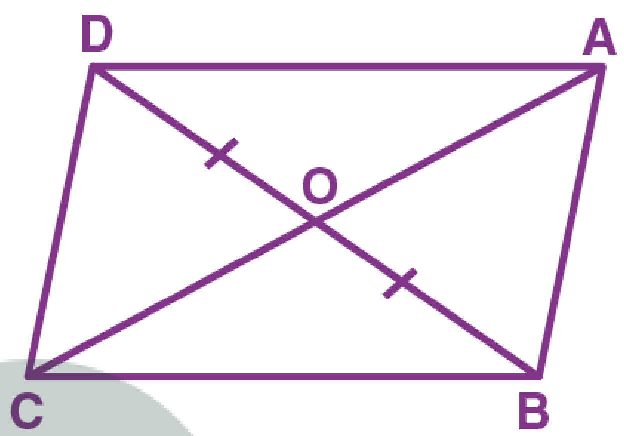
Exercise 16(C)

1. In the given figure, the diagonals AC and BD intersect at point O. If OB = OD and AB//DC, show that:
(i) Area (Δ DOC) = Area (Δ AOB).
(ii) Area (Δ DCB) = Area (Δ ACB).
(iii) ABCD is a parallelogram.



Solution:

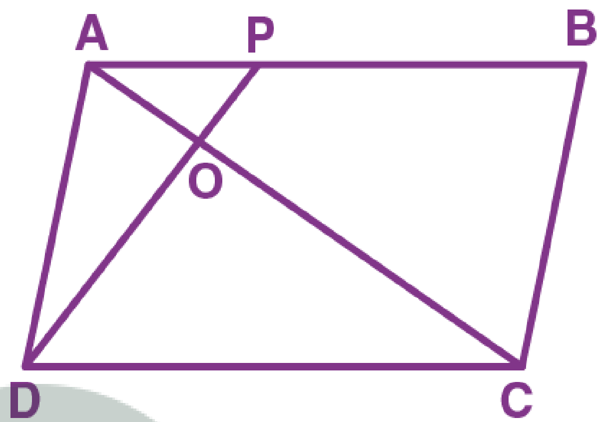
(i) Ratio of area of triangles with same vertex and bases along the same line is equal to the ratio of their respective bases.
So, we have
Area of ΔDOC/Area of ΔBOC = DO/BO = 1 … (i)
Similarly,
Area of ΔDOA/Area of ΔBOA = DO/BO = 1 … (ii)
We know that area of triangles on the same base and between same parallel lines are equal.
Area of Δ ACD = Area of Δ BCD
Area of Δ AOD + Area of Δ DOC = Area of Δ DOC + Area of Δ BOC
⇒ Area of Δ AOD = Area of Δ BOC … (iii)
From (i), (ii) and (iii) we have
Area (Δ DOC) = Area (Δ AOB)
- Hence Proved.

(ii) Similarly, from 1, 2 and 3, we also have
Area of Δ DCB = Area of Δ DOC + Area of Δ BOC = Area of Δ AOB + Area of Δ BOC = Area of Δ ABC
So, Area of Δ DCB = Area of Δ ABC
- Hence Proved.

(iii) We know that area of triangles on the same base and between same parallel lines are equal.
Given: triangles are equal in area on the common base, so it indicates AD|| BC.

So, ABCD is a parallelogram.
- Hence Proved

2. The given figure shows a parallelogram ABCD with area 324 sq. cm. P is a point in AB such that AP: PB = 1:2 Find The area of ΔAPD.



Solution:

Ratio of area of triangles with the same vertex and bases along the same line is equal to the ratio of their respective bases.
So, we have
Area of ΔAPD/Area of ΔBPD = AP/BP = ½
Area of parallelogram ABCD = 324 sq.cm

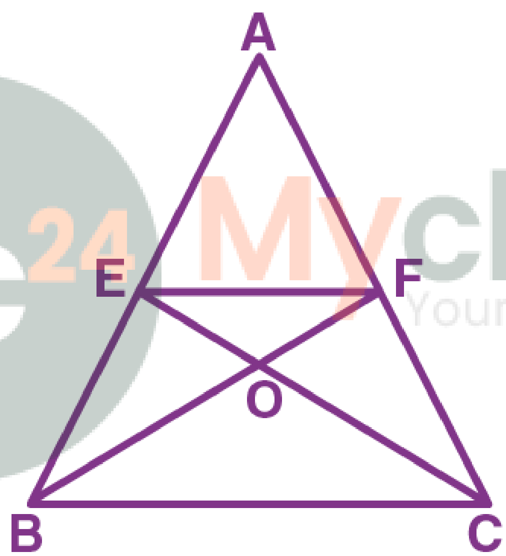
Area of the triangles with the same base and between the same parallels are equal.
We know that area of the triangle is half the area of the parallelogram if they lie on the same base and between the parallels.
Therefore, we have,
Area(ΔABD) = ½ × Area(||gm ABCD)
= 324/2
= 162 sq. cm
Also, from the diagram it is clear that
Area(ΔABD) = Area(ΔAPD) + Area(ΔBPD)
162 = Area(ΔAPD) + 2 × Area(ΔAPD)
162 = 3 × Area(ΔAPD)
Area(ΔAPD) = 162/3
= 54 sq. cm

(ii) Consider ΔAOP and ΔCOD
∠AOP = ∠COD [Vertically opposite angles]
∠CDO = ∠APD [AB || DC and DP is the transversal, alternate interior angles are equal]
Thus, ΔAOP ~ ΔCOD by AA similarity

Hence the corresponding sides are proportional.
AP/CD = OP/OD
= AP/AB
= AP/(AP+PB)
= AP/3AP
= 1/3
Therefore, OP: OD = 1: 3

3. In △ABC, E and F are mid-points of sides AB and AC respectively. If BF and CE intersect each other at point O, prove that the △OBC and quadrilateral AEOF are equal in area.
Solution:

Given, E and F are the midpoints of the sides AB and AC
Let's consider the following figure.



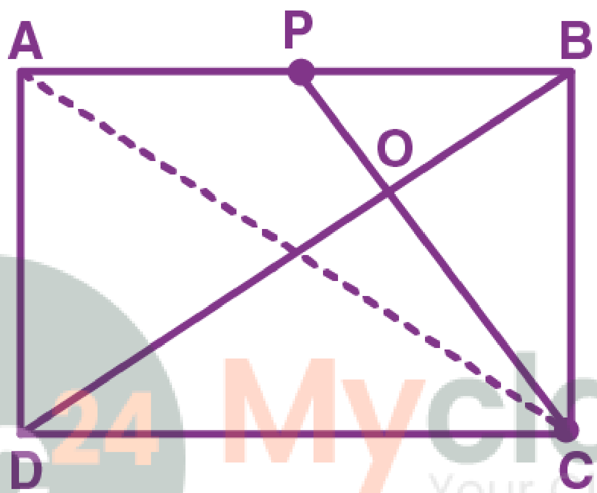
Therefore, by midpoint theorem
We have, EF || BC
Triangles BEF and CEF lie on the common base EF and between the parallels, EF and BC
Therefore, Ar(∆BEF) = Ar(∆CEF)
⇒ Ar(∆BOE) + Ar(∆EOF) = Ar(∆EOF) + Ar(∆COF)
Ar(∆BOE) = Ar(∆COF)

Now, BF and CE are the medians of the triangle ABC
And, median of the triangle divide it into two equal areas of triangles.
Thus, Ar(∆ABF) = Ar(∆CBF)
Now, subtracting Ar(∆BOE) on the both the sides, we get
Ar(∆ABF) – Ar(∆BOE) = Ar(∆CBF) – Ar(∆BOE)
Since, Ar(∆BOE) = Ar(∆COF)
⇒ Ar(∆ABF) – Ar(∆BOE) = Ar(∆CBF) – Ar(∆COF)
Ar(quad. AEOF) = Ar(∆OBC)

- Hence proved

4. In parallelogram ABCD, P is mid-point of AB. CP and BD intersect each other at point O. If area of ΔPOB = 40 cm^2, and OP: OC = 1: 2, find:
(i) Areas of ΔBOC and ΔPBC
(ii) Areas of ΔABC and parallelogram ABCD.
Solution:

(i) On joining AC, we have the following figure as below



Let's consider ΔPOB and ΔCOD
∠POB = ∠DOC [Vertically opposite angles]
∠OPB = ∠ODC [Since, AB || DC; CP and BD are the transversal, alternate interior angles are equal]
Therefore, ΔPOB ~ ΔCOD by AA similarity criterion
As, P is the midpoint
AP = BP and AB = CD, we have CD = 2BP
Therefore, we have
BP/CD = OP/OC = OB/OD = ½
OP: OC = 1: 2

(ii) From (i), we have
BP/CD = OP/OC = OB/OD = ½,
Ratio between the areas of two similar triangles is equal to the ratio between the square of the corresponding sides
Here, ΔDOC and ΔPOB are similar triangles.
Thus, we have
Ar(ΔDOC)/Ar(ΔPOB) = DC^2/PB^2
Ar(ΔDOC)/Ar(ΔPOB) = $(2PB)^2/PB^2$
Ar(ΔDOC)/Ar(ΔPOB) = $4PB^2/PB^2$
Ar(ΔDOC)/Ar(ΔPOB) = 4
Ar(ΔDOC) = 4 × Ar(ΔPOB)

= 4 × 40

=160 cm^2

Now, consider Ar(∆DBC) = Ar(∆DOC) + Ar(BOC)

= 160 + 80

= 240 cm^2

Two triangles are equal in area if they are on the equal bases and between the same parallels

Therefore,

Ar(∆DBC) = Ar(∆ABC) = 240 cm^2

We know that,

Median divides the triangle into areas of two equal triangles

Thus, CP is the median of the triangle ABC.

Hence, Ar(∆ABC) = 2 × Ar(∆PBC)

Ar(∆PBC) = Ar(∆ABC)/2

= 240/2

= 120cm^2

(iii) From (ii), we have

Ar(∆ABC) = 2 × Ar(∆PBC) 240 cm^2

Area of a triangle is half the area of the parallelogram

If both are on equal bases and between the same parallels

Thus, Ar(∆ABC) = ½ Ar(||gm ABCD)

⇒ Ar(llgm ABCD) = 2 Ar(∆ABC)

= 2 × 240

= 480 cm^2

5. The medians of a triangle ABC intersect each other at point G. If one of its medians is AD, prove that:

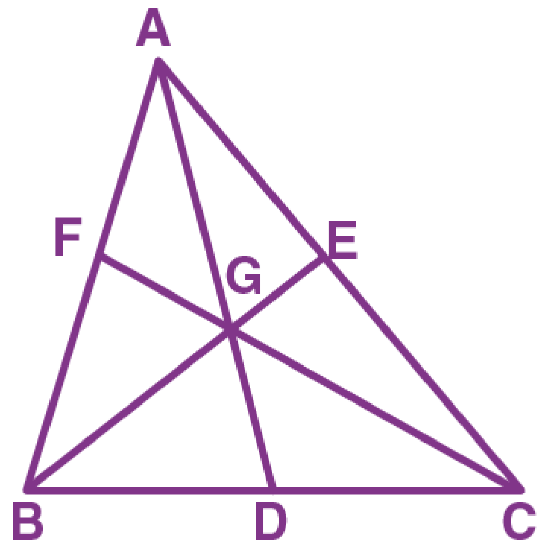
(i) Area (∆ABD) = 3 × Area (∆BGD)

(ii) Area (∆ACD) = 3 × Area (∆CGD)

(iii) Area (∆BGC) = 1/3 × Area (∆ABC)

Solution:

The figure is as shown below



We know that, medians interest at the centroid
Given that G is the point of intersect of medians
So, G is the centroid of the triangle ABC.
Now, the centroid divides the medians in the ratio 2:1

(i) We have, AG:GD = 2:1
So, Area(ΔAGB)/Area(ΔBGD) = 2/1
Area(ΔAGB) = 2Area(ΔBGD)
Now, from the figure, it is clearly seen that
Area(ΔABD) = Area(ΔAGB) + Area(ΔBGD)
Area(ΔABD) = 2 × Area(ΔBGD) + Area(ΔBGD)
Thus, Area(ΔABD) = 3 × Area(ΔBGD) … (1)

(ii) Similarly, CG divides AD in the ratio 2:1
So, Area(ΔAGC) / Area(ΔCGD) = 2/1
Area(ΔAGC) = 2 × Area(ΔCGD)
Now, from the figure, it is clearly seen that
Area(ΔACD) = Area(ΔAGC) + Area(ΔCGD)
Area(ΔACD) = 2 × Area(ΔCGD) + Area(ΔCGD)
Thus, Area(ΔACD) = 3 × Area(ΔCGD) … (2)

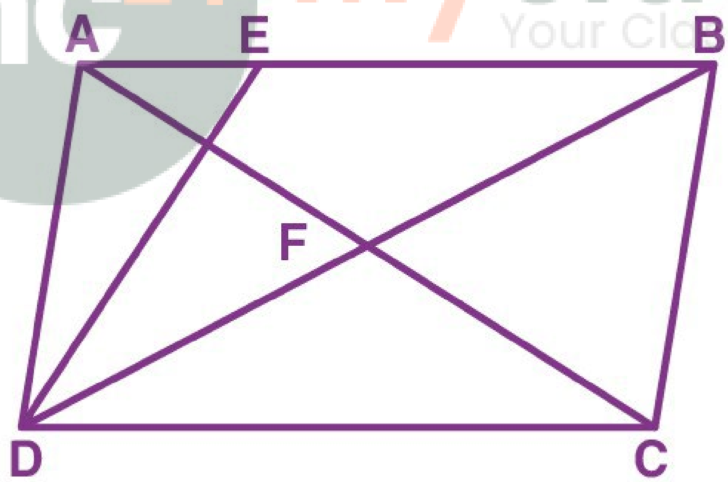
(iii) Adding equation (1) and (2), we have
Area(ΔABD) + Area(ΔACD) = 3Area(ΔBGD) + 3Area(ΔCGD)
Area(ΔABC) = 3 × [Area(ΔBGD0 + Area(ΔCGD)]
Area(ΔABC) = 3 × Area(ΔBGC)
Area(ΔABC)/3 = Area(ΔBGC)
Thus, Area(ΔBGC) = 1/3 × Area(ΔABC)

6. The perimeter of a triangle ABC is 37 cm and the ratio between the lengths of its altitudes be 6 : 5 : 4. Find the lengths of its sides.

Solution:

Let's consider the sides of ∆ABC to be x cm, y cm and (37 – x – y) cm
Also, let the lengths of its altitudes be 6a cm, 5a cm and 4a cm
We know that,
Area of a triangle= ½ × base × altitude
⇒ ½ × 6a = ½ × y × 5a = ½ × (37 – x – y) × 4a
6x = 5y = 148 – 4x – 4y
6x = 5y and 6x = 148 – 4x – 4y
6x - 5y = 0 and 10x + 4y – 148 = 0
Now, by solving both the equations, we have
x = 10 cm and y = 12 cm
And, (37 – x – y) = 15cm

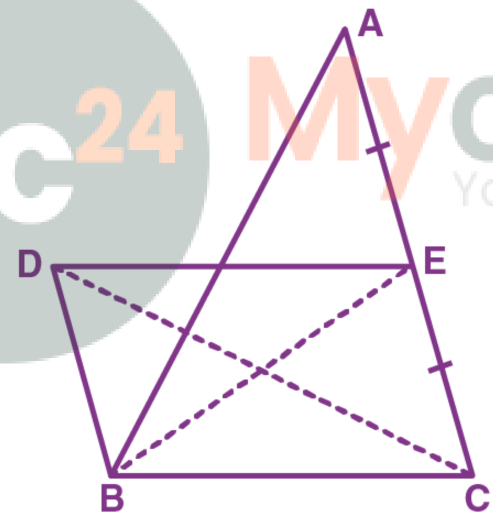
7. In parallelogram ABCD, E is a point in AB and DE meets diagonal AC at point F. If DF:FE = 5:3 and area of ∆ADF is 60 cm^2; find
(i) Area of ∆ADE
(ii) If AE:EB = 4:5, find the area of ∆ADB
(iii) Also, find area of parallelogram ABCD
Solution:



Triangles ADF and AFE have the same vertex A and their bases are on the same straight line DE
Hence, Ar(∆ADF)/Ar(∆AFE) = DF/FE
60/Ar(∆AFE) = 5/3
Ar(∆AFE) = (60 × 3)/5
= 36 cm^2
Now, Ar(∆ADE) = Ar(∆ADF) + Ar(∆AFE)
= 60 cm^2 + 36 cm^2
= 96 cm^2

ΔADE and ΔEDB have their bases are on the same straight line AB
∴ Ar(ΔADE)/Ar(ΔEDB) = AE/EB
96/Ar(ΔEDB) = 4/5
Ar(ΔEDB) = (96 × 5)/4
= 120cm^2
Now, Ar(ΔADB) = Ar(ΔADE) + Ar(ΔEDB)
= 96 cm^2 + 120 cm^2
= 216 cm^2
Now,
ΔADB and ||^m ABCD are on the same base AB and between the same parallels AB and DC
∴ Ar(ADB) = ½ Ar(||^m ABCD)
216 = ½ Ar(||^m ABCD)
Thus, Ar(||^m ABCD) = 2 × 216
= 432 cm^2

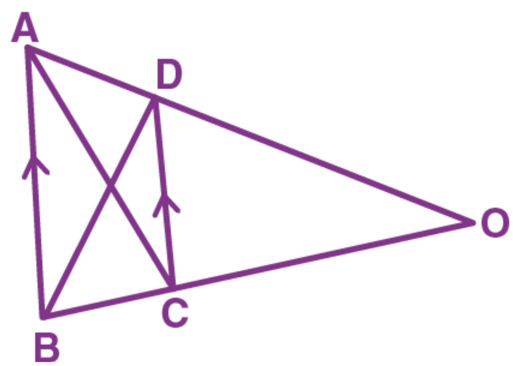
8. In the following figure, BD is parallel to CA, E is mid-point of CA and BD = ½ CA. Prove that: Ar (ABC) = 2 x Ar (DBC)



Solution:

Here, BCED is a parallelogram, since BD = CE and BD || CE.
Now,
Ar(ΔDBC) = Ar(ΔEBC) … (i) [Since they have the same base and area between the same parallels]
In ΔABC, we have
BE as the median
So, Ar(ΔEBC) = ½ [Ar(ΔEBC) + Ar(ΔABE)] = ½ Ar(ΔABC)
⇒ Ar(ΔABC) = 2Ar(ΔEBC)
Therefore, Ar(ΔABC) = 2Ar(ΔDBC)   [From (i)]

9. In the following figure, OAB is a triangle and AB ∥ DC.

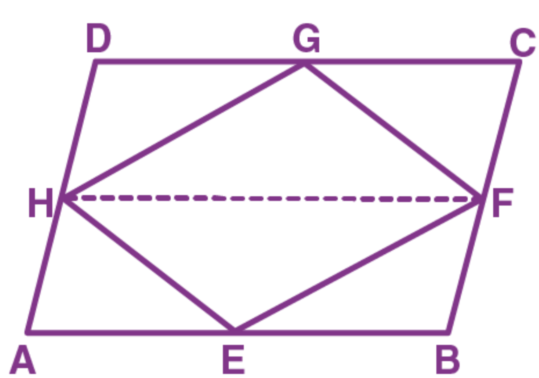


If the area of ΔCAD = 140 cm^2 and the area of ΔODC = 172 cm^2, find
(i) the area of ΔDBC
(ii) the area of ΔOAC
(iii) the area of ΔODB.
Solution:

Given: ΔCAD = 140cm^2, ΔODC = 172cm^2 and AB || CD
As triangles DBC and CAD have the same base CD and between the same parallel lines
Hence,
Ar(ΔDBC) = Ar(ΔCAD) = 140cm^2
Ar(ΔOAC) = Ar(ΔCAD) + Ar(ΔODC)
= 140cm^2 + 172cm^2
= 312cm^2
Ar(ΔODB) = Ar(ΔDBC) + Ar(ΔODC)
= 140cm^2 + 172cm^2
= 312cm^2

10. E, F, G and H are the mid- points of the sides of a parallelogram ABCD. Show that area of quadrilateral EFGH is half of the area of parallelogram ABCD.
Solution:

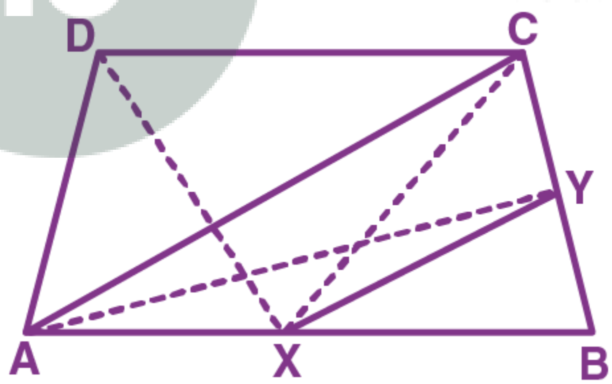
Let's join HF.



Since H and F are mid-points of AD and BC respectively, we have
AH = ½ AD and BF = ½ BC
Now, ABCD is a parallelogram
AD = BC and AD ∥ BC
⇒ ½ AD = ½ BC and AD ∥ BC
So, AH = BF and AH ∥ BF
Thus, ABFH is a parallelogram
Now,
Since parallelogram FHAB and ΔFHE are on the same base FH and between the same parallels HF and AB, we have
Ar(ΔFHE) = ½ A(∥^{m}FHAB) …(i)
Similarly,
Ar(ΔFHG) = ½ Ar(∥^{m}FHDC) …(ii)
Adding (i) and (ii), we get
Ar(ΔFHE) + Ar(ΔFHG) = ½ Ar(∥^{m}FHAB) + ½ Ar(∥^{m}FHDC)
Ar(EFGH) = ½ [Ar(∥^{m}FHAB) + ½ Ar(∥^{m}FHDC)]
∴ Ar(EFGH) = ½ Ar(∥^{m}ABCD)

11. ABCD is a trapezium with AB parallel to DC. A line parallel to AC intersects AB at X and BC at Y. Prove that area of ΔADX = area of ΔACY.
Solution:

Let's join CX, DX and AY.



Now, triangles ADX and ACX are on the same base AX and between the parallels AB and DC.
∴ Ar(ΔADX) = Ar(ΔACX) ….(i)
Also, triangles ACX and ACY are on the same base AC and between the parallels AC and XY.
∴ Ar(ΔACX) = Ar(ΔACY) ….(ii)
From (i) and (ii), we get
Ar(ΔADX) = Ar(ΔACY)
- Hence proved