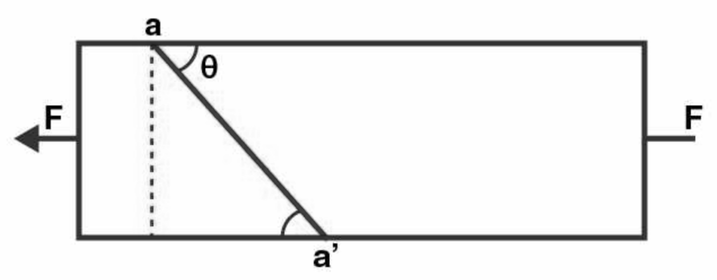
Class 11 Physics Chapter 9: Mechanical Properties of Solids

Long Answers

24. Consider a long steel bar under tensile stress due to forces F acting at the edges along the length of the bar. Consider a plane making an angle θ with the length. What are the tensile and shearing stresses on this plane?



Given: Force F is applied along the horizontal direction.

Analysis:

- Component perpendicular to plane aa' = F sin θ
- Component parallel to plane aa' = F cos θ
- Cross-sectional area of bar = A
- Area of plane aa' = A/sin θ

Tensile stress on plane aa': $\sigma = (F \sin\theta)/(A/\sin\theta) = (F \sin^2\theta)/A$

Shear stress on plane aa': $\tau = (F \cos\theta)/(A/\sin\theta) = (F \sin 2\theta)/(2A)$

a) **For maximum tensile stress:** θ = 90° b) **For maximum shearing stress:** θ = 45°

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25. a) A steel wire of mass μ per unit length with a circular cross-section has a radius of 0.1 cm. The wire is of length 10 m when measured lying horizontal, and hangs from a hook on the wall. A mass of 25 kg is hung from the free end of the wire. Find the extension in the length of the wire. The density of steel is 7860 kg/m³.

Given:

- $r = 0.1$ cm $= 10^{-3}$ m
- L = 10 m
- M = 25 kg
- $\rho = 7860$ kg/m^3
- $A = \pi(10^{-3})^2 = \pi\times10^{-6}$ m^2
- $\mu = \rho A = 7860 \times \pi\times10^{-6}$ kg/m

Solution: Consider a small element dx at distance x from the free end. Mass of wire below this element = μx Total force at this section = Mg + μgx

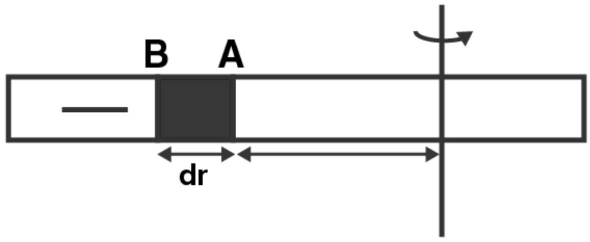
Extension of element dx = (Mg + μgx)dx/(AY)

Total extension = $\int_0^L (Mg + \mu g x)dx/(AY) = (1/AY)[Mgx + \mu g x^2/2]_0^L = (1/AY)[MgL + \mu g L^2/2]$

Substituting values: Mass of wire = $\mu L = 7860 \times \pi\times10^{-6} \times 10 = 0.25$ kg

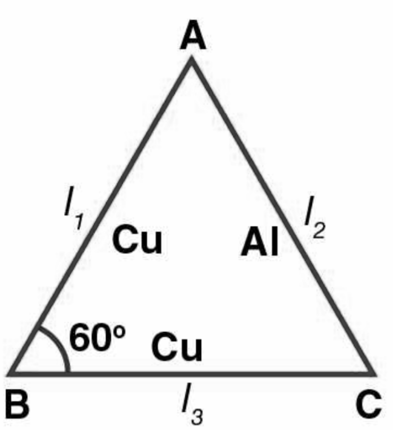
Extension = $(25\times9.8\times10 + 0.25\times9.8\times10^2/2)/(\pi\times10^{-6} \times 2\times10^{11}) \approx 0.4$ cm
b) **Maximum weight calculation:** For yield strength = 2.5×10^8 N/m² Maximum stress occurs at the top = $(M_{max}\, g + \mu g L)/A$ $2.5\times10^8 = (M_{max} \times 9.8 + 0.25\times9.8\times10)/(\pi\times10^{-6})$ $M_{max} \approx 78.25$ kg

26. A steel rod of length 2l, cross-sectional area A and mass M is set rotating in a horizontal plane about an axis passing through the centre. If Y is the Young's modulus for steel, find the extension in the length of the rod.



Analysis: Consider an element at distance r from the center with thickness dr. Mass of element = $dm = (M/2l)dr$ Centrifugal force = $dm\cdot r\cdot\omega^2 = (M/2l)r\cdot\omega^2\cdot dr$
The tension varies along the rod. At distance r from center: $T(r) = \int_r^l (M/2l)r'\omega^2dr' = (M\omega^2/2l)\int_r^l r'dr' = (M\omega^2/4l)(l^2 - r^2)$
Stress at distance r = $T(r)/A = (M\omega^2/4lA)(l^2 - r^2)$
Extension of element dr = $[\text{Stress} \times dr]/Y = (M\omega^2/4lAY)(l^2 - r^2)dr$
Total extension of half rod = $\int_0^l (M\omega^2/4lAY)(l^2 - r^2)dr = (M\omega^2/4lAY)[l^2r - r^3/3]_0^l = (M\omega^2/4lAY)(l^3 - l^3/3) = (M\omega^2l^2)/(6AY)$
Total extension of complete rod = $2 \times (M\omega^2l^2)/(6AY) = (M\omega^2l^2)/(3AY)$

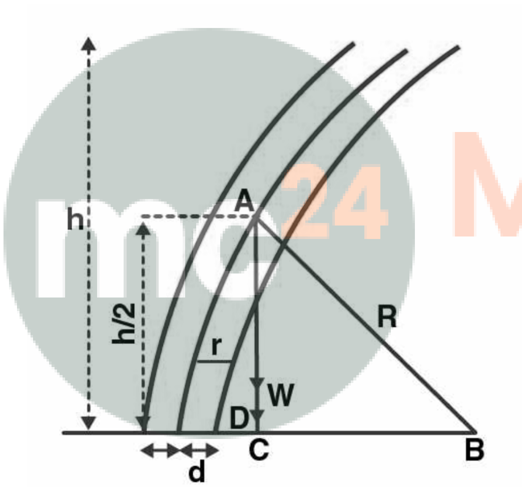
27. An equilateral triangle ABC is formed by two Cu rods AB and BC, and one Al rod AC. It is heated such that the temperature of each rod increases by ΔT. Find the change in the angle ABC.
Analysis: Let each side have original length l. After heating:



- Cu rods: $l_1 = l_2 = l(1 + \alpha_{Cu}\cdot\Delta T)$
- Al rod: $l_3 = l(1 + \alpha_{Al}\cdot\Delta T)$

Using cosine rule for triangle ABC: $\cos\theta = (l_1^2 + l_3^2 - l_2^2)/(2l_3l_1)$
Initially: $\cos 60° = 1/2$
After heating: $\cos(60° + d\theta) = [l^2(1 + \alpha\_Cu \cdot \Delta T)^2 + l^2(1 + \alpha\_Al \cdot \Delta T)^2 - l^2(1 + \alpha\_Cu \cdot \Delta T)^2]/[2l^2(1 + \alpha\_Cu \cdot \Delta T)(1 + \alpha\_Al \cdot \Delta T)]$
Simplifying and differentiating: $d\theta = 2(\alpha_2 - \alpha_1)\Delta T/\sqrt{3}$
where $\alpha_1 = \alpha\_Cu$ and $\alpha_2 = \alpha\_Al$

28. In nature, the failure of structural members usually results from large torque because of twisting or bending rather than due to tensile or compressive strains. This process of structural breakdown is called buckling, and in cases of tall cylindrical structures like trees, the torque is caused by its own weight bending the structure. Thus the vertical through the centre of gravity does not fall within the base. The elastic torque caused because of this bending about the central axis if the tree is given by T*pi*r⁴/4R. Y is the Young's modulus, r is the radius of the trunk, and R is the radius of curvature of the bent surface along the height of the trees containing the centre of gravity. Estimate the critical height of the trees for a given radius of the trunk.



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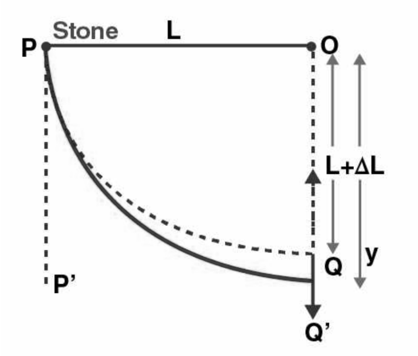
Given: Elastic torque = $T\pi r^4/(4R)$

Analysis: For a tree of height h and radius r:

- Weight = $\rho \cdot \pi \cdot r^2 \cdot h \cdot g$ (where ρ is wood density)
- Bending moment at base ≈ Weight × h/2
- Radius of curvature $R \approx h^2/(8d)$ where d is deflection

At critical height, bending moment equals elastic torque: $\rho \cdot \pi \cdot r^2 \cdot h \cdot g \cdot h/2 = Y \cdot \pi \cdot r^4/(4R)$
Substituting R and solving: $h\_critical = [2Y/\rho g]^{(1/3)} \times r^{(2/3)}$
This shows that taller trees need disproportionately larger trunk radius to avoid buckling.

29. A stone of mass m is tied to an elastic string of negligible mass and spring constant k. The unstretched length is L. The stone is dropped vertically from point P.



a) Distance y when mass first comes to rest:

Using energy conservation: PE lost = PE gained by spring mgy = ½k(y-L)²

This gives: ky² - 2(kL + mg)y + kL² = 0

Using quadratic formula: y = [kL + mg ± √(mg(mg + 2kL))]/k

Taking the larger root (first rest position): y = [kL + mg + √(mg(mg + 2kL))]/k

b) Maximum velocity:

Maximum velocity occurs when net force = 0 At extension x from natural length: mg = kx

Therefore: x = mg/k

Using energy conservation from P to this point: mg(L + mg/k) = ½mv² + ½k(mg/k)² v² = 2gL + (mg)²/k v = √[2gL + (mg)²/k]^(1/2)

c) Nature of motion after reaching lowest point:

After reaching the lowest point, the stone will oscillate in simple harmonic motion about the equilibrium position z_0 = L + mg/k with angular frequency ω = √(k/m).

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