

EXERCISE 21A

1. The length, breadth and height of a rectangular solid are in the ratio 5: 4: 2. If the total surface area is 1216 cm², find the length, the breadth and the height of the solid.

Solution:

Consider the length as $5x$, $4x$ and $2x$.

$$\text{Total Surface Area} = 2 (lb + bh + hl) = 1216$$

Substituting the values

$$2 (20x^2 + 8x^2 + 10x^2) = 1216$$

$$76x^2 = 1216$$

$$x^2 = 1216/76$$

$$x^2 = 16$$

$$x = 4 \text{ cm}$$

So we get

$$5x = 5 \times 4 = 20 \text{ cm}$$

$$4x = 4 \times 4 = 16 \text{ cm}$$

$$2x = 2 \times 4 = 8 \text{ cm}$$

Hence, the dimensions are 20 cm, 16 cm and 8 cm.

Here

$$\text{Volume} = lbh$$

$$= 20 \times 16 \times 8$$

$$= 2560 \text{ cm}^3$$

2. The volume of a cube is 729 cm³. Find its total surface area.

Solution:

Consider one edge of a cube = a

We know that

$$\text{Volume} = a^3$$

$$729 = a^3$$

$$9^3 = a^3$$

$$9 = a$$

$$a = 9 \text{ cm}$$

So we get

$$\text{Total surface area} = 6a^2 = 6 \times 9^2 = 486 \text{ cm}^2$$

3. The dimensions of a Cinema Hall are 100 m, 60 m and 15 m. How many persons can sit in the hall, if each requires 150 m³ of air?

Solution:

We know that

$$\text{Volume of cinema hall} = 100 \times 60 \times 15 = 90000 \text{ m}^3$$

It is given that

$$150 \text{ m}^3 = 1 \text{ person}$$

$$90000 \text{ m}^3 = (1/150) \times 90000 = 600 \text{ persons}$$

Hence, 600 persons can sit in the hall.

4. 75 persons can sleep in a room 25 m by 9.6 m. If each person requires 16 m³ of air; find the height of the room.

Solution:

Consider the height of the room = h

$$1 \text{ person} = 16 \text{ m}^3$$

$$75 \text{ person} = 75 \times 16 = 1200 \text{ m}^3$$

We know that

Volume of room is 1200 m^3

$$1200 = 25 \times 9.6 \times h$$

By further calculation

$$h = 1200 / (25 \times 9.6)$$

$$h = 5 \text{ m}$$

5. The edges of three cubes of metal are 3 cm, 4 cm and 5 cm. They are melted and formed into a single cube. Find the edge of the new cube.

Solution:

We know that

$$\text{Volume of melted single cube} = 3^3 + 4^3 + 5^3$$

$$= 27 + 64 + 125$$

$$= 216 \text{ cm}^3$$

Consider the edge of the new cube as a

$$\text{Volume} = 216 \text{ cm}^3$$

$$a^3 = 216$$

$$a^3 = 6^3$$

$$a = 6 \text{ cm}$$

Hence, 6 cm is the edge of cube.

6. Three cubes, whose edges are x cm, 8 cm and 10 cm respectively, are melted and recasted into a single cube of edge 12 cm. Find 'x'.

Solution:

We know that

$$\text{Volume of a melted single cube} = x^3 + 8^3 + 10^3 \text{ cm}^3$$

$$= x^3 + 512 + 1000 \text{ cm}^3$$

$$= x^3 + 1512 \text{ cm}^3$$

From the question, 12 cm is the edge of the single cube.

$$12^3 = x^3 + 1512 \text{ cm}^3$$

It can be written as

$$x^3 = 12^3 - 1512$$

$$x^3 = 1728 - 1512$$

$$x^3 = 216$$

So we get

$$x^3 = 6^3$$

$$x = 6 \text{ cm}$$

7. Three equal cubes are placed adjacently in a row. Find the ratio of the total surfaced area of the resulting cuboid to that of the sum of the total surface areas of the three cubes.

Solution:

Consider the side of a cube = a units

$$\text{Total surface area of one cube} = 6a^2$$

$$\text{Total surface area of 3 cubes} = 3 \times 6a^2 = 18a^2$$

So after joining 3 cubes in a row,

Length of cuboid becomes 3a

Breadth and height of cuboid = a

$$\text{Here the total surface area of cuboid} = 2(3a^2 + a^2 + 3a^2) = 14a^2$$

We know that

$$\text{Ratio of total surface area of cuboid to the total surface area of 3 cubes} = 14a^2/18a^2 = 7/9$$

8. The cost of papering the four walls of a room at 75 paisa per square meter is Rs. 240. The height of the room is 5 metres. Find the length and the breadth of the room, if they are in the ratio 5: 3.

Solution:

Consider the length and breadth as 5x and 3x respectively.

From the question,

The cost of papering the four walls of a room at 75 paisa per square meter is Rs. 240.

$$240 = \text{Area} \times 0.75$$

$$\text{Area} = 240/0.75$$

$$\text{Area} = 24000/75$$

$$\text{Area} = 320 \text{ m}$$

We know that

$$\text{Area} = 2 \times \text{Height} (\text{Length} + \text{Breadth})$$

Substituting the values

$$320 = 2 \times 5 (5x + 3x)$$

$$320 = 10 \times 8x$$

$$32 = 8x$$

$$x = 4$$

So we get

$$\text{Length} = 5x = 5 \times 4 = 20 \text{ m}$$

$$\text{Breadth} = 3x = 3 \times 4 = 12 \text{ m}$$

9. The area of a playground is 3650 m^2 . Find the cost of covering it with gravel 1.2 cm deep, if the gravel costs Rs. 6.40 per cubic metre.

Solution:

Area of the playground = 3650 m^2

Gravels are 1.2 cm deep

So the total volume to be covered = $3650 \times 0.012 = 43.8 \text{ m}^3$

As the cost of per cubic metre is Rs. 6.40

Total cost = $43.8 \times \text{Rs. } 6.40 = \text{Rs. } 280.32$

10. A square plat of side 'x' cm is 8 mm thick. If its volume is 2880 cm^3 ; find the value of x.

Solution:

We know that

1 mm = $1/10 \text{ cm}$

8 mm = $8/10 \text{ cm}$

Here

Volume = Base area \times Height

Substituting the values

$$2880 \text{ cm}^3 = x \times x \times 8/10$$

So we get

$$2880 \times 10/8 = x^2$$

$$x^2 = 3600$$

$$x = 60 \text{ cm}$$

