 0$

At (-a, -a)

$\tan\theta = \frac{m_1 - m_2}{1 + m_1m_2}$

$\tan\theta = \frac{(-1)-(-1)}{1+(-1)(-1)} = 0 \Rightarrow \theta = 0$

So, we can say that two curves touch each other because the angle between two tangent at their intersecting point is equal to 0.

28. Question

Show that the curves $x^3 - 3xy^2 + 2 = 0$ and $3x^2y - y^3 - 2 = 0$ cut orthogonally.

Answer

If the two curve cut orthogonally then angle between their tangent at intersecting point is equal to 90°.

We have to find their intersecting point.

$x^3 - 3xy^2 + 2 = 0$ ...(1) and $3x^2y - y^3 - 2 = 0$ ...(2)

On adding eq (1) and eq (2)

$x^3 - 3xy^2 + 2 + 3x^2y - y^3 - 2 = 0$

$x^3 - y^3 - 3xy^2 + 3x^2y = 0$

$(x - y)^3 = 0 \Rightarrow x = y$

Put x = y in eq (1)

$y^3 - 3y^3 + 2 = 0 \Rightarrow y = 1$

At y = 1, x = 1

$m_1 : 3x^2 - 3\left(x \times 2y\frac{dy}{dx} + y^2\right) = 0$

m_1 at (1, 1) = 0

$m_2 : 3\left(x^2\frac{dy}{dx} + 2xy\right) - 3y^2\frac{dy}{dx} = 0$

m_2 at (1, 1) = -2/0

At (1, 1)

$$\tan\theta = \frac{m_1 - m_2}{1 + m_1m_2}$$

$$\tan\theta = \frac{m_2\left(1 - \frac{m_1}{m_2}\right)}{m_2\left(\frac{1}{m_2} + m_1\right)}$$

$$\tan\theta = \frac{(1-0)}{(0+0)} = \text{not defined} \Rightarrow \theta = \frac{\pi}{2}$$

So, we can say that two curve cut each other orthogonally because angle between two tangent at their intersecting point is equal to 90°.

29. Question

Find the equation of tangent to the curve $x = (\theta + \sin\theta), y = (1 + \cos\theta)$ at $\theta = \frac{\pi}{4}$.

Answer

$$m : \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-\sin\theta}{1+\cos\theta}$$

$$m \text{ at } \left(\theta = \frac{\pi}{4}\right) = \frac{-1}{1+\sqrt{2}} = 1 - \sqrt{2}$$

At $\theta = \frac{\pi}{4}, x = \left(\frac{\pi}{4} + \frac{1}{\sqrt{2}}\right)$ and $y = \left(1 + \frac{1}{\sqrt{2}}\right)$

$y - b = m(x - a)$

$$y - \left(1 + \frac{1}{\sqrt{2}}\right) = (1 - \sqrt{2})\left(x - \left(\frac{\pi}{4} + \frac{1}{\sqrt{2}}\right)\right)$$

$y = (1-\sqrt{2})x + \frac{(\sqrt{2}-1)\pi}{4} + 2$

30. Question

Find the equation of the tangent at $t = \frac{\pi}{4}$ for the curve $x = \sin 3t$, $y = \cos 2t$.

Answer

$m : \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-2\sin 2t}{3\cos 3t}$

$m \text{ at } \left(t = \frac{\pi}{4}\right) = \frac{2\sqrt{2}}{3}$

At $t = \frac{\pi}{4}, x = \frac{1}{\sqrt{2}}$ and $y = 0$

y - b = m(x - a)

$y - 0 = \frac{2\sqrt{2}}{3}\left(x - \frac{1}{\sqrt{2}}\right)$

$4x - 3\sqrt{2}y - 2\sqrt{2} = 0$

Objective Questions

1. Question

Mark (√) against the correct answer in the following:

If $y = 2^x$ then $\frac{dy}{dx} = ?$

A. $x\left(2^{x-1}\right)$

B. $\frac{2^x}{(\log 2)}$

C. $2^x(\log 2)$

D. none of these

Answer

Given that $y=2^x$

Taking log both sides, we get

$\log_e y = x\log_e 2$ (Since $\log_a b^c = c\log_a b$)

Differentiating with respect to x, we get

$\frac{1}{y}\frac{dy}{dx} = \log_e 2 \text{ or } \frac{dy}{dx} = \log_e 2 \times y$

Hence $\frac{dy}{dx} = 2^x \log_e 2$

2. Question

Mark (√) against the correct answer in the following:

If $y = \log_{10} x$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{x}$

B. $\frac{1}{x}(\log 10)$

C. $\frac{1}{x(\log 10)}$

D. none of these

Answer

Given that $y = \log_{10} x$

Using the property that $\log_a b = \frac{\log_e b}{\log_e a}$, we get

$y = \frac{\log_e x}{\log_e 10}$

Differentiating with respect to x, we get

$\frac{dy}{dx} = \frac{1}{x\log_e 10}$

3. Question

Mark (√) against the correct answer in the following:

If $y = e^{1/x}$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{x} \cdot e^{(1/x-1)}$

B. $\frac{-e^{1/x}}{x^2}$

C. $e^{1/x} \log x$

D. none of these

Answer

Given that $y = e^{\frac{1}{x}}$

Taking log both sides, we get

$\log_e y = \frac{1}{x}$ (Since $\log_a b^c = c\log_a b$)

Differentiating with respect to x, we get

$\frac{1}{y}\frac{dy}{dx} = -\frac{1}{x^2}$ or $\frac{dy}{dx} = -\frac{1}{x^2} \times y$

Hence $\frac{dy}{dx} = -\frac{1}{x^2} \times e^{\frac{1}{x}}$

4. Question

Mark (√) against the correct answer in the following:

If $y = x^x$ then $\frac{dy}{dx} = ?$

A. $x^x \log x$

B. $x^x(1+\log x)$

C. $x(1+\log x)$

D. none of these

Answer

Let $y=f(x)=x^x$

Taking log both sides, we get

$\log_e y = x \times \log_e x$-(1) (Since $\log_a b^c = c\log_a b$)

Differentiating (1) with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = x \times \frac{1}{x} + \log_e x \times 1$$

$$\Rightarrow \frac{dy}{dx} = y \times (1 + \log_e x)$$

$$\Rightarrow \frac{dy}{dx} = f'(x) = x^x(1 + \log_e x)$$

5. Question

Mark (√) against the correct answer in the following:

If $y = x^{\sin x}$ then $\frac{dy}{dx} = ?$

A. $(\sin x) \cdot x^{(\sin x - 1)}$

B. $(\sin x \cos x) \cdot x^{(\sin x - 1)}$

C. $x^{\sin x}\left\{\frac{\sin x + x \log x \cdot \cos x}{x}\right\}$

D. none of these

Answer

Let $y=f(x)=x^{\sin x}$

Taking log both sides, we get

$\log_e y = \sin x \times \log_e x$-(1) (Since $\log_a b^c = c\log_a b$)

Differentiating (1) with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx} = \sin x \times \frac{1}{x} + \log_e x \times \cos x$$

$\Rightarrow \frac{dy}{dx} = y \times \left(\frac{\sin x}{x} + \log_e x \cos x\right)$

$\Rightarrow \frac{dy}{dx} = f'(x) = x^x\left(\frac{\sin x + x\log_e x \cos x}{x}\right)$

6. Question

Mark (√) against the correct answer in the following:

f $y = x^{\sqrt{x}}$ then $\frac{dy}{dx} = ?$

A. $\sqrt{x} \cdot x^{\left(\sqrt{x}-1\right)}$

B. $\frac{x^{\sqrt{x}} \log x}{2\sqrt{x}}$

C. $x^{\sqrt{x}}\left\{\frac{2 + \log x}{2\sqrt{x}}\right\}$

D. none of these

Answer

Let $y = f(x) = x^{\sqrt{x}}$

Taking log both sides, we get

$\log_e y = \sqrt{x} \times \log_e x$ -(1)

(Since $\log_a b^c = c\log_a b$)

Differentiating (1) with respect to x, we get

$\frac{1}{y}\frac{dy}{dx} = \sqrt{x} \times \frac{1}{x} + \log_e x \times \frac{1}{2\sqrt{x}}$

$\Rightarrow \frac{dy}{dx} = y \times \left(\frac{2 + \log_e x}{2\sqrt{x}}\right)$

$\Rightarrow \frac{dy}{dx} = f'(x)$

$= x^{\sqrt{x}}\left(\frac{2 + \log_e x}{2\sqrt{x}}\right)$

7. Question

Mark (√) against the correct answer in the following:

If $y = e^{\sin\sqrt{x}}$ then $\frac{dy}{dx} = ?$

A. $e^{\sin\sqrt{x}} \cdot \cos\sqrt{x}$

B. $\frac{e^{\sin\sqrt{x}} \cos\sqrt{x}}{2\sqrt{x}}$

C. $\frac{e^{\sin\sqrt{x}}}{2\sqrt{x}}$

D. none of these

Answer

Given that $y = e^{\sin\sqrt{x}}$

Taking log both sides, we get

$\log_e y = \sin\sqrt{x}$

$(\text{Since } \log_a b^c = c\log_a b)$

Differentiating with respect to x, we get

$\frac{1}{y}\frac{dy}{dx} = \cos\sqrt{x} \times \frac{1}{2\sqrt{x}}$

Or

$\frac{dy}{dx} = \cos\sqrt{x} \times \frac{1}{2\sqrt{x}} \times y$

Hence $\frac{dy}{dx} = \frac{e^{\sin\sqrt{x}}\cos\sqrt{x}}{2\sqrt{x}}$

8. Question

Mark (√) against the correct answer in the following:

If $y = (\tan x)^{\cot x}$ then $\frac{dy}{dx} = ?$

A. $\cot x \cdot (\tan x)^{\cot x - 1} \cdot \sec^2 x$

B. $-(\tan x)^{\cot x} \cdot \operatorname{cosec}^2 x$

C. $(\tan x)^{\cot x} \cdot \operatorname{cosec}^2 x(1 - \log \tan x)$

D. none of these

Answer

Given that $y = (\tan x)^{\cot x}$

Taking log both sides, we get

$\log_e y = \cot x \times \log_e \tan x$ (Since $\log_a b^c = c\log_a b$)

Differentiating with respect to x, we get

$\frac{1}{y}\frac{dy}{dx} = \cot x \times \frac{1}{\tan x} \times \sec^2 x - \log_e \tan x \times \operatorname{cosec}^2 x = \operatorname{cosec}^2 x(1 - \log_e \tan x)$

Hence, $\frac{dy}{dx} = \operatorname{cosec}^2 x(1 - \log_e \tan x \times y = \operatorname{cosec}^2 x(1 - \log_e \tan x)(\tan x)^{\cot x}$

9. Question

Mark (√) against the correct answer in the following:

If $y=(\sin x)^{\log x}$ then $\frac{dy}{dx}=?$

A. $(\log x)\cdot(\sin x)^{(\log x-1)}\cdot\cos x$

B. $(\sin x)^{\log x}\cdot\left\{\frac{x\log x+\log\sin x}{x}\right\}$

C. $(\sin x)^{\log x}\cdot\left\{\frac{(x\log x)\cot x+\log\sin x}{x}\right\}$

D. none of these

Answer

Given that $y=(\sin x)^{\log_e x}$

Taking log both sides, we get

$\log_e y=\log_e x\times\log_e \sin x$ (Since $\log_a b^c=c\log_a b$)

Differentiating with respect to x, we get

$$\frac{1}{y}\frac{dy}{dx}=\log_e x\times\frac{1}{\sin x}\times\cos x+\log_e \sin x\times\frac{1}{x}$$

$$=\frac{x\cot x\log_e x+\log_e \sin x}{x}$$

Hence, $\frac{dy}{dx}=\frac{x\cot x\log_e x+\log_e \sin x}{x}\times y$

$$=\frac{x\cot x\log_e x+\log_e \sin x}{x}(\sin x)^{\log_e x}$$

10. Question

Mark (√) against the correct answer in the following:

If $y=\sin(x^x)$ then $\frac{dy}{dx}=?$

A. $x^x\cos(x^x)$

B. $x^x\cos x^x(1+\log x)$

C. $x^x\cos x^x\log x$

D. none of these

Answer

Given that $y=\sin(x^x)$

Let $x^x=u$, then $y=\sin u$

Differentiating with respect to x, we get

$\frac{dy}{dx}=\cos u\times\frac{du}{dx}=\cos(x^x)\frac{du}{dx}$ –(1)

Also, $u=x^x$

Taking log both sides, we get

$\log_e u = x \times \log_e x$

(Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we get

$\frac{1}{u}\frac{du}{dx} = x \times \frac{1}{x} + \log_e x \times 1$

$\Rightarrow \frac{du}{dx} = u \times (1 + \log_e x)$

$\Rightarrow \frac{du}{dx} = x^x(1 + \log_e x)$ -(2)

From (1) and (2), we get

$\frac{dy}{dx} = \cos(x^x)\, x^x(1 + \log_e x)$

11. Question

Mark (√) against the correct answer in the following:

If $y = \sqrt{x \sin x}$ then $\frac{dy}{dx} = ?$

A. $\frac{(x \cos x + \sin x)}{2\sqrt{x \sin x}}$

B. $\frac{1}{2}(x \cos x + \sin x) \cdot \sqrt{x \sin x}$

C. $\frac{1}{2\sqrt{x \sin x}}$

D. none of these

Answer

Given that $y = \sqrt{x\sin x}$

Squaring both sides, we get

$y^2 = x\sin x$

Differentiating with respect to x, we get

$2y\frac{dy}{dx} = x\cos x + \sin x$ or $\frac{dy}{dx} = \frac{x\cos x + \sin x}{2y}$

Hence, $\frac{dy}{dx} = \frac{x\cos x+\sin x}{2\sqrt{x\sin x}}$

12. Question

Mark (√) against the correct answer in the following:

If $e^{x+y} = xy$ then $\frac{dy}{dx} = ?$

A. $\frac{x(1-y)}{y(x-1)}$

B. $\frac{y(1-x)}{x(y-1)}$

C. $\frac{(x-xy)}{(xy-y)}$

D. none of these

Answer

Given that $xy=e^{x+y}$

Taking log both sides, we get

$\log_e xy = x + y$ (Since $\log_a b^c = c\log_a b$)

Since $\log_a bc = \log_a b + \log_a c$, we get

$\log_e x + \log_e y = x + y$

Differentiating with respect to x, we get

$$\frac{1}{x}+\frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$$

Or

$$\frac{dy}{dx}\left(\frac{y-1}{y}\right) = \frac{1-x}{x}$$

Hence, $\frac{dy}{dx} = \frac{y(1-x)}{x(y-1)}$

13. Question

Mark (√) against the correct answer in the following:

If $(x+y) = \sin(x+y)$ then $\frac{dy}{dx} = ?$

A. -1

B. 1

C. $\frac{1-\cos(x+y)}{\cos^2(x+y)}$

D. none of these

Answer

Given that x+y=sin(x+y)

Differentiating with respect to x, we get

$1+\frac{dy}{dx} = \cos(x+y)\left(1+\frac{dy}{dx}\right)$ or $(\cos(x+y)-1)\left(1+\frac{dy}{dx}\right) = 0$

Hence, cos(x+y)=1 or $\frac{dy}{dx} = -1$

If cos(x+y)=1 then, x+y=2nπ, n∈ℤ

Hence x+y=sin(2nπ)=0 or y=-x

Differentiating with respect to x, we get

$\frac{dy}{dx} = -1$

Hence, $\frac{dy}{dx} = -1$

14. Question

Mark (√) against the correct answer in the following:

If $\sqrt{x} + \sqrt{y} = \sqrt{a}$ then $\frac{dy}{dx} = ?$

A. $\frac{-\sqrt{x}}{\sqrt{y}}$

B. $-\frac{1}{2} \cdot \frac{\sqrt{y}}{\sqrt{x}}$

C. $\frac{-\sqrt{y}}{\sqrt{x}}$

D. None of these

Answer

Given that $\sqrt{x} + \sqrt{y} = \sqrt{a}$

Differentiating with respect to x, we get

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \frac{dy}{dx} = 0$$

Or

$$\frac{dy}{dx} = -\sqrt{\frac{y}{x}}$$

15. Question

Mark (√) against the correct answer in the following:

If $x^y = y^x$ then $\frac{dy}{dx} = ?$

A. $\frac{(y - x \log y)}{(x - y \log x)}$

B. $\frac{y(y - x \log y)}{x(x - y \log x)}$

C. $\frac{y(y + x \log y)}{x(x + y \log x)}$

D. none of these

Answer

Given that $x^y = y^x$

Taking log both sides, we get

$y \log_e x = x \log_e y$

(Since $\log_a b^c = c \log_a b$)

Differentiating with respect to x, we get

$$\frac{y}{x} + \log_e x \frac{dy}{dx} = \frac{x}{y}\frac{dy}{dx} + \log_e y$$

$$\Rightarrow \frac{x - y\log_e x}{y}\frac{dy}{dx} = \frac{y - x\log_e y}{x}$$

Hence $\frac{dy}{dx} = \frac{y(y - x\log_e y)}{x(x - y\log_e x)}$

16. Question

Mark (√) against the correct answer in the following:

If $x^p y^q = (x + y)^{(p+q)}$ then $\frac{dy}{dx} = ?$

A. $\frac{x}{y}$

B. $\frac{y}{x}$

C. $\frac{x^{p-1}}{y^{q-1}}$

D. none of these

Answer

Given that $x^p y^q = (x+y)^{p+q}$

Taking log both sides, we get

$\log_e x^p y^q = (p + q) \log_e(x + y)$

(Since $\log_a b^c = c \log_a b$)

Since $\log_a bc = \log_a b + \log_a c$, we get

$\log_e x^p + \log_e y^q = (p + q) \log_e(x + y)$

$p \log_e x + q \log_e y = (p + q) \log_e(x + y)$

Differentiating with respect to x, we get

$$\frac{p}{x} + \frac{q}{y}\frac{dy}{dx} = \frac{p + q}{x + y}\left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx}\left(\frac{xq - yp}{y(x + y)}\right) = \frac{xq - yp}{x(x + y)}$$

Hence, $\frac{dy}{dx} = \frac{y}{x}$

17. Question

Mark (√) against the correct answer in the following:

If $y = x^2 \sin\frac{1}{x}$ then $\frac{dy}{dx} = ?$

A. $x\sin\frac{1}{x} - \cos\frac{1}{x}$

B. $-\cos\frac{1}{x} + 2x\sin\frac{1}{x}$

C. $-x\sin\frac{1}{x} + \cos\frac{1}{x}$

D. None of these

Answer

Given that $y = x^2 \sin\frac{1}{x}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = x^2\cos\frac{1}{x} \times -\frac{1}{x^2} + 2x\sin\frac{1}{x} = 2x\sin\frac{1}{x} - \cos\frac{1}{x}$$

18. Question

Mark (√) against the correct answer in the following:

If $y = \cos^2 x^3$ then $\frac{dy}{dx} = ?$

A. $-3x^2 \sin$

B. $-3x^2 \sin^2 x^3$

C. $-3x^2 \cos^2\left(2x^3\right)$

D. none of these

Answer

$y=\cos^2x^3=(\cos(x^3))^2$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = 2\cos(x^3) \times -\sin(x^3) \times 3x^2$$

Using 2sinAcosA=sin2A

$$\frac{dy}{dx} = -3x^2\sin(2x^3)$$

19. Question

Mark (√) against the correct answer in the following:

If $y = \log\left(x + \sqrt{x^2 + a^2}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{2\left(x + \sqrt{x^2 + a^2}\right)}$

B. $\dfrac{-1}{\sqrt{x^2+a^2}}$

C. $\dfrac{1}{\sqrt{x^2+a^2}}$

D. none of these

Answer

Given that $y=\log_e\left(x+\sqrt{x^2+a^2}\right)$

Differentiating with respect to x, we get

$$\frac{dy}{dx}=\frac{1}{x+\sqrt{x^2+a^2}}\left(1+\frac{1}{2\sqrt{x^2+a^2}}\times 2x\right)$$

Hence, $\frac{dy}{dx}=\frac{1}{x+\sqrt{x^2+a^2}}\times\frac{x+\sqrt{x^2+a^2}}{\sqrt{x^2+a^2}}=\frac{1}{\sqrt{x^2+a^2}}$

20. Question

Mark (√) against the correct answer in the following:

If $y=\log\left(\dfrac{1+\sqrt{x}}{1-\sqrt{x}}\right)$ then $\dfrac{dy}{dx}=?$

A. $\dfrac{1}{\sqrt{x}\,(1-x)}$

B. $\dfrac{-1}{x\left(1-\sqrt{x}\right)^2}$

C. $\dfrac{-\sqrt{x}}{2\left(1-\sqrt{x}\right)}$

D. none of these

Answer

Given that $y=\log_e\frac{1+\sqrt{x}}{1-\sqrt{x}}$

Differentiating with respect to x, we get

$$\frac{dy}{dx}=\frac{1-\sqrt{x}}{1+\sqrt{x}}\times\frac{(1-\sqrt{x})\times\frac{1}{2\sqrt{x}}-(1+\sqrt{x})\times-\frac{1}{2\sqrt{x}}}{(1-\sqrt{x})^2}=\frac{1}{(1-x)\sqrt{x}}$$

21. Question

Mark (√) against the correct answer in the following:

If $y=\log\left(\dfrac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right)$ then $\dfrac{dy}{dx}=?$

A. $\frac{2}{\sqrt{1+x^2}}$

B. $\frac{2\sqrt{1+x^2}}{x^2}$

C. $\frac{-2}{\sqrt{1+x^2}}$

D. none of these

Answer

Given that $y = \log_e\left(\frac{\sqrt{1+x^2}+x}{\sqrt{1+x^2}-x}\right)$

Differentiating with respect to x, we get

$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{1+x^2}-x}{\sqrt{1+x^2}+x} \times \frac{\left(\sqrt{1+x^2}-x\right)\times\left(\frac{1}{2\sqrt{1+x^2}}\times 2x+1\right)-\left(\sqrt{1+x^2}+x\right)\times\left(\frac{1}{2\sqrt{1+x^2}}\times 2x-1\right)}{\left(\sqrt{1+x^2}-x\right)^2}$

Hence, $\frac{dy}{dx} = \frac{2}{\sqrt{1+x^2}}$

22. Question

Mark (√) against the correct answer in the following:

If $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{2}\sec^2\left(\frac{\pi}{4}-\frac{\pi}{2}\right)$

B. $\frac{1}{2}\text{cosec}^2\left(\frac{\pi}{4}-\frac{\pi}{2}\right)$

C. $\frac{1}{2}\text{cosec}^2\left(\frac{\pi}{4}-\frac{\pi}{2}\right)\cot\left(\frac{\pi}{4}-\frac{\pi}{2}\right)$

D. none of these

Answer

Given that $y = \sqrt{\frac{1+\sin x}{1-\sin x}}$

Using, $\cos^2\theta + \sin^2\theta = 1$ and $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$

$$y = \sqrt{\frac{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{\sin^2\frac{x}{2}+\cos^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}}$$

$$= \frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}$$

Dividing by $\sin\frac{x}{2}$ in numerator and denominator, we get

$$y = \frac{\cot\frac{x}{2} + 1}{\cot\frac{x}{2} - 1} = \cot\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$\left(\text{Using } \cot\left(\frac{\pi}{4} - A\right) = \frac{\cot A + 1}{\cot A - 1}\right)$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = -\mathrm{cosec}^2\left(\frac{\pi}{4} - \frac{x}{2}\right) \times -\frac{1}{2}$$

Hence, $\frac{dy}{dx} = \frac{1}{2}\mathrm{cosec}^2\left(\frac{\pi}{4} - \frac{x}{2}\right)$

23. Question

Mark (√) against the correct answer in the following:

If $y = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$ then $\frac{dy}{dx} = ?$

A. $\sec^2 x$

B. $\frac{1}{2}\sec^2\frac{x}{2}$

C. $\frac{-1}{2}\mathrm{cosec}^2\frac{x}{2}$

D. none of these

Answer

Given that $y = \sqrt{\frac{\sec x - 1}{\sec x + 1}}$

Multiplying by cos x in numerator and denominator, we get

$$y = \sqrt{\frac{1 - \cos x}{1 + \cos x}}$$

Using $1 - \cos x = 2\sin^2\frac{x}{2}$ and $1 + \cos x = 2\cos^2\frac{x}{2}$, we get

$$y = \sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}$$

$$= \tan\left(\frac{x}{2}\right)$$

Differentiating with respect to x, we get

$$y = \sec^2\frac{x}{2} \times \frac{1}{2}$$

$$= \frac{1}{2}\sec^2\frac{x}{2}$$

24. Question

Mark (√) against the correct answer in the following:

If $y = \sqrt{\dfrac{1 + \tan x}{1 - \tan x}}$ then $\dfrac{dy}{dx} = ?$

A. $\dfrac{1}{2}\sec^2 x \cdot \tan\left(x + \dfrac{\pi}{4}\right)$

B. $\dfrac{\sec^2\left(x + \frac{\pi}{4}\right)}{2\sqrt{\tan\left(x + \frac{\pi}{4}\right)}}$

C. $\dfrac{\sec^2\left(\frac{x}{4}\right)}{\sqrt{\tan\left(x + \frac{\pi}{4}\right)}}$

D. none of these

Answer

Given that $y = \sqrt{\frac{1+\tan x}{1-\tan x}}$

Using $\tan\left(\frac{\pi}{4} + x\right) = \frac{1+\tan x}{1-\tan x}$, we get

$y = \sqrt{\tan\left(\frac{\pi}{4} + x\right)}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{2\sqrt{\tan\left(\frac{\pi}{4} + x\right)}} \times \sec^2\left(\frac{\pi}{4} + x\right) \times 1$$

Hence, $\frac{dy}{dx} = \frac{\sec^2\left(\frac{\pi}{4}+x\right)}{2\sqrt{\tan\left(\frac{\pi}{4}+x\right)}}$

25. Question

Mark (√) against the correct answer in the following:

If $y = \tan^{-1}\left(\dfrac{1 - \cos x}{\sin x}\right)$ then $\dfrac{dy}{dx} = ?$

A. 1

B. -1

C. $\dfrac{1}{2}$

D. $\dfrac{-1}{2}$

Answer

Given that $y = \tan^{-1}\left(\frac{1-\cos x}{\sin x}\right)$

Using $1-\cos x = 2\sin^2\frac{x}{2}$ and Using $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$, we get

$y = \tan^{-1}\left(\frac{2\sin^2\frac{x}{2}}{2\sin\frac{x}{2}\cos\frac{x}{2}}\right)$ or $y = \tan^{-1}\tan\frac{x}{2}$

$$y = \frac{x}{2}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{2}$$

26. Question

Mark (√) against the correct answer in the following:

If $y = \tan^{-1}\left\{\frac{\cos x + \sin x}{\cos x - \sin x}\right\}$ then $\frac{dy}{dx} = ?$

A. 1

B. -1

C. $\frac{1}{2}$

D. $\frac{-1}{2}$

Answer

Given that $y = \tan^{-1}\left(\frac{\cos x+\sin x}{\cos x-\sin x}\right)$

Dividing numerator and denominator with cosx, we get

$$y = \tan^{-1}\left(\frac{1+\tan x}{1-\tan x}\right)$$

Using $\tan\left(\frac{\pi}{4}+x\right) = \frac{1+\tan x}{1-\tan x}$, we get

$$y = \tan^{-1}\tan\left(\frac{\pi}{4}+x\right) = \frac{\pi}{4}+x$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = 1$$

27. Question

Mark (√) against the correct answer in the following:

If $y = \tan^{-1}\left\{\frac{\cos x}{1+\sin x}\right\}$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{2}$

B. $\frac{-1}{2}$

C. 1

D. -1

Answer

Given that $y = \tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)$

Using $\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$, $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$ and $\cos^2\theta + \sin^2\theta = 1$

Hence, $y = \tan^{-1}\left(\frac{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}{\cos^2\frac{x}{2} + \sin^2\frac{x}{2} + 2\sin\frac{x}{2}\cos\frac{x}{2}}\right) = \tan^{-1}\left(\frac{\left(\cos\frac{x}{2} - \sin\frac{x}{2}\right)\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)}{\left(\cos\frac{x}{2} + \sin\frac{x}{2}\right)^2}\right)$

$$\Rightarrow y = \tan^{-1}\frac{\cos\frac{x}{2} - \sin\frac{x}{2}}{\cos\frac{x}{2} + \sin\frac{x}{2}}$$

Dividing by $\cos\frac{x}{2}$ in numerator and denominator, we get

$$y = \tan^{-1}\frac{1 - \tan\frac{x}{2}}{1 + \tan\frac{x}{2}}$$

Using $\tan\left(\frac{\pi}{4} - x\right) = \frac{1-\tan x}{1+\tan x}$, we get

$$y = \tan^{-1}\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$= \frac{\pi}{4} - \frac{x}{2}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = -\frac{1}{2}$$

28. Question

Mark (√) against the correct answer in the following:

If $y = \tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$ then $\frac{dy}{dx} =$

A. $\frac{1}{2}$

B. $\frac{-1}{2}$

C. $\frac{1}{(1+x^2)}$

D. none of these

Answer

Given that $y = \tan^{-1}\sqrt{\frac{1-\cos x}{1+\cos x}}$

Using $1 - \cos x = 2\sin^2\frac{x}{2}$ and $1 + \cos x = 2\cos^2\frac{x}{2}$, we get

$$y = \tan^{-1}\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}} = \tan^{-1}\tan\left(\frac{x}{2}\right) = \frac{x}{2}$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{1}{2}$$

29. Question

Mark (√) against the correct answer in the following:

If $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{a}{b}$

B. $\frac{-b}{a}$

C. 1

D. -1

Answer

Given that $y = \tan^{-1}\left(\frac{a\cos x - b\sin x}{b\cos x + a\sin x}\right)$

Dividing by bcosx in numerator and denominator, we get

$$y = \tan^{-1}\left(\frac{\frac{a}{b} - \tan x}{1 + \frac{a}{b}\tan x}\right)$$

Let $\frac{a}{b} = \tan\alpha \Rightarrow \alpha = \tan^{-1}\frac{a}{b}$

Then $y = \tan^{-1}\left(\frac{\tan\alpha - \tan x}{1 + \tan\alpha\tan x}\right)$

Using $\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$, we get

$$y = \tan^{-1}\tan(\alpha - x) = \alpha - x = \tan^{-1}\frac{a}{b} - x$$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = -1$$

30. Question

Mark (√) against the correct answer in the following:

If $y = \sin^{-1}(3x - 4x^3)$ then $\frac{dy}{dx} = ?$

A. $\frac{3}{\sqrt{1 - x^2}}$

B. $\frac{-4}{\sqrt{1-x^2}}$

C. $\frac{3}{\sqrt{1+x^2}}$

D. none of these

Answer

Given that $y=\sin^{-1}(3x-4x^3)$

Let $x=\sin\theta$

$\Rightarrow \theta=\sin^{-1}x$

Then, $y=\sin^{-1}(3\sin\theta-4\sin^3\theta)$

Using $\sin3\theta=3\sin\theta-4\sin^3\theta$, we get

$y=\sin^{-1}(\sin3\theta)=3\theta=3\sin^{-1}x$

Differentiating with respect to x, we get

$$\frac{dy}{dx}=\frac{3}{\sqrt{1-x^2}}$$

31. Question

Mark (√) against the correct answer in the following:

If $y=\cos^{-1}\left(4x^3-3x\right)$ then $\frac{dy}{dx}=?$

A. $\frac{3}{\sqrt{1-x^2}}$

B. $\frac{-3}{\sqrt{1-x^2}}$

C. $\frac{4}{\sqrt{1-x^2}}$

D. $\frac{4}{\left(3x^2-1\right)}$

Answer

Given that $y=\cos^{-1}(4x^3-3x)$

Let $x=\cos\theta$

$\Rightarrow \theta=\cos^{-1}x$

Then, $y=\cos^{-1}(4\cos^3\theta-3\cos\theta)$

Using $\cos3\theta=4\cos^3\theta-3\cos\theta$, we get

$y=\cos^{-1}(\cos3\theta)=3=3\cos^{-1}x$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{-3}{\sqrt{1-x^2}}$$

32. Question

Mark (√) against the correct answer in the following:

If $y = \tan^{-1}\left(\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{1}{(1+x)}$

B. $\frac{1}{\sqrt{x}(1+x)}$

C. $\frac{2}{\sqrt{x}(1+x)}$

D. $\frac{1}{2\sqrt{x}(1+x)}$

Answer

Given that $y = \tan^{-1}\frac{\sqrt{a}+\sqrt{x}}{1-\sqrt{ax}}$

Let $\sqrt{a} = \tan A$ and $\sqrt{x} = \tan B$, then $A = \tan^{-1}\sqrt{a}$ and $B = \tan^{-1}\sqrt{x}$

Hence, $y = \tan^{-1}\frac{\tan A+\tan B}{1-\tan A\tan B}$

Using $\tan(A+B) = \frac{\tan A+\tan B}{1-\tan A\tan B}$, we get

$y=\tan^{-1}\tan(A+B)=A+B$

$= \tan^{-1}\sqrt{a} + \tan^{-1}\sqrt{x}$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = 0 + \frac{1}{1+(\sqrt{x})^2} \times \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}(1+x)}$$

33. Question

Mark (√) against the correct answer in the following:

If $y = \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{2}{(1+x^2)}$

B. $\frac{-2}{(1+x^2)}$

C. $\frac{2x}{\left(1+x^2\right)}$

D. none of these

Answer

Given that $y = \cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)$

$\Rightarrow \cos y = \frac{x^2-1}{x^2+1}$ or $\sec y = \frac{x^2+1}{x^2-1}$

Since $\tan^2 x = \sec^2 x - 1$, therefore

$$\tan^2 y = \left(\frac{x^2+1}{x^2-1}\right)^2 - 1$$

$$= \frac{4x^2}{(x^2-1)^2}$$

Hence, $\tan y = -\frac{2x}{1-x^2}$ or $y = \tan^{-1}\left(-\frac{2x}{1-x^2}\right)$

Let $x = \tan\theta$

$\Rightarrow \theta = \tan^{-1} x$

Hence, $y = \tan^{-1}\left(-\frac{2\tan\theta}{1-\tan^2\theta}\right)$

Using $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$, we get

$y = \tan^{-1}(-\tan 2\theta)$

Using $-\tan x = \tan(-x)$, we get

$y = \tan^{-1}(\tan(-2\theta))$

$= -2\theta$

$= -2\tan^{-1} x$

Differentiating with respect to x, we get

$$\frac{dy}{dx} = \frac{-2}{1+x^2}$$

34. Question

Mark (√) against the correct answer in the following:

If $y = \tan^{-1}\left(\frac{1+x^2}{1-x^2}\right)$ then $\frac{dy}{dx} = ?$

A. $\frac{2x}{\left(1+x^4\right)}$

B. $\frac{-2x}{\left(1+x^4\right)}$

C. $\frac{x}{\left(1+x^4\right)}$

D. none of these

Answer

Given that $y=\tan^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

Let $x^2=\tan\theta$

$\Rightarrow \theta=\tan^{-1}x^2$

Hence, $y=\tan^{-1}\left(\frac{1+\tan\theta}{1-\tan\theta}\right)$

Using $\tan\left(\frac{\pi}{4}+x\right)=\frac{1+\tan x}{1-\tan x}$, we get

$$y=\tan^{-1}\tan\left(\frac{\pi}{4}+\theta\right)=\frac{\pi}{4}+\theta=\frac{\pi}{4}+\tan^{-1}(x^2)$$

Differentiating with respect to x, we get

$$\frac{dy}{dx}=\frac{1}{1+x^4}\times 2x=\frac{2x}{1+x^4}$$

35. Question

Mark (√) against the correct answer in the following:

If $y=\tan^{-1}\left(-\sqrt{x}\right)$

A. $\frac{-1}{(1+x)}$

B. $\frac{2}{\sqrt{(1+x)}}$

C. $\frac{-1}{2\sqrt{x}\,(1+x)}$

D. none of these

Answer

Given that $y=\tan^{-1}(-\sqrt{x})$

Differentiating with respect to x, we get

$$\frac{dy}{dx}=\frac{1}{1+\left(-\sqrt{x}\right)^2}\times\frac{-1}{2\sqrt{x}}=\frac{-1}{2\sqrt{x}(1+x)}$$

36. Question

Mark (√) against the correct answer in the following:

If $y=\cos^{-1}x^3$ then $\frac{dy}{dx}=?$

A. $\frac{-1}{\sqrt{1-x^6}}$

B. $\frac{-3x^2}{\sqrt{1-x^6}}$

C. $\frac{-3}{x^2\sqrt{1-x^6}}$

D. none of these

Answer

Given that $y=\cos^{-1}x^3$

Differentiating with respect to x, we get

$$\frac{dy}{dx}=\frac{-1}{\sqrt{1-(x^3)^2}}\times 3x^2=\frac{-3x^2}{\sqrt{1-x^6}}$$

37. Question

Mark (√) against the correct answer in the following:

If $y=\tan^{-1}(\sec x+\tan x)$ then $\frac{dy}{dx}=?$

A. $\frac{1}{2}$

B. $\frac{-1}{2}$

C. 1

D. none of these

Answer

Given that $y=\tan^{-1}(\sec x + \tan x)$

Hence, $y=\tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$

Using $\cos x=\cos^2\frac{x}{2}-\sin^2\frac{x}{2}$, $\sin x=2\sin\frac{x}{2}\cos\frac{x}{2}$ and $\cos^2\theta+\sin^2\theta=1$

Hence, $y=\tan^{-1}\left(\frac{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}{\cos^2\frac{x}{2}-\sin^2\frac{x}{2}}\right)=\tan^{-1}\left(\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)}\right)$

$\Rightarrow y=\tan^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right)$

Dividing by $\cos\frac{x}{2}$ in numerator and denominator, we get

$$y=\tan^{-1}\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}$$

Using $\tan\left(\frac{\pi}{4}+x\right)=\frac{1+\tan x}{1-\tan x}$, we get

$$y=\tan^{-1}\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)=\frac{\pi}{4}+\frac{x}{2}$$

Differentiating with respect to x, we get