Chapter 1
Real numbers
Exercise 1.4

1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

(i) $\frac{13}{3125}$

Solution:

$$\frac{13}{3125}$$

Factorize the denominator we get,

$3125 = 5\times5\times5\times5\times5 = 5^5$

The denominator is of the form 5^m

Hence, the decimal expansion of $\frac{13}{3125}$ is terminating.

(ii) $\frac{17}{8}$

Solution:

$$\frac{17}{8}$$

Factorize the denominator we get,

$8 = 2 \times 2 \times 2 = 2^3$

The denominator is of the form 2^m

Hence, the decimal expansion of $\frac{17}{8}$ is terminating.

(iii) $\frac{64}{455}$

Solution:

$$\frac{64}{455}$$

Factorize the denominator we get,

455 = 5×7×13

Since, the denominator is not in the form of $2^m \times 5^n$, and it also contains 7 and 13 as its factors,

Its decimal expansion will be non-terminating repeating.

(iv) $\frac{15}{1600}$

Solution:

$$\frac{15}{1600}$$

Factorize the denominator we get,

$1600 = 2^6 \times 5^2$

The denominator is in the form $2^m \times 5^n$

Hence, the decimal expansion of $\frac{15}{1600}$ is terminating.

(v) $\frac{29}{343}$

Solution:

$$\frac{29}{343}$$

Factorize the denominator we get,

$343 = 7^3$

Since the denominator is not in the form of $2^m \times 5^n$, it has 7 as its factors.

So, the decimal expansion of $\frac{29}{343}$ non-terminating repeating.

(vi) $\frac{23}{2^3 5^2}$

Solution:

$$\frac{23}{2^3 5^2}$$

The denominator is in the form $2^m \times 5^n$

Hence, the decimal expansion of $\frac{23}{2^3 5^2}$ is terminating.

(vii) $\frac{129}{2^2 5^7 7^5}$

Solution:

$$\frac{23}{2^2 5^7 7^5}$$

Since, the denominator is not in the form of $2^m \times 5^n$, as it has 7 in denominator.

So, the decimal expansion of $\frac{23}{2^2 5^7 7^5}$ is non-terminating repeating.

(viii) $\frac{6}{15}$

Solution:

$$\frac{6}{15} = \frac{2 \times 3}{3 \times 5} = \frac{2}{5}$$

The denominator is in the form 5^n

Hence, the decimal expansion of $\frac{6}{15}$ is terminating.

(ix) $\frac{35}{50}$

Solution:

$$\frac{35}{50} = \frac{7 \times 5}{10 \times 5} = \frac{7}{10}$$

Factorize the denominator we get,

$10 = 2 \times 5$

The denominator is in the form $2^m \times 5^n$

Hence, the decimal expansion of $\frac{35}{50}$ is terminating.

(x) $\frac{77}{210}$

Solution:

$$\frac{77}{210} = \frac{11 \times 7}{30 \times 7} = \frac{11}{30}$$

Factorize the denominator we get,

30 = 2×3×5

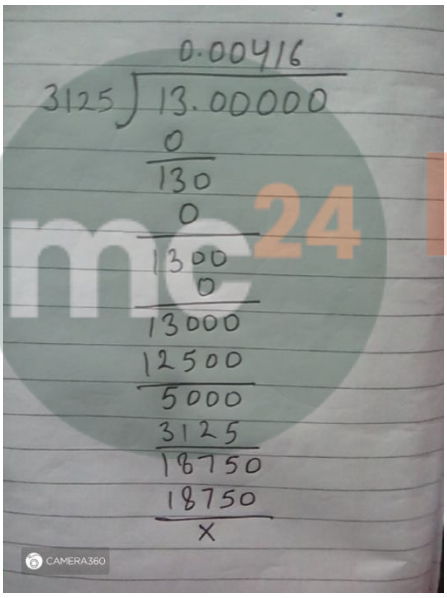
Since the denominator is not in the form of $2^m \times 5^n$, as it has 3 in denominator.

So, the decimal expansion of $\frac{77}{210}$ is a non-terminating repeating.

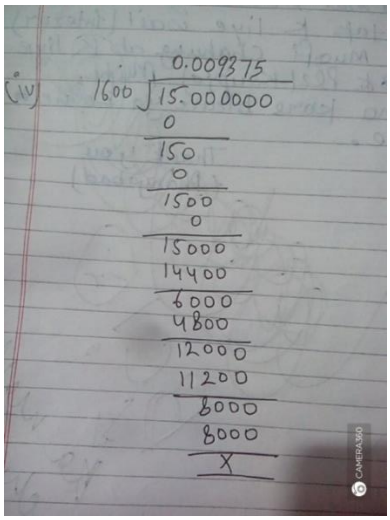
2. Write down the decimal expansions of those rational numbers in Question 1 above which have terminating decimal expansions.

Solution:

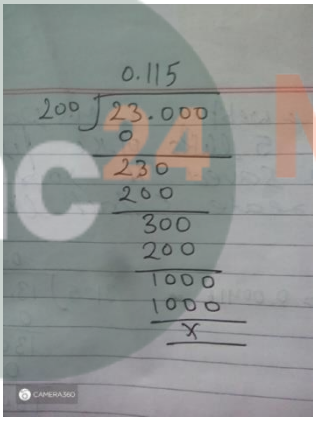
(i) $\frac{13}{3125} = 0.00416$



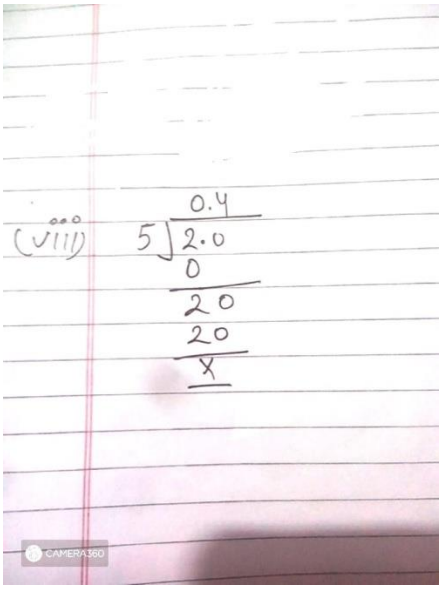
(ii) $\frac{15}{1600} = 0.009375$



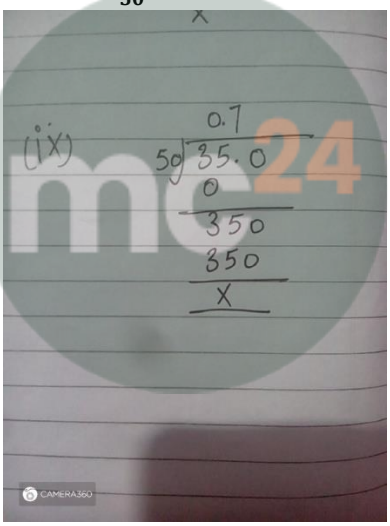
(i) $\frac{23}{2^3 \times 5^2} = 0.115$



(viii) $\frac{6}{15} = \frac{2 \times 3}{2 \times 5} = \frac{2}{5} = 0.4$



(ix) $\frac{35}{50} = 0.7$



3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If they are rational, and of the form $\frac{p}{q}$, what can you say about the prime factors of q?

Solution:

(i) 43.123456789

Since this number has a terminating decimal expansion, it is a rational number of the form $\frac{p}{q}$ and q is of the form $2^m \times 5^n$

That is, the prime factor of q will be 2 or 5 or both.

(ii) 0.120120012000120000...

The decimal expansion is neither terminating nor recurring.

Therefore, the given number is an irrational number.

(iii) 43.123456789

Since the decimal expansion is non-terminating recurring, the given number is a rational number of the form $\frac{p}{q}$ and q is not of the form $2^m \times 5^n$ that is, the prime factors of q will also have a factor other than 2 or 5.