Solution 1: **Exercise 21(C)**

The perimeter of a cube formula is, Perimeter = 4a where (a= length)

Given that perimeter of the face of the cube is 32 cm

$\Rightarrow 4a = 32\ cm$

$\Rightarrow a = \frac{32}{4}$

$\Rightarrow a = 8\ cm$

We know that surface area of a cube with side 'a' $= 6a^2$

Thus, Surface area $= 6 \times 8^2 = 6 \times 64 = 384\ cm^2$

We know that the volume of a cube with side 'a' $= a^3$

Thus, volume $= 8^3 = 512\ cm^3$

Solution 2:

Given dimensions of the auditorium are: $40\ m \times 30\ m \times 12\ m$

The area of the floor $= 40 \times 30$

Also given that each student requires $1.2\ m^2$ of the floor area.

Thus, Maximum number of students $= \frac{40 \times 30}{1.2} = 1000$

Volume of the auditorium

$= 40 \times 30 \times 12\ m^3$

= Volume of air available for 1000 students

Therefore, Air available for each student $= \frac{40 \times 30 \times 12}{1000}\ m^3 = 14.4\ m^3$

Solution 3:

Length of longest rod=Length of the diagonal of the box

$17 = \sqrt{12^2 + x^2 + 9^2}$

$17^2 = 12^2 + x^2 + 9^2$

$x^2 = 17^2 - 12^2 - 9^2$

$x^2 = 289 - 144 - 81$

$x^2 = 64$

$x = 8$ cm

Solution 4:

(i)

No. of cube which can be placed along length $= \frac{30}{3} = 10.$

No. of cube along the breadth $= \frac{24}{3} = 8$

No. of cubes along the height $= \frac{15}{3} = 5.$

$\therefore$ The total no. of cubes placed $= 10 \times 8 \times 5 = 400$

(ii)

Cubes along length $= \frac{30}{4} = 7.5 = 7$

Cubes along width $= \frac{24}{4} = 6$ and cubes along height $= \frac{15}{4} = 3.75 = 3$

$\therefore$ The total no. of cubes placed $= 7 \times 6 \times 3 = 126$

(iii)

Cubes along length $= \frac{30}{5} = 6$

Cubes along width $= \frac{24}{5} = 4.5 = 4$ and cubes along height $= \frac{15}{5} = 3$

$\therefore$ The total no. of cubes placed $= 6 \times 4 \times 3 = 72$

Solution 5:

Vol. of the tank= vol. of earth spread

$$4 \times 6^3\ m^3 = \left(112 \times 62 - 4 \times 6^2\right)\ m^2 \times \text{Rise in level}$$

$$\begin{aligned} \text{Rise in level} &= \frac{4 \times 6^3}{112 \times 62 - 4 \times 6^2} \\ &= \frac{864}{6800} \\ &= 0.127\ m \\ &= 12.7\ cm \end{aligned}$$

Solution 6:

Let a be the side of the cube.
Side of the new cube=a+3
Volume of the new cube=$a^3 + 2457$
That is, $(a+3)^3 = a^3 + 2457$

$\Rightarrow a^3 + 3 \times a \times 3(a+3) + 3^3 = a^3 + 2457$

$\Rightarrow 9a^2 + 27a + 27 = 2457$

$\Rightarrow 9a^2 + 27a - 2430 = 0$

$\Rightarrow a^2 + 3a - 270 = 0$

$\Rightarrow a^2 + 18a - 15a - 270 = 0$

$\Rightarrow a(a+18) - 15(a+18) = 0$

$\Rightarrow (a-15)(a+18) = 0$

$\Rightarrow a - 15 = 0$ or $a + 18 = 0$

$\Rightarrow a = 15$ or $a = -18$

$\Rightarrow$ a=15 cm [since side cannot be negative]

Volume of the cube whose side is 15 cm $= 15^3 = 3375$ cm^3
Suppose the length of the given cube is reduced by 20%.

Thus new side $a_{new} = a - \frac{20}{100} \times a$

$= a\left(1 - \frac{1}{5}\right)$

$= \frac{4}{5} \times 15$

$= 12$ cm

Volume of the new cube whose side is 12 cm=$12^3 = 1728$ cm^3
Decrease in volume=$3375 - 1728 = 1647$ cm^3

Solution 7:

The dimensions of rectangular tank: $30\text{ cm} \times 20\text{ cm} \times 12\text{ cm}$

Side of the cube $= 10$ cm

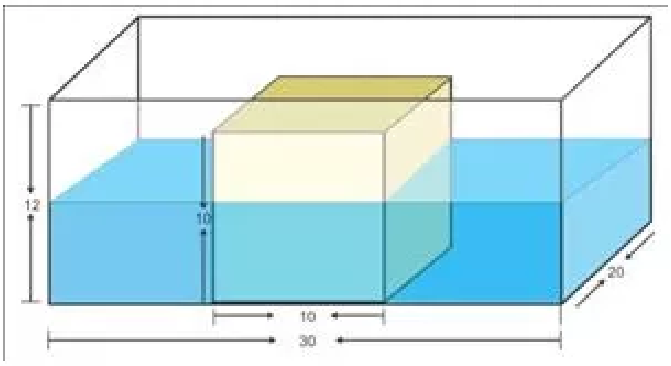
Volume of the cube $= 10^3 = 1000\text{ cm}^3$

The height of the water in the tank is 6 cm.

Volume of the cube till 6cm $= 10 \times 10 \times 6 = 600\text{cm}^3$

Hence when the cube is placed in the tank,

then the volume of the water increases by 600 cm^3.



The surface area of the water level

is $30\text{ cm} \times 20\text{ cm} = 600\text{ cm}^2$

Out of this area, let us subtract the

surface area of the cube.

Thus, the surface area of the

shaded part in the above figure is 500 cm^2

The displaced water is spreaded out

in 500 cm^2 to a height of 'h' cm.

And hence the volume of the water

displaced is equal to the volume

of the part of the cube in water.

Thus, we have,

$500 \times h = 600\text{ cm}^3$

$\Rightarrow h = \frac{600}{500}\text{ cm}$

$\Rightarrow h = 1.2\text{ cm}$

Thus, now the level of the water in the tank

is $= 6 + 1.2 = 7.2$ cm

Remaining height of the water level,

so that the metal cube is just

submerged in the water $= 10 - 7.2 = 2.8$ cm

Thus the volume of the water that must be

poured in the tank so that the metal

cube is just submerged in the water $= 2.8 \times 500 = 1400\text{ cm}^3$

We know that 1000 cc = 1 litre

Thus, the required volume of water $= \frac{1400}{1000} = 1.4$ litres.

Solution 8:

The dimensions of a solid cuboid are: 72 cm, 30 cm, 75 cm

Volume of the cuboid $= 72 \text{ cm} \times 30 \text{ cm} \times 75 \text{ cm} = 162000 \text{ cm}^3$

Side of a cube = 6 cm

Volume of a cube $= 6^3 = 216 \text{ cm}^3$

The number of cubes $= \frac{162000}{216} = 750$

The surface area of a cube $= 6a^2 = 6 \times 6^2 = 216 \text{ cm}^2$

Total surface area of 750 cubes $= 750 \times 216 = 162000 \text{ cm}^2$

Total surface area in square metres $= \frac{162000}{10000}$

$= 16.2$ square metres

Rate of polishing the surface per square metre = Rs.150

Total cost of polishing the surfaces $= 150 \times 16.2 =$ Rs.2430

Solution 9:

The dimensions of a car petrol tank are: $50 \text{ cm} \times 32 \text{ cm} \times 24 \text{ cm}$

Volume of the tank $= 38400 \text{ cm}^3$

We know that $1000 \text{ cm}^3 = 1$ litre

Thus volume of the tank $= \frac{38400}{1000} = 38.4$ litres

The average consumption of the car = 15 Km/litre

Thus, the total distance that can be
covered by the car $= 38.4 \times 15 = 576$ Km

Solution 10:

Given dimensions of a rectangular
box are in the ratio 4:2:3

Therefore, the total surface area of
the box $= 2[4x \times 2x + 2x \times 3x + 4x \times 3x]$

$= 2(8x^2 + 6x^2 + 12x^2) \text{ m}^2$

Difference between cost of covering the box
with paper at Rs.12 per m^2 and with paper
at Rs.13.50 per $\text{m}^2 = Rs.1,248$

$\Rightarrow 52x^2[13.5 - 12] = 1248$

$\Rightarrow 52 \times x^2 \times 1.5 = 1248$

$\Rightarrow 78 \times x^2 = 1248$

$\Rightarrow x^2 = \frac{1248}{78}$

$\Rightarrow x^2 = 16$

$\Rightarrow x = 4$ [Length, width and height cannot be negative]

Thus, the dimensions of the rectangular
box are: 4×4 m, 2×4 m, 3×4 m

Thus, the dimensions are 16 m, 8 m and 12 m.