

EXERCISE 1.4

Show that the cube of a positive integer of the form $6q + r$, q is an integer and $r = 0, 1, 2, 3, 4, 5$ is also of the form $6m + r$.

Solution:

$6q + r$ is a positive integer, where q is an integer and $r = 0, 1, 2, 3, 4, 5$

Then, the positive integers are of the form $6q, 6q+1, 6q+2, 6q+3, 6q+4$ and $6q+5$.

Taking cube on L.H.S and R.H.S,

For $6q$,

$$\begin{aligned}(6q)^3 &= 216q^3 = 6(36q^3) + 0 \\ &= 6m + 0, \text{ (where } m \text{ is an integer} = (36q^3)\end{aligned}$$

For $6q+1$,

$$\begin{aligned}(6q+1)^3 &= 216q^3 + 108q^2 + 18q + 1 \\ &= 6(36q^3 + 18q^2 + 3q) + 1 \\ &= 6m + 1, \text{ (where } m \text{ is an integer} = 36q^3 + 18q^2 + 3q)\end{aligned}$$

For $6q+2$,

$$\begin{aligned}(6q+2)^3 &= 216q^3 + 216q^2 + 72q + 8 \\ &= 6(36q^3 + 36q^2 + 12q + 1) + 2 \\ &= 6m + 2, \text{ (where } m \text{ is an integer} = 36q^3 + 36q^2 + 12q + 1)\end{aligned}$$

For $6q+3$,

$$\begin{aligned}(6q+3)^3 &= 216q^3 + 324q^2 + 162q + 27 \\ &= 6(36q^3 + 54q^2 + 27q + 4) + 3 \\ &= 6m + 3, \text{ (where } m \text{ is an integer} = 36q^3 + 54q^2 + 27q + 4)\end{aligned}$$

For $6q+4$,

$$\begin{aligned}(6q+4)^3 &= 216q^3 + 432q^2 + 288q + 64 \\ &= 6(36q^3 + 72q^2 + 48q + 10) + 4 \\ &= 6m + 4, \text{ (where } m \text{ is an integer} = 36q^3 + 72q^2 + 48q + 10)\end{aligned}$$

For $6q+5$,

$$\begin{aligned}(6q+5)^3 &= 216q^3 + 540q^2 + 450q + 125 \\ &= 6(36q^3 + 90q^2 + 75q + 20) + 5 \\ &= 6m + 5, \text{ (where } m \text{ is an integer} = 36q^3 + 90q^2 + 75q + 20)\end{aligned}$$

Hence, the cube of a positive integer of the form $6q + r$, q is an integer and $r = 0, 1, 2, 3, 4, 5$ is also of the form $6m + r$.

1. Prove that one and only one out of $n, n + 2$ and $n + 4$ is divisible by 3, where n is any positive integer.

Solution:

According to Euclid's division Lemma,

Let the positive integer = n

And $b=3$

$n = 3q+r$, where q is the quotient and r is the remainder

$0 \leq r < 3$ implies remainders may be 0, 1 and 2

Therefore, n may be in the form of $3q, 3q+1, 3q+2$

When $n=3q$

$n+2=3q+2$

$$n+4=3q+4$$

Here n is only divisible by 3

$$\text{When } n = 3q+1$$

$$n+2=3q+3$$

$$n+4=3q+5$$

Here only $n+2$ is divisible by 3

$$\text{When } n=3q+2$$

$$n+2=3q+4$$

$$n+4=3q+2+4=3q+6$$

Here only $n+4$ is divisible by 3

So, we can conclude that one and only one out of n , $n + 2$ and $n + 4$ is divisible by 3.

Hence Proved



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