## Exercise 16.3

**A bucket has top and bottom diameters of 40 cm and 20 cm respectively. Find the volume of the bucket if its depth is 12 cm. Also, find the cost of tin sheet used for making the bucket at the rate of Rs 1.20 per $dm^2$.**
**Solution:**

Given,
Diameter to top of bucket = 40 cm
So, the radius ($r_1$) = 40/2 = 20 cm
Diameter of bottom part of the bucket = 20 cm
So, the radius ($r_2$) = 30/2 = 10cm
Depth of the bucket (h) = 12 cm
Volume of the bucket = $1/3\ \pi(r_2^2 + r_1^2 + r_1 r_2)h$
$= \pi/3(20^2 + 10^2 + 20 \times 10)12$
$= 8800\ cm^3$
Now,

Let 'L' be slant height of the bucket $\Rightarrow L = \sqrt{(r_2 - r_1)^2 + h^2}$

$\Rightarrow L = \sqrt{(20-10)^2 + 12^2}$

$\Rightarrow L = 15.620$ cm

Total surface area of bucket $= \pi(r_1 + r_2) \times L + \pi \times r_2^2$

$= \pi(20 + 10) \times 15.620 + \pi \times 10^2$

$= \frac{1320\sqrt{61} + 2200}{7} = 17.87 dm^2$

Given that the cost of tin sheet used for making bucket per $dm^2$ = Rs 1.20
So, the total cost for $17.87 dm^2$ = 1.20 × 17.87 = Rs 21.40
Therefore, the cost of tin sheet used for making the bucket is Rs 21.40

**1. A frustum of a right circular cone has a diameter of base 20 cm, of top 12 cm and height 3 cm. Find the area of its whole surface and volume.**
**Solution:**

Given,
Base diameter of cone ($d_1$) = 20 cm
So the radius ($r_1$) = 20/2 cm = 10 cm
Top diameter of Cone ($d_2$) = 12 cm
So, the radius ($r_2$) = 12/2 cm = 6 cm
Height of the cone (h) = 3 cm
Volume of the frustum of a right circular cone = $1/3\ \pi(r_2^2 + r_1^2 + r_1 r_2)h$

$= \pi/3(10^2 + 6^2 + 10 \times 6)^3$
$= 616\ cm^3$

Let 'L' be the slant height of cone, then we know that
$L = \sqrt{(r_1 - r2_1)^2 + h^2}$
$L = \sqrt{(10 - 6)^2 + 3^2}$
$L = \sqrt{(16 + 9)}$
$L = 5cm$
So, the slant height of cone (L) = 5 cm
Thus,
Total surface area of the frustum $= \pi(r_1 + r_2) \times L + \pi r_1^2 + \pi r_2^2$
$= \pi(10 + 6) \times 5 + \pi \times 10^2 + \pi \times 6^2$
$= \pi(80 + 100 + 36)$
$= \pi(216)$
$= 678.85\ cm^2$

**2. The slant height of the frustum of a cone is 4 cm and the perimeters of its circular ends are 18 cm and 6 cm. Find the curved surface of the frustum.**
**Solution:**

Given,
Slant height of frustum of cone (l) = 4 cm
Let ratio of the top and bottom circles be $r_1$ and $r_2$
And given perimeters of its circular ends as 18 cm and 6 cm
$\Longrightarrow 2\pi r_1 = 18$ cm; $2\pi r_2 = 6$ cm
$\Longrightarrow \pi r_1 = 9$ cm and $\pi r_2 = 3$ cm
We know that,
Curved surface area of frustum of a cone $= \pi(r_1 + r_2)l$
$= \pi(r_1 + r_2)l$
$= (\pi r_1 + \pi r_2)l = (9 + 3) \times 4 = (12) \times 4 = 48\ cm^2$
Therefore, the curved surface area of the frustum = $48\ cm^2$

**3. The perimeters of the ends of a frustum of a right circular cone are 44 cm and 33 cm. If the height of the frustum be 16 cm, find its volume, the slant surface and the total surface.**
**Solution:**

Given,
Perimeter of the upper end = 44 cm
$2 \pi r_1 = 44$
$2(22/7)\ r_1 = 44$
$r_1 = 7$ cm
Perimeter of the lower end = 33 cm
$2 \pi r_2 = 33$
$2(22/7)\ r_2 = 33$
$r_2 = 21/4$ cm
Now,
Let the slant height of the frustum of a right circular cone be L

$L = \sqrt{(r_1 - r_2)^2 + h^2}$

$L = \sqrt{(7 - 5.25)^2 + 16^2}$

L = 16.1 cm

So, the curved surface area of the frustum cone = $\pi(r_1 + r_2)l$

$= \pi(7 + 5.25)16.1$

Curved surface area of the frustum cone = 619.65 $cm^3$

Next,

The volume of the frustum cone = $1/3\ \pi(r_2^2 + r_1^2 + r_1 r_2)h$

$= 1/3\ \pi(7^2 + 5.25^2 + (7)(5.25)) \times 16$

$= 1898.56\ cm^3$

Thus, volume of the cone = 1898.56 $cm^3$

Finally, the total surface area of the frustum cone

$= \pi(r_1 + r_2) \times L + \pi r_1^2 + \pi r_2^2$

$= \pi(7 + 5.25) \times 16.1 + \pi 7^2 + \pi 5.25^2$

$= \pi(7 + 5.25) \times 16.1 + \pi(7^2 + 5.25^2) = 860.27\ cm^2$

Therefore, the total surface area of the frustum cone is 860.27 $cm^2$

**4. If the radii of the circular ends of a conical bucket which is 45 cm high be 28 cm and 7 cm, find the capacity of the bucket.**

**Solution:**

Given,

Height of the conical bucket = 45 cm

Radii of the 2 circular ends of the conical bucket are 28 cm and 7 cm

So, $r_1$ = 28 cm $r_2$ = 7 cm

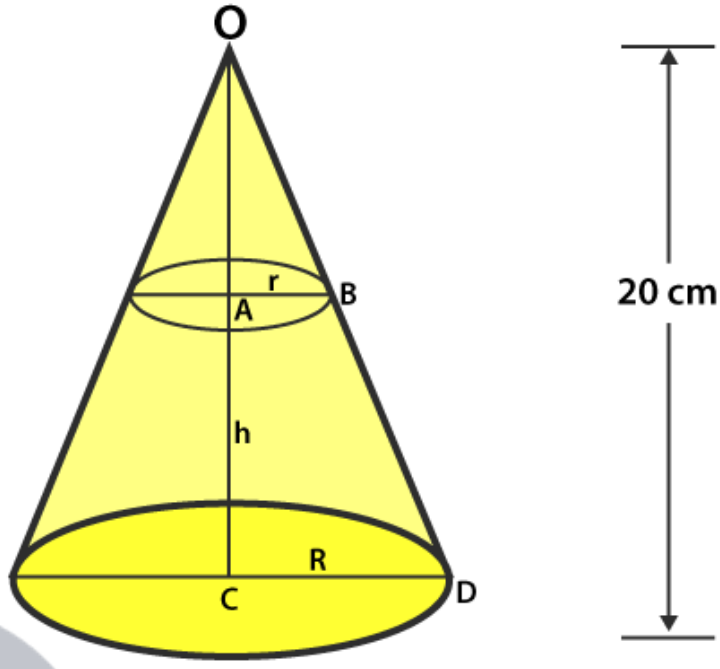
Volume of the conical bucket = $1/3\ \pi(r_1^2 + r_2^2 + r_1 r_2)h$

$= 1/3\ \pi(28^2 + 7^2 + 28 \times 7)45 = 15435\pi$

Therefore, the volume/ capacity of the bucket is 48510 $cm^3$.

**5. The height of a cone is 20 cm. A small cone is cut off from the top by a plane parallel to the base. If its volume be 1/125 of the volume of the original cone, determine at what height above the base the section is made.**

**Solution:**






Let the radius of the small cone be r cm
Radius if the big cone = R cm
Given, height of the big cone = 20 cm
Let the height of section made = h cm
Then, the height of small cone will be = (20 - h) cm
Now,
In △OAB and △OCD
∠AOB = ∠COD [common]
∠OAB = ∠OCD [each 90º]
Then, OAB ~ △OCD [by AA similarity]
So, by C.P.S.T we have
OA/ OC = AB/ CD
(20 – h)/ 20 = r/ R …… (i)

Also given,
Volume of small cone = 1/125 x volume of big cone
$1/3\ \pi r^2(20 - h) = 1/125 \times 1/3\ \pi R^2 \times 20$
$r^2/ R^2 = 1/125 \times 20/ (20 - h)$ [From (i)]
$(20 - h)^2/ 20^2 = 1/125 \times 20/20 – h$
$(20 - h)^3 = 20^3/ 125$
20 – h = 20/5
20 – h = 4
h = 20 - 4 = 16 cm
Therefore, it’s found that the section was made at a height of 16 cm above the base.

**6. If the radii of the circular ends of a bucket 24 cm high are 5 and 15 cm respectively, find the surface area of the bucket.**
**Solution:**

Given,
Height of the bucket (h) = 24 cm
Radius of the circular ends of the bucket 5 cm and 15 cm
So, $r_1 = 5$ cm; $r_2 = 15$ cm
Let 'L' be the slant height of the bucket
Then, we know that

$$L = \sqrt{(r_1 - r_2)^2 + h^2}$$

$$\Rightarrow L = \sqrt{(5 - 15)^2 + 24^2}$$

$$\Rightarrow L = \sqrt{(100 + 576)}$$

$$\Rightarrow L = 26 \text{ cm}$$

Now,
Curved surface area of the bucket
$= \pi(r_1 + r_2)l + \pi r_1^2$
$= \pi(5 + 15)26 + \pi 5^2 = \pi(520 + 25) = 545\pi$ cm$^2$
Therefore, the curved surface area of the bucket = $545\pi$ cm$^2$

**7. The radii of circular bases of a frustum of a right circular cone are 12 cm and 3 cm and the height is 12 cm. Find the total surface area and volume of frustum.**
**Solution:**

Given the height of frustum cone = 12 cm
Radii of a frustum cone are 12 cm and 3 cm
So, $r_1 = 12$ cm; $r_2 = 3$ cm
Let slant height of the frustum cone be 'L'
Then, we know that

$$L = \sqrt{(r_1 - r_2)^2 + h^2}$$

$$L = \sqrt{81 + 144} = 15 \text{ cm}$$

L = 15 cm
Now, the total surface area of frustum of a cone = $\pi (r_1 + r_2) \times L + \pi r_1^2 + \pi r_2^2$
$= \pi (12 + 3)15 + \pi 12^2 + \pi 3^2$
Thus, total surface area of the frustum = $378\ \pi$ cm$^2$
Next,
Volume of frustum cone = $1/3\ \pi(r_2^2 + r_1^2 + r_1 r_2)h$
$= 1/3\ \pi(12^2 + 3^2 + 12 \times 3) \times 12 = 756\pi$ cm$^3$

Therefore, the volume of the frustum cone = 756 π $cm^3$

**8. A tent consists of a frustum of a cone capped by a cone. If radii of the ends of the frustum be 13 m and 7 m, the height of frustum be 8 m and the slant height of the conical cap be 12 m, find the canvas required for the tent.**
**Solution:**

Given,
Height of frustum (h) = 8 m
Radii of the frustum cone are 13 cm and 7 cm
So, $r_1$ = 13 cm and $r_2$ = 7 cm
Let 'L' be slant height of the frustum cone
Then, we know that

$$\Rightarrow L = \sqrt{(r_1 - r_2)^2 + h^2}$$

$$\Rightarrow L = \sqrt{(13 - 7)^2 + 8^2}$$

$$\Rightarrow L = \sqrt{36 + 64}$$

$$\Rightarrow L = 10 \text{ m}$$

Curved surface area of the frustum ($s_1$) = $\pi(r_1 + r_2) \times L = \pi(13 + 7) \times 10 = 200\pi$ $m^2$
Then, given slant height of conical cap = 12 m
Base radius of upper cap cone = 7 m
So, the curved surface area of upper cap cone ($s_2$) = $\pi rl = \pi \times 7 \times 12 = 264$ $m^2$
Thus, the total canvas required for tent (S) = $s_1 + s_2$
S = 200π + 264 = 892.57 $m^2$
Therefore, the canvas required for the tent is 892.57 $m^2$.

**9. A milk container of height 16 cm is made of metal sheet in the form of frustum of a cone with radii of its lower and upper ends as 8 cm and 20 cm respectively. Find the cost of milk at the rate of Rs.44 per litre which the container can hold.**
**Solution:**

Given,
A milk container of the form of frustum of a cone
The radius of the lower end ($r_1$) = 8 cm
The radius of the upper end ($r_2$) = 20 cm
Let h be its height, h = 16 cm
Then, the capacity of the container = Volume of frustum of the cone

$$= 1/3\ \pi(r_2^2 + r_1^2 + r_1 r_2)h$$
$$= 1/3\ \pi(20^2 + 8^2 + (20)(8)) \times 16$$
$$= 10459.42\ cm^3$$
$$= 10.46 \text{ litres}$$

Now, given that cost of 1 litre of milk = Rs 44
Then the cost of 10.46 litres of milk = Rs (44 x 10.46) = Rs 460.24

**10. A bucket is in the form of a frustum of a cone of height 30 cm with radii of its lower and upper ends as 10 cm and 20 cm respectively. Find the capacity and surface area of the bucket. Also, find the cost of milk which can completely fill the container, at the rate of Rs.25 per litre.**
**Solution:**

Let R and r be the radii of the top and base of the bucket respectively,
Let h be its height.
Then, we have R = 20 cm, r = 10 cm, h = 30 cm
Capacity of the bucket = Volume of the frustum of the cone

$= 1/3\ \pi(R^2 + r^2 + R\,r)h$
$= 1/3\ \pi(20^2 + 10^2 + 20 \times 10) \times 30$
$= 3.14 \times 10\ (400 + 100 + 200)$
$= 21980\ cm^3 = 21.98$ litres

Now,
Surface area of the bucket = CSA of the bucket + Surface area of the bottom

$= \pi\, l\, (R + r) + \pi r^2$

We know that,
$l = \sqrt{h^2 + (R - r)^2}$
$= \sqrt{[30^2 + (20 - 10)^2]} = \sqrt{(900 + 100)}$
$= \sqrt{1000} = 31.62$ cm
So,
The Surface area of the bucket $= (3.14) \times 31.62 \times (20 + 10) + (3.14) \times 10^2$

$= 2978.60 + 314$
$= 3292.60\ cm^2$

Next, given that the cost of 1 litre milk = Rs 25
Thus, the cost of 21.98 litres of milk = Rs (25 x 21.98) = Rs 549.50