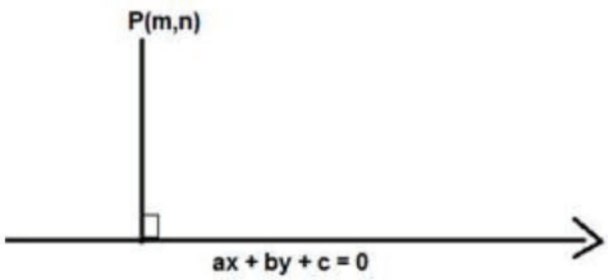
Q. 4. Find the distance of the point (2, 3) from the line y = 4.

Answer : Given: Point (2,3) and line y = 4

To find: The distance of the point (2, 3) from the line y = 4

Formula used: We know that the distance between a point P (m,n) and a line ax + by + c = 0 is given by,

$$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$



The equation of the line is y - 4 = 0

Here m= 2 and n = 3 , a = 0 , b = 1 , c = -4

$$D=\frac{|1(3)-4|}{\sqrt{0^2+1^2}}$$

$$D=\frac{|3-4|}{\sqrt{0+1}}=\frac{|-1|}{\sqrt{1}}=1$$

D = 1

The distance of the point (2, 3) from the line y = 4 is 1 units

Q. 5. Find the distance of the point (4, 2) from the line joining the points (4, 1) and (2, 3)

Answer : Given: Point (4,2) and the line joining the points (4, 1) and (2, 3)

To find: The distance of the point (4,2) from the line joining the points (4, 1) and (2, 3)

Formula used: The equation of the line joining the points (x_1,y_1) and (x_2,y_2) is given by

$$\frac{y - y_1}{y_2 - y_1} = \frac{x - x_1}{x_2 - x_1}$$

Here $x_1 = 4$ $y_1 = 1$ and $x_2 = 2$ $y_2 = 3$

$$\frac{y-1}{x-4} = \frac{3-1}{2-4} = \frac{2}{-2} = -1$$

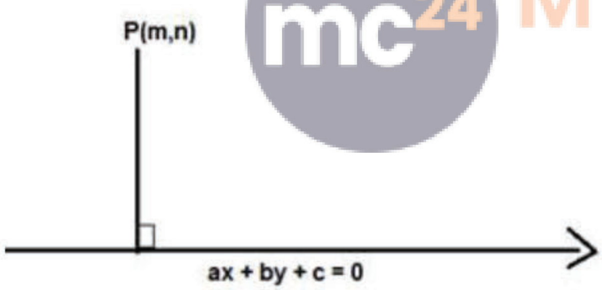
$$y - 1 = -x + 4$$

$$x + y - 5 = 0$$

The equation of the line is $x + y - 5 = 0$

Formula used: We know that the distance between a point P (m,n) and a line ax + by + c = 0 is given by,

$$D = \frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$



The equation of the line is $x + y - 5 = 0$
Here m = 4 and n = 2 , a = 1 , b = 1 , c = -5

$$D = \frac{|1(4)+1(2)-5|}{\sqrt{1^2+1^2}}$$

$$D = \frac{|4+2-5|}{\sqrt{1+1}} = \frac{|6-5|}{\sqrt{2}} = \frac{|1|}{\sqrt{2}}$$

$$D = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

The distance of the point (4,2) from the line joining the points (4, 1) and (2, 3) is $\frac{\sqrt{2}}{2}$ units

Q. 6. Find the length of the perpendicular from the origin to each of the following lines :

(i) 7x + 24y = 50
(ii) 4x + 3y = 9
(iii) x = 4

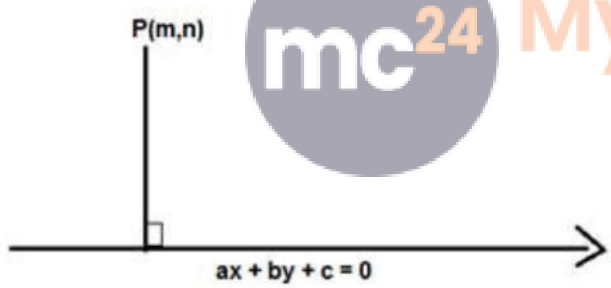
Answer : Given: Point (0,0) and line 7x + 24y = 50

To find: The length of the perpendicular from the origin to the line 7x + 24y = 50

Formula used:

We know that the length of the perpendicular from P (m,n) to the line ax + by + c = 0 is given by,

$$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$



The given equation of the line is 7x + 24y - 50=0

Here m= 0 and n = 0 , a = 7 , b = 24 , c = -50

$$D=\frac{|7(0)+24(0)-50|}{\sqrt{7^2+24^2}}$$

$$D=\frac{|0+0-50|}{\sqrt{49+576}}=\frac{|-50|}{\sqrt{625}}=\frac{|-50|}{25}=\frac{50}{25}=2$$

D = 2

The length of perpendicular from the origin to the line 7x + 24y = 50 is 2 units

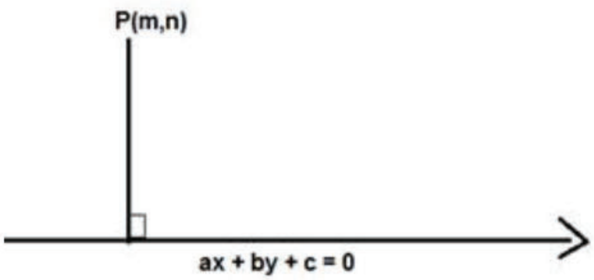
(ii) Given: Point (0,0) and line 4x + 3y = 9

To find: The length of perpendicular from the origin to the line 4x + 3y = 9

Formula used:

We know that the length of perpendicular from P (m,n) to the line ax + by + c = 0 is given by,

$$D = \frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$



The given equation of the line is 4x + 3y - 9=0

Here m= 0 and n = 0 , a = 4 , b = 3 , c = -9

$$D = \frac{|4(0)+3(0)-9|}{\sqrt{4^2+3^2}}$$

$$D = \frac{|0+0-9|}{\sqrt{16+9}} = \frac{|-9|}{\sqrt{25}} = \frac{|-9|}{5} = \frac{9}{5}$$

$$D = \frac{9}{5}$$

The length of perpendicular from the origin to the line 4x + 3y = 9 is $\frac{9}{5}$ units

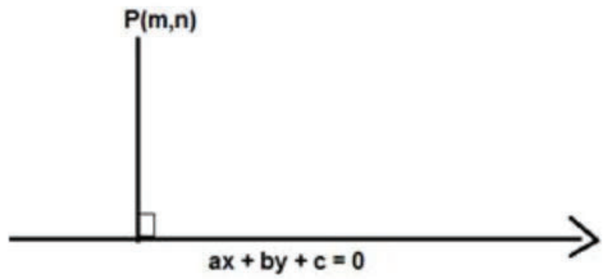
(iii) Given: Point (0,0) and line x = 4

To find: The length of perpendicular from the origin to the line x = 4

Formula used: We know that the length of perpendicular from (m,n) to the line ax + by + c = 0 is given by,

$$D = \frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$

$D = \frac{|am+bn+c|}{\sqrt{a^2+b^2}}$



The given equation of the line is x - 4=0

Here m= 0 and n = 0 , a = 1 , b = 0 , c = -4

$D = \frac{|1(0)+0(0)-4|}{\sqrt{1^2+0^2}}$

$D = \frac{|0+0-4|}{\sqrt{1+0}} = \frac{|-4|}{\sqrt{1}} = \frac{|-4|}{1} = 4$

D = 4

The length of perpendicular from the origin to the line x = 4 is 4 units

Q. 7. Prove that the product of the lengths of perpendiculars drawn from the points

$A(\sqrt{a^2-b^2},0)$ **and** $B(-\sqrt{a^2-b^2},0)$ **to the line** $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$**, is** b^2

Answer :

Given: Point $A(\sqrt{a^2-b^2},0)$, $B(-\sqrt{a^2-b^2},0)$ and line $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$

To Prove: The product of the lengths of perpendiculars drawn from the points

$A(\sqrt{a^2-b^2},0)$ and $B(-\sqrt{a^2-b^2},0)$ to the line $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$, is b^2

Formula used:

We know that the length of the perpendicular from (m,n) to the line ax + by + c = 0 is given by,

$$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$

The equation of the line is $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta-1=0$

At point A, m= $\sqrt{a^2-b^2}$ and n= 0, $a=\frac{\cos\theta}{a}$ $b=\frac{\sin\theta}{b}$ $c=-1$

$$D_1=\frac{\left|\frac{\cos\theta}{a}\left(\sqrt{a^2-b^2}\right)+\frac{\sin\theta}{b}(0)-1\right|}{\sqrt{(\frac{\cos\theta}{a})^2+(\frac{\sin\theta}{b})^2}}$$

$$D_1=\frac{\left|\frac{\cos\theta}{a}\left(\sqrt{a^2-b^2}\right)-1\right|}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}$$

At point B, m= $-\sqrt{a^2-b^2}$ and n= 0, $a=\frac{\cos\theta}{a}$ $b=\frac{\sin\theta}{b}$ $c=-1$

$$D_2=\frac{\left|\frac{\cos\theta}{a}\left(-\sqrt{a^2-b^2}\right)+\frac{\sin\theta}{b}(0)-1\right|}{\sqrt{(\frac{\cos\theta}{a})^2+(\frac{\sin\theta}{b})^2}}$$

$$D_2=\frac{\left|\frac{\cos\theta}{a}\left(-\sqrt{a^2-b^2}\right)-1\right|}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}=\frac{\left|\frac{\cos\theta}{a}\left(\sqrt{a^2-b^2}\right)+1\right|}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}}$$

Product of the lengths of perpendiculars drawn from the points A and B is $D_1 \times D_2$

$$D_1 \times D_2 = \frac{\left|\frac{\cos\theta}{a}\left(\sqrt{a^2-b^2}\right)-1\right|}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}} \times \frac{\left|\frac{\cos\theta}{a}\left(\sqrt{a^2-b^2}\right)+1\right|}{\sqrt{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}} = \frac{\left|\frac{\cos^2\theta}{a^2}(a^2-b^2)-1\right|}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}$$

(In the numerator we have $(x-y) \times (x+y) = x^2 + y^2$ and $\sin^2\theta + \cos^2\theta$)

$$D_1 \times D_2 = \frac{\left|\frac{\cos^2\theta \times a^2}{a^2}+\frac{\cos^2\theta \times (-b^2)}{a^2}-\cos^2\theta-\sin^2\theta\right|}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}} = \frac{\left|\cos^2\theta+\frac{\cos^2\theta \times (-b^2)}{a^2}-\cos^2\theta-\sin^2\theta\right|}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}$$

$$D_1 \times D_2 = \frac{\left|\frac{\cos^2\theta \times (-b^2)}{a^2}-\sin^2\theta\right|}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}} = b^2 \times \frac{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}}{\frac{\cos^2\theta}{a^2}+\frac{\sin^2\theta}{b^2}} = b^2$$

$D_1 \times D_2 = b^2$

Product of the lengths of perpendiculars drawn from the points A and B is b^2

Q. 8. Find the values of k for which the length of the perpendicular from the point (4, 1) on the line 3x – 4y + k = 0 is 2 units

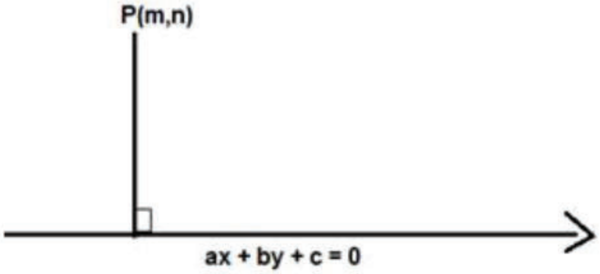
Answer : Given: Point (4,1) , line 3x – 4y + k = 0 and length of perpendicular is 2 units

To find: The values of k

Formula used:

We know that the length of the perpendicular from (m,n) to the line ax + by + c = 0 is given by,

$$D = \frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$



The equation of the line is 3x – 4y + k =0

Here m= 4 and n= 1 , a = 3 , b = -4 , c = k and D=2 units

$$D=\frac{|3(4)-4(1)+k|}{\sqrt{3^2+4^2}}=2$$

$$D=\frac{|12-4+k|}{\sqrt{9+16}}=\frac{|8+k|}{\sqrt{25}}=\frac{|8+k|}{5}=2$$

$$|8+k|=2\times 5=10$$

$8+k=10.$ or $8+k=-10$

$k=10-8$ or $k=-10-8$

$k=2$ or $k=-18$

The values of k are 2 and -18

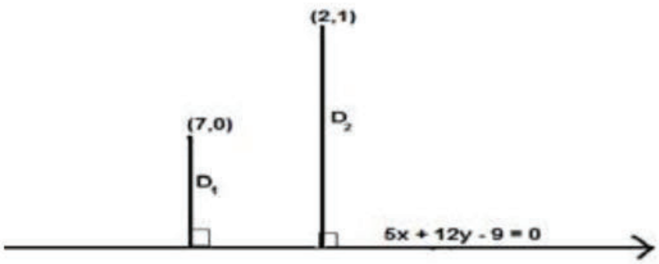
Q. 9. Show that the length of the perpendicular from the point (7, 0) to the line 5x + 12y – 9 = 0 is double the length of perpendicular to it from the point (2, 1)

Answer : Given: Points (7,0) and (2,1) , line 5x + 12y – 9 = 0

To Prove : length of the perpendicular from the point (7, 0) to the line 5x + 12y – 9 = 0 is double the length of perpendicular to it from the point (2, 1)

Formula used: We know that the length of the perpendicular from (m,n) to the line ax + by + c = 0 is given by,

$$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$



Let D_1 be the length of perpendicular from the point (7, 0) to the line 5x + 12y – 9 = 0

The given equation of the line is 5x + 12y – 9 = 0

Here at point (7,0) m= 7 and n= 0 , a = 5 , b = 12 , c = -9

$$D_1=\frac{|5(7)+12(0)-9|}{\sqrt{5^2+12^2}}$$

$$D_1=\frac{|35+0-9|}{\sqrt{25+144}}=\frac{26}{\sqrt{169}}=\frac{26}{13}=2$$

D_1=2

Let D_2 be the length of perpendicular from the point (2, 1) to the line 5x + 12y – 9 = 0

The given equation of the line is 5x + 12y – 9 = 0

Here at point (2,1) m= 2 and n= 1 , a = 5 , b = 12 , c = -9

$$D_2 = \frac{|5(2)+12(1)-9|}{\sqrt{5^2+12^2}}$$

$$D_2 = \frac{|10+12-9|}{\sqrt{25+144}} = \frac{22-9}{\sqrt{169}} = \frac{13}{13} = 1$$

$D_2 = 1$

$D_1 = 2D_2 = 2$

Thus the length of the perpendicular from the point (7, 0) to the line 5x + 12y – 9 = 0 is double the length of perpendicular to it from the point (2, 1)

Q. 10. The points A(2, 3), B(4, -1) and C(-1, 2) are the vertices of ΔABC. Find the length of the perpendicular from C on AB and hence find the area of ΔABC

Answer : Given: points A(2, 3), B(4, -1) and C(-1, 2) are the vertices of ΔABC

To find : length of the perpendicular from C on AB and the area of ΔABC

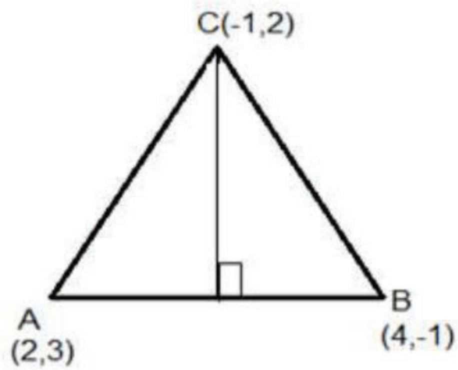
Formula used:

We know that the length of the perpendicular from (m,n) to the line ax + by + c = 0 is given by,

$$D = \frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$

The equation of the line joining the points (x_1,y_1) and (x_2,y_2) is given by

$$\frac{y-y_1}{y_2-y_1} = \frac{x-x_1}{x_2-x_1}$$



The equation of the line joining the points A(2,3) and B(4,-1) is

Here x_1 =2 y_1 =3 and x_2=4 y_2=-1

$$\frac{y-3}{x-2}=\frac{-1-3}{4-2}=\frac{-4}{2}=-2$$

$$y-3=-2x+4$$

$$2x+y-7=0$$

The equation of the line is 2x + y – 7 =0

The length of perpendicular from C(-1, 2) to the line AB

The given equation of the line is 2x + y – 7 =0

Here m= -1 and n= 2 , a = 2 , b = 1 , c = -7

$$D=\frac{|2(-1)+1(2)-7|}{\sqrt{2^2+1^2}}$$

$$D=\frac{-2+2-7}{\sqrt{4+1}}=\frac{|-7|}{\sqrt{5}}=\frac{|-7|}{\sqrt{5}}=\frac{7}{\sqrt{5}}$$

$$D=\frac{7}{\sqrt{5}}$$

The length of the perpendicular from C on AB is $\frac{7}{\sqrt{5}}$ units.

Height of the triangle is $\frac{7}{\sqrt{5}}$ units

The distance between points A(x_1, y_1) and B(x_2, y_2) is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Here x1=2 and y1=3 ,x2=4 and y2=-1

$$AB = \sqrt{(4-2)^2 + (-1-3)^2} = \sqrt{2^2 + (-4)^2} = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

Base AB = $2\sqrt{5}$ units

Area of the triangle = $\frac{1}{2} \times \text{BASE} \times \text{HEIGHT}$

Area of the triangle ABC = $\frac{1}{2} \times \text{AB} \times \text{HEIGHT} = \frac{1}{2} \times 2\sqrt{5} \times \frac{7}{\sqrt{5}} = 7$

Area of the triangle ABC = 7 square units

Q. 11. What are the points on the x-axis whose perpendicular distance from the line $\frac{x}{3} + \frac{y}{4} = 1$ is 4 units?

Answer : Given: perpendicular distance is 4 units and line

$$\frac{x}{3} + \frac{y}{4} = 1$$

To find : points on the x-axis

Formula used:

We know that the length of the perpendicular from (m,n) to the line ax + by + c = 0 is given by,

$$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$

The equation of the line is 4x + 3y - 12=0

Any point on the x-axis is given by (x,0)

Here m= x and n= 0 , a = 4 , b = 3 , c = -12 and D=4 units

$$D=\frac{|4(x)+3(0)-12|}{\sqrt{4^2+3^2}}=4$$

$$D=\frac{|4x-12|}{\sqrt{16+9}}=\frac{|4x-12|}{\sqrt{25}}=\frac{|4x-12|}{5}=4$$

|4x-12| = 4 × 5 = 20

4x - 12 = 20 or 4x - 12 = -20

4x = 20 + 12 or 4x = -20 + 12

4x = 32 or 4x = -8

x = 32/4 = 8 or x = (-8)/4 = -2

(8,0) and (2,0)are the points on the x-axis whose perpendicular distance from the line is 4 units

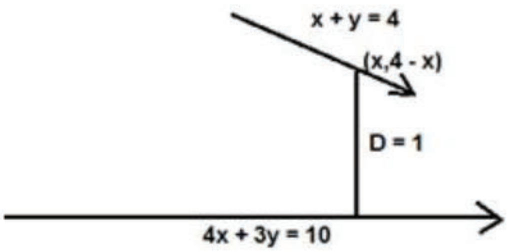
Q. 12. Find all the points on the line x + y = 4 that lie at a unit distance from the line 4x+3y=10.

Answer : Given: points lie on the line x + y = 4 , perpendicular distance = 1 units

To find : points on the line x + y = 4

Formula used: We know that the distance between a point (m,n) and a line ax + by + c = 0 is given by,

$$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$



The equation of the line is 4x + 3y – 10 = 0 and D=1 units

Here m= x and n= 4 - x (from the equation x + y = 4),a = 4 , b = 3 , c = -10

$$D=\frac{|4(x)+3(4-x)-10|}{\sqrt{4^2+3^2}}=1$$

$$D=\frac{|4x+12-3x-10|}{\sqrt{16+9}}=\frac{|x-2|}{\sqrt{25}}=\frac{|x-2|}{5}=1$$

$$|x-2|=1\times5=5$$

$x-2=5$ or $x-2=-5$

$x=5+2$ or $x=-5+2$

$x=7$ or $x=-3$

We know that the points lie on the line x + y = 4

y = 4 – 7 = -3 or y=4-(-3) =7

(7,-3) and (-3,7) are the points on the line x + y = 4 that lie at a unit distance from

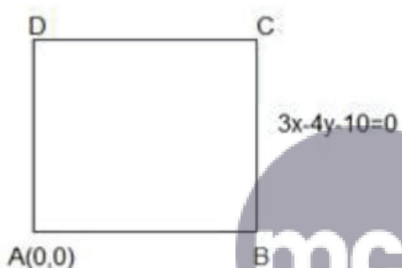
4x + 3y = 10.

Q. 13. A vertex of a square is at the origin and its one side lies along the line 3x – 4y – 10 = 0.

Find the area of the square.

Answer : Given: ABCD is a square and equation of BC is 3x – 4y – 10 = 0

To find : Area of the square



Formula used:

We know that the length of perpendicular from (m,n) to the line ax + by + c = 0 is given by,

$$D=\frac{|am+bn+c|}{\sqrt{a^2+b^2}}$$

The given equation of the line is 3x – 4y – 10 = 0

Here m= 0 and n= 0 , a = 3 , b = -4 , c = -10

The given equation of the line is 3x – 4y – 10 = 0

Here m= 0 and n= 0 , a = 3 , b = -4 , c = -10

$$D = \frac{|3(0) - 4(0) - 10|}{\sqrt{3^2 + 4^2}}$$

$$D = \frac{|0 + 0 - 10|}{\sqrt{9 + 16}} = \frac{|-10|}{\sqrt{25}} = \frac{|-10|}{5} = \frac{10}{5} = 2$$

D = 2

Side of the square=D=2

Area of the square = 2×2 = 4 square units

Area of the square = 4 square units

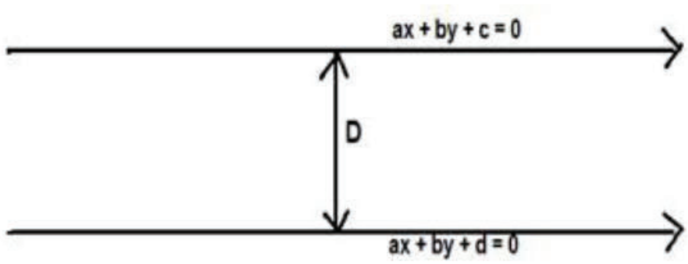
Q. 14. Find the distance between the parallel lines 4x – 3y + 5 = 0 and 4x – 3y + 7 = 0

Answer : Given: parallel lines 4x – 3y + 5 = 0 and 4x – 3y + 7 = 0

To find : distance between the parallel lines

Formula used : The distance between the parallel lines ax + by + c =0 and ax + by + d =0 is,

$$D = \frac{|d - c|}{\sqrt{a^2 + b^2}}$$



Here a = 4 ,b = -3 ,c = 5 ,d = 7

$$D = \frac{|7-5|}{\sqrt{4^2 + (-3)^2}} = \frac{|2|}{\sqrt{16+9}} = \frac{2}{\sqrt{25}} = \frac{2}{5}$$

The distance between the parallel lines 4x – 3y + 5 = 0 and 4x – 3y + 7 = 0 is $\frac{2}{5}$ units

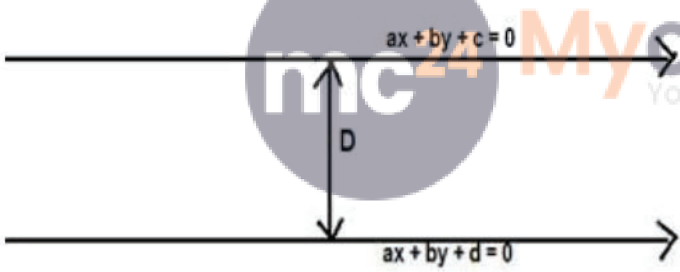
Q. 15. Find the distance between the parallel lines 8x + 15y – 36 = 0 and 8x + 15y + 32 = 0.

Answer : Given: parallel lines 8x + 15y – 36 = 0 and 8x + 15y + 32 = 0.

To find : distance between the parallel lines

Formula used : The distance between the parallel lines ax + by + c =0 and ax + by + d =0 is,

$$D = \frac{|d-c|}{\sqrt{a^2 + b^2}}$$



Here a = 8 ,b = 15 ,c = -36 ,d = 32

$$D = \frac{|32-(-36)|}{\sqrt{8^2 + 15^2}} = \frac{|32+36|}{\sqrt{64+225}} = \frac{68}{\sqrt{289}} = \frac{68}{17} = 4$$

The distance between the parallel lines 8x + 15y – 36 = 0 and 8x + 15y + 32 =0 is 4 Units

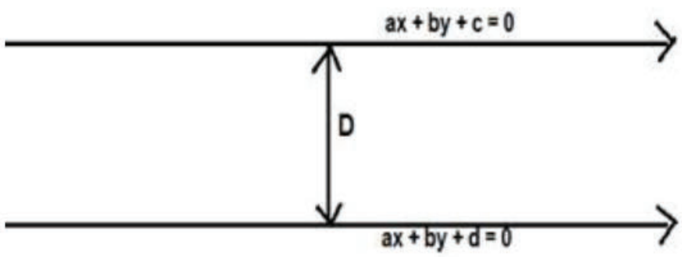
Q. 16. Find the distance between the parallel lines y = mx + c and y = mx + d

Answer : Given: parallel lines y = mx + c and y = mx + d

To find : distance between the parallel lines

Formula used : The distance between the parallel lines ax + by + c =0 and ax + by + d =0 is,

$$D = \frac{|d - c|}{\sqrt{a^2 + b^2}}$$



The parallel lines are mx – y + c = 0 and mx – y + d=0

Here a = m ,b = -1 ,c = c ,d = d

$$D = \frac{|d - c|}{\sqrt{m^2 + 1^2}} = \frac{|d - c|}{\sqrt{m^2 + 1}}$$

The distance between the parallel lines y = mx + c and y = mx + d is $\frac{|d - c|}{\sqrt{m^2 + 1}}$ units

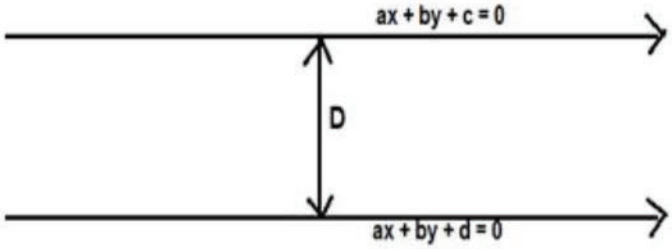
Q. 17. Find the distance between the parallel lines p(x + y) = q = 0 and p(x + y) – r =0

Answer : Given: parallel lines p(x + y) = q = 0 and p(x + y) – r =0

To find : distance between the parallel lines p(x + y) - q = 0 and p(x + y) – r =0

Formula used : The distance between the parallel lines ax + by + c =0 and ax + by + d =0 is,

$$D = \frac{|d - c|}{\sqrt{a^2 + b^2}}$$



The parallel lines are p(x + y) - q = 0 and p(x + y) – r =0

The parallel lines are px + py - q = 0 and px + py – r = 0

Here a = p ,b = p ,c = -q ,d = -r

$$D = \frac{|-r-(-q)|}{\sqrt{p^2+p^2}} = \frac{|-r+q|}{\sqrt{2p^2}} = \frac{|q-r|}{p\sqrt{2}}$$

The distance between the parallel lines p(x + y) =q = 0 and p(x + y) – r = 0 is $\frac{|q-r|}{p\sqrt{2}}$ units

Q. 18. Prove that the line 12x – 5y – 3 = 0 is mid-parallel to the lines 12x – 5y + 7 = 0 and 12x – 5y – 13 = 0

Answer : Given: parallel lines 12x – 5y – 3 = 0, 12x – 5y + 7 = 0, 12x – 5y – 13 = 0

To Prove : line 12x – 5y – 3 = 0 is mid-parallel to the lines 12x – 5y + 7 = 0 and 12x – 5y – 13 = 0

Formula used : The distance between the parallel lines ax + by + c =0 and ax + by + d =0 is,

$$D = \frac{|d-c|}{\sqrt{a^2+b^2}}$$

The equation of line l is 12x – 5y + 7 = 0

The equation of line m is 12x – 5y – 3 = 0

The equation of line n is 12x – 5y – 13 = 0

Let D_1 be the distance between the lines l and m .

Here a = 12 ,b = -5 ,c = 7 ,d = -3

$$D_1 = \frac{|-3-7|}{\sqrt{12^2+(-5)^2}} = \frac{|-10|}{\sqrt{144+25}} = \frac{10}{\sqrt{169}} = \frac{10}{13}.$$

The distance between the parallel lines l and m is $\frac{10}{13}$ units

Let D_2 be the distance between the lines m and n .

Here a = 12 ,b = -5 ,c = 7 ,d = -3

$$D_2 = \frac{|-13-(-3)|}{\sqrt{12^2+(-5)^2}} = \frac{|-13+3|}{\sqrt{144+25}} = \frac{|-10|}{\sqrt{169}} = \frac{10}{13}$$

The distance between the parallel lines m and n is $\frac{10}{13}$ units

Distance between the parallel lines l and m = Distance between the parallel lines m and n

Thus the line 12x – 5y – 3 = 0 is mid-parallel to the lines 12x – 5y + 7 = 0 and 12x – 5y – 13 = 0

Q. 19. The perpendicular distance of a line from the origin is 5 units, and its slope is -1. Find the equation of the line.

Answer :

Given: perpendicular distance from orgin is 5 units, and the slope is -1

To find : the equation of the line

Formula used :

We know that the perpendicular distance from a point (x_0, y_0) to the line ax + by + c = 0 is given by

$$D = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$$

The equation of a straight line is given by y=mx+c where m denotes the slope of the line.

The equation of the line is mx – y + c =0

Here x_0 = 0 and y_0= 0 , a = m , b = -y , c = c and D=5 units

$$D = \frac{|m(0) - 1(0) + c|}{\sqrt{m^2 + 1^2}} = \frac{|c|}{\sqrt{m^2 + 1}} = \frac{c}{\sqrt{m^2 + 1}} = 5$$

Slope of the line = m = -1 ,Substituting in the above equation we get,

$$\frac{c}{\sqrt{(-1)^2 + 1^2}} = 5$$

$$\frac{c}{\sqrt{1+1}} = \frac{c}{\sqrt{2}} = 5$$

$$c = 5\sqrt{2}$$

Thus the equation of the straight line is $y = -x + 5\sqrt{2}$ or $x + y - 5\sqrt{2} = 0$

Exercise 20I

Q. 1. Find the points of intersection of the lines 4x + 3y = 5 and x = 2y – 7.

Answer : Suppose the given two lines intersect at a point $P(x_1, y_1)$. Then, (x_1, y_1) satisfies each of the given equations.

∴ 4x + 3y = 5

or 4x + 3y – 5 = 0 …(i)

and x = 2y – 7

or x – 2y + 7 = 0 …(ii)

Now, we find the point of intersection of eq. (i) and (ii)

Multiply the eq. (ii) by 4, we get

4x – 8y + 28 = 0 …(iii)

On subtracting eq. (iii) from (i), we get

4x – 8y + 28 – 4x – 3y + 5 = 0

⇒ -11y + 33 = 0

⇒ -11y = -33

$$\Rightarrow y = \frac{33}{11} = 3$$

Putting the value of y in eq. (i), we get

4x + 3(3) – 5 = 0

⇒ 4x + 9 – 5 = 0

⇒ 4x + 4 = 0

⇒ 4x = -4

⇒ x = -1

Hence, the point of intersection $P(x_1, y_1)$ is (-1, 3)

Q. 2. Show that the lines x + 7y = 23 and 5x + 2y = a 16 intersect at the point (2, 3).

Answer : Suppose the given two lines intersect at a point P(2, 3). Then, (2, 3) satisfies each of the given equations.

So, taking equation $x + 7y = 23$

Substituting $x = 2$ and $y = 3$

Lhs = $x + 7y$

$= 2 + 7(3)$

$= 2 + 21$

$= 23$

= RHS

Now, taking equation $5x + 2y = 16$

Substituting $x = 2$ and $y = 3$

LHS = $5x + 2y$

$= 5(2) + 2(3)$

$= 10 + 6$

$= 16$

= RHS

In both the equations pair (2, 3) for (x, y) satisfies the given equations, therefore both lines pass through (2, 3).

Q. 3. Show that the lines $3x – 4y + 5 = 0$, $7x – 8y + 5 = 0$ and $4x + 5y = 45$ are concurrent. Also find their point of intersection.

Answer : Given: $3x – 4y + 5 = 0$,

$7x – 8y + 5 = 0$

and $4x + 5y = 45$

or $4x + 5y – 45 = 0$

To show: Given lines are concurrent

The lines $a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + c_1 = 0$ and $a_1x + b_1y + c_1 = 0$ are concurrent if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

We know that,

We have,

$a_1 = 3$, $b_1 = -4$, $c_1 = 5$

$a_2 = 7$, $b_2 = -8$, $c_2 = 5$

$a_3 = 4$, $b_2 = 5$, $c_3 = -45$

$$\Rightarrow \begin{vmatrix} 3 & -4 & 5 \\ 7 & -8 & 5 \\ 4 & 5 & -45 \end{vmatrix}$$

Now, expanding along first row, we get

$\Rightarrow 3[(-8)(-45) – (5)(5)] – (-4)[(7)(-45) – (4)(5)] + 5[(7)(5) – (4)(-8)]$

$\Rightarrow 3[360 – 25] + 4[-315 – 20] + 5[35 + 32]$

$\Rightarrow 3[335] + 4[-335] + 5[67]$

$\Rightarrow 1005 – 1340 + 335$

$\Rightarrow 1340 – 1340$

$= 0$

So, the given lines are concurrent.

Now, we have to find the point of intersection of the given lines

$3x – 4y + 5 = 0$,

$7x – 8y + 5 = 0$

and $4x + 5y - 45 = 0$ …(A)

We know that, if three lines are concurrent the point of intersection of two lines lies on the third line.

So, firstly, we find the point of intersection of two lines

$3x - 4y + 5 = 0$, …(i)

$7x - 8y + 5 = 0$ …(ii)

Multiply the eq. (i) by 2, we get

$6x - 8y + 10 = 0$ …(iii)

On subtracting eq. (iii) from (ii), we get

$7x - 8y + 5 - 6x + 8y - 10 = 0$

$\Rightarrow x - 5 = 0$

$\Rightarrow x = 5$

Putting the value of x in eq. (i), we get

$3(5) - 4y + 5 = 0$

$\Rightarrow 15 - 4y + 5 = 0$

$\Rightarrow 20 - 4y = 0$

$\Rightarrow -4y = -20$

$\Rightarrow y = 5$

Thus, the first two lines intersect at the point (5, 5). Putting x = 5 and y = 5 in eq. (A), we get

$4(5) + 5(5) - 45$

$= 20 + 25 - 45$

$= 45 - 45$

$= 0$

So, point (5, 5) lies on line $4x + 5y - 45 = 0$

Hence, the point of intersection is (5, 5)

Q. 4. Find the value of k so that the lines $3x - y - 2 = 0$, $5x + ky - 3 = 0$ and $2x + y - 3 = 0$ are concurrent.

Answer : Given that $3x - y - 2 = 0$,

$5x + ky - 3 = 0$

and $2x + y - 3 = 0$ are concurrent

We know that,

The lines $a_1x + b_1y + c_1 = 0$, $a_1x + b_1y + c_1 = 0$ and $a_1x + b_1y + c_1 = 0$ are concurrent if

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$$

It is given that the given lines are concurrent.

$$\Rightarrow \begin{vmatrix} 3 & -1 & -2 \\ 5 & k & -3 \\ 2 & 1 & -3 \end{vmatrix} = 0$$

Now, expanding along first row, we get

$\Rightarrow 3[(k)(-3) - (-3)(1)] - (-1)[(5)(-3) - (-3)(2)] + (-2)[5 - 2k] = 0$

$\Rightarrow 3[-3k + 3] + 1[-15 + 6] - 2[5 - 2k] = 0$

$\Rightarrow -9k + 9 - 9 - 10 + 4k = 0$

$\Rightarrow -5k - 10 = 0$

$\Rightarrow -5k = 10$

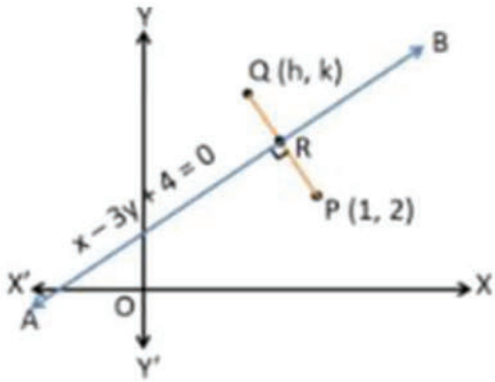
$\Rightarrow k = -2$

Hence, the value of k = -2

Q. 5. Find the image of the point P(1, 2) in the line x – 3y + 4 = 0.

Answer : Let line AB be x – 3y + 4 = 0 and point P be (1, 2)

Let the image of the point P(1, 2) in the line mirror AB be Q(h, k).



Since line AB is a mirror. Then PQ is perpendicularly bisected at R.

Since R is the midpoint of PQ.

We know that,

Midpoint of a line joining (x_1, y_1) & $(x_2, y_2) = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$

So, Midpoint of the line joining (1, 2) & (h, k) $= \frac{1+h}{2}, \frac{2+k}{2}$

Since point R lies on the line AB. So, it will satisfy the equation of line AB x – 3y + 4 = 0

Substituting the $x = \frac{1+h}{2}$ & $y = \frac{2+k}{2}$ in abthe ove equation, we get

$$\frac{1+h}{2} - 3\left(\frac{2+k}{2}\right) + 4 = 0$$

$$\Rightarrow \frac{1+h-6-3k+8}{2} = 0$$

⇒ 3 + h – 3k = 0

⇒ h – 3k = -3 …(i)

Also, PQ is perpendicular to AB

We know that, if two lines are perpendicular then the product of their slope is equal to -1

∴ Slope of AB × Slope of PQ = -1

$$\Rightarrow \text{Slope of PQ} = \frac{-1}{\text{Slope of AB}}$$

Now, we find the slope of line AB i.e. x – 3y + 4 = 0

We know that, the slope of an equation is

$$m = -\frac{a}{b}$$

and here, a = 1 & b = -3

$$\Rightarrow m = -\frac{1}{(-3)} = \frac{1}{3}$$

$$= \frac{-1}{\frac{1}{3}}$$

= - 3

Now, Equation of line PQ formed by joining the points P(1, 2) and Q(h, k) and having the slope – 3 is

$y_2 - y_1 = m(x_2 - x_1)$

⇒ k – 2 = (-3)(h – 1)

⇒ k – 2 = -3h + 3

⇒ 3h + k = 5 …(ii)

Now, we will solve the eq. (i) and (ii) to find the value of h and k

h – 3k = -3 …(i)

and 3h + k = 5 …(ii)

From eq. (i), we get

h = -3 + 3k

Putting the value of h in eq. (ii), we get

3(-3 + 3k) + k = 5

⇒ -9 + 9k + k = 5

⇒ -9 + 10k = 5

⇒ 10k = 5 + 9

⇒ 10k = 14

$$\Rightarrow k = \frac{14}{10} = \frac{7}{5}$$

Putting the value of k in eq. (i), we get

$$h-3\left(\frac{7}{5}\right)=-3$$

⇒ 5h – 21 = -3 × 5

⇒ 5h – 21 = -15

⇒ 5h = -15 + 21

⇒ 5h = 6

$$\Rightarrow h=\frac{6}{5}$$

Q. 6. Find the area of the triangle formed by the lines x + y = 6, x – 3y = 2 and 5x – 3y + 2 = 0.

Answer : The given equations are

x + y = 6 …(i)

x – 3y = 2 …(ii)

and 5x – 3y + 2 = 0

or 5x – 3y = -2 …(iii)

Let eq. (i), (ii) and (iii) represents the sides AB, BC and AC respectively of ΔABC