

## EXERCISE 25

### 1. Evaluate:

(i)  $\cos 22^\circ / \sin 68^\circ$

(ii)  $\tan 47^\circ / \cot 43^\circ$

(iii)  $\sec 75^\circ / \operatorname{cosec} 15^\circ$

(iv)  $\frac{\cos 55^\circ}{\sin 35^\circ} + \frac{\cot 35^\circ}{\tan 55^\circ}$

(v)  $\sin^2 40^\circ - \cos^2 50^\circ$

(vi)  $\sec^2 18^\circ - \operatorname{cosec}^2 72^\circ$

(vii)  $\sin 15^\circ \cos 15^\circ - \cos 75^\circ \sin 75^\circ$

(viii)  $\sin 42^\circ \cos 48^\circ - \cos 42^\circ \sin 48^\circ$

### Solution:

(i) According to the question we have,  
 $\cos 22^\circ / \sin 68^\circ = \cos (90^\circ - 68^\circ) / \sin 68^\circ$   
 by using complementary angle identity,  
 $= \sin 68^\circ / \sin 68^\circ$   
 $= 1$

(ii) According to the question we have,  
 $\tan 47^\circ / \cot 43^\circ = \tan (90^\circ - 43^\circ) / \cot 43^\circ$   
 by using complementary angle identity,  
 $= \cot 43^\circ / \cot 43^\circ$   
 $= 1$

(iii) According to the question we have,  
 $\sec 75^\circ / \operatorname{cosec} 15^\circ = \sec (90^\circ - 15^\circ) / \operatorname{cosec} 15^\circ$   
 by using complementary angle identity,  
 $= \operatorname{cosec} 15^\circ / \operatorname{cosec} 15^\circ$   
 $= 1$

(iv) According to the question we have,  

$$\frac{\cos 55^\circ}{\sin 35^\circ} + \frac{\cot 35^\circ}{\tan 55^\circ}$$

$$= \frac{\cos(90^\circ - 35^\circ)}{\sin 35^\circ} + \frac{\cot(90^\circ - 55^\circ)}{\tan 55^\circ}$$

$$= \frac{\sin 35^\circ}{\sin 35^\circ} + \frac{\tan 55^\circ}{\tan 55^\circ}$$

$$= 1 + 1$$

$$= 2$$

(v)  $\sin^2 40^\circ - \cos^2 50^\circ$   
 $= \sin^2 (90^\circ - 50^\circ) - \cos^2 50^\circ$

$$= \cos^2 50^\circ - \cos^2 50^\circ$$

$$= 0$$

$$(vi) \sec^2 18^\circ - \operatorname{cosec}^2 72^\circ$$

$$= [\sec(90^\circ - 72^\circ)]^2 - \operatorname{cosec}^2 72^\circ$$

$$= \operatorname{cosec}^2 72^\circ - \operatorname{cosec}^2 72^\circ$$

$$= 0$$

$$(vii) \sin 15^\circ \cos 15^\circ - \cos 75^\circ \sin 75^\circ$$

$$= \sin(90^\circ - 75^\circ) \cos 15^\circ - \cos 75^\circ \sin(90^\circ - 15^\circ)$$

$$= \cos 75^\circ \cos 15^\circ - \cos 75^\circ \cos 15^\circ$$

$$= 0$$

$$(viii) \sin 42^\circ \sin 48^\circ - \cos 42^\circ \cos 48^\circ$$

$$= \sin(90^\circ - 48^\circ) \sin 48^\circ - \cos(90^\circ - 48^\circ) \cos 48^\circ$$

$$= \cos 48^\circ \sin 48^\circ - \sin 48^\circ \cos 48^\circ$$

$$= 0$$

**2. Evaluate:**

(i)  $\sin(90^\circ - A) \sin A - \cos(90^\circ - A) \cos A$

(ii)  $\sin^2 35^\circ - \cos^2 55^\circ$

(iii)  $\frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2$

(iv)  $\frac{2 \tan 53^\circ}{\cot 37^\circ} - \frac{\cot 80^\circ}{\tan 10^\circ}$

(v)  $\cos^2 25^\circ - \sin^2 65^\circ - \tan^2 45^\circ$

(vi)  $\left(\frac{\sin 77^\circ}{\cos 13^\circ}\right)^2 + \left(\frac{\cos 77^\circ}{\sin 13^\circ}\right)^2 - 2 \cos^2 45^\circ$

**Solution:**

$$(i) \text{ Given } \sin(90^\circ - A) \sin A - \cos(90^\circ - A) \cos A$$

$$= \cos A \sin A - \sin A \cos A$$

$$= 0$$

$$(ii) \text{ Given } \sin^2 35^\circ - \cos^2 55^\circ$$

$$= \sin^2 35^\circ - [\cos(90^\circ - 35^\circ)]^2$$

$$= \sin^2 35^\circ - \sin^2 35^\circ$$

$$= 0$$

(iii) According to the question we have,

$$\begin{aligned} & \frac{\cot 54^\circ}{\tan 36^\circ} + \frac{\tan 20^\circ}{\cot 70^\circ} - 2 \\ &= \frac{\cot(90^\circ - 36^\circ)}{\tan 36^\circ} + \frac{\tan(90^\circ - 70^\circ)}{\cot 70^\circ} - 2 \\ &= \frac{\tan 36^\circ}{\tan 36^\circ} + \frac{\cot 70^\circ}{\cot 70^\circ} - 2 \end{aligned}$$

On simplifying we get

$$\begin{aligned} &= 1 + 1 - 2 \\ &= 2 - 2 \\ &= 0 \end{aligned}$$

(iv) According to the question we have

$$\begin{aligned} & \frac{2 \tan 53^\circ}{\cot 37^\circ} - \frac{\cot 80^\circ}{\tan 10^\circ} \\ &= \frac{2 \tan(90^\circ - 37^\circ)}{\cot 37^\circ} - \frac{\cot(90^\circ - 10^\circ)}{\tan 10^\circ} \\ &= \frac{2 \cot 37^\circ}{\cot 37^\circ} - \frac{\tan 10^\circ}{\tan 10^\circ} \end{aligned}$$

On simplifying we get

$$\begin{aligned} &= 2 - 1 \\ &= 1 \end{aligned}$$

(v) According to the question we have

$$\begin{aligned} & \cos^2 25^\circ - \sin^2 65^\circ - \tan^2 45^\circ \\ &= [\cos(90^\circ - 65^\circ)]^2 - \sin^2 65^\circ - (\tan^2 45^\circ)^2 \\ &= \sin^2 65^\circ - \sin^2 65^\circ - 1^2 \\ &= 0 - 1 \\ &= -1 \end{aligned}$$

(vi) According to the question we have,

$$\begin{aligned} & \left( \frac{\sin 77^\circ}{\cos 13^\circ} \right)^2 + \left( \frac{\cos 77^\circ}{\sin 13^\circ} \right)^2 - 2 \cos^2 45^\circ \\ &= \left( \frac{\sin(90^\circ - 13^\circ)}{\cos 13^\circ} \right)^2 + \left( \frac{\cos(90^\circ - 13^\circ)}{\sin 13^\circ} \right)^2 - 2 (\cos 45^\circ)^2 \\ &= \left( \frac{\cos 13^\circ}{\cos 13^\circ} \right)^2 + \left( \frac{\sin 13^\circ}{\sin 13^\circ} \right)^2 - 2 \left( \frac{1}{\sqrt{2}} \right)^2 \end{aligned}$$

On simplifying we get

$$\begin{aligned} &= 1^2 + 1^2 - 2 \left( \frac{1}{2} \right) \\ &= 1 + 1 - 1 \\ &= 1 \end{aligned}$$

**3. Show that:**

(i)  $\tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ = 1$

(ii)  $\sin 42^\circ \sec 48^\circ + \cos 42^\circ \operatorname{cosec} 48^\circ = 2$

**Solution:**

(i) L.H.S.

$$= \tan 10^\circ \tan 15^\circ \tan 75^\circ \tan 80^\circ$$

$$= \tan (90^\circ - 80^\circ) \tan (90^\circ - 75^\circ) \tan 75^\circ \tan 80^\circ$$

$$= \cot 80^\circ \cot 75^\circ \tan 75^\circ \tan 80^\circ$$

$$= (\cot 80^\circ \tan 80^\circ) (\cot 75^\circ \tan 75^\circ)$$

$$= (1)(1)$$

$$= 1$$

= R.H.S.

(ii) consider LHS

$$= \sin 42^\circ \sec 48^\circ + \cos 42^\circ \operatorname{cosec} 48^\circ$$

$$= \sin(90^\circ - 48^\circ) \times \frac{1}{\cos 48^\circ} + \cos(90^\circ - 48^\circ) \times \frac{1}{\sin 48^\circ}$$

$$= \cos 48^\circ \times \frac{1}{\cos 48^\circ} + \sin 48^\circ \times \frac{1}{\sin 48^\circ}$$

On simplifying we get,

$$= 1 + 1$$

$$= 2$$

= RHS

**4. Express each of the following in terms of angles between  $0^\circ$  and  $45^\circ$ :**

(i)  $\sin 59^\circ + \tan 63^\circ$

(ii)  $\operatorname{cosec} 68^\circ + \cot 72^\circ$

(iii)  $\cos 74^\circ + \sec 67^\circ$

**Solution:**

(i) Given  $\sin 59^\circ + \tan 63^\circ$

It can be written as

$$\sin 59^\circ + \tan 63^\circ = \sin (90 - 31)^\circ + \tan(90 - 27)^\circ$$

$$= \cos 31^\circ + \cos 27^\circ$$

(ii) Given  $\operatorname{cosec} 68^\circ + \cot 72^\circ$

It can be written as

$$\operatorname{cosec} 68^\circ + \cot 72^\circ = \operatorname{cosec} (90 - 22)^\circ + \cot (90 - 18)^\circ$$

$$= \sec 22^\circ + \tan 18^\circ$$

(iii) Given  $\cos 74^\circ + \sec 67^\circ$

It can be written as

$$\cos 74^\circ + \sec 67^\circ = \cos (90 - 16)^\circ + \sec(90 - 23)^\circ$$

$$= \sin 16^\circ + \operatorname{cosec} 23^\circ$$

**5. Question 5**

For triangle ABC, show that:

$$(i) \quad \sin\left(\frac{A+B}{2}\right) = \cos\frac{C}{2}$$

$$(ii) \quad \tan\left(\frac{B+C}{2}\right) = \cot\frac{A}{2}$$

**Solution:**

(i) We know that for  $\triangle ABC$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle B + \angle A = 180^\circ - \angle C$$

$$(\angle B + \angle A) / 2 = 90^\circ - \angle C / 2$$

$$\sin [(A+B)/2] = \sin (90^\circ - C/2)$$

$$= \cos (C/2)$$

(ii) We know that for  $\triangle ABC$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$\angle B + \angle C = 180^\circ - \angle A$$

$$(\angle B + \angle C) / 2 = 90^\circ - \angle A / 2$$

$$\tan [(B+C)/2] = \tan (90^\circ - A/2)$$

$$= \cot (A/2)$$

**6. Evaluate:**

$$(i) \quad 3 \frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ}$$

$$(ii) \quad 3 \cos 80^\circ \operatorname{cosec} 10^\circ + 2 \sin 59^\circ \sec 31^\circ$$

$$(iii) \quad \frac{\sin 80^\circ}{\cos 10^\circ} + \sin 59^\circ \sec 31^\circ$$

$$(iv) \quad \tan (55^\circ - A) - \cot (35^\circ + A)$$

$$(v) \quad \operatorname{cosec} (65^\circ + A) - \sec (25^\circ - A)$$

$$(vi) \quad 2 \frac{\tan 57^\circ}{\cot 33^\circ} - \frac{\cot 70^\circ}{\tan 20^\circ} - \sqrt{2} \cos 45^\circ$$

$$(vii) \quad \frac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2 \frac{\sin^2 75^\circ}{\cos^2 15^\circ}$$

$$(viii) \quad \frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8 \sin^2 30^\circ$$

$$(ix) \quad 14 \sin 30^\circ + 6 \cos 60^\circ - 5 \tan 45^\circ$$

**Solution:**

(i) Given

$$\begin{aligned} & 3 \frac{\sin 72^\circ}{\cos 18^\circ} - \frac{\sec 32^\circ}{\operatorname{cosec} 58^\circ} \\ &= 3 \frac{\sin(90^\circ - 18^\circ)}{\cos 18^\circ} - \frac{\sec(90^\circ - 58^\circ)}{\operatorname{cosec} 58^\circ} \\ &= 3 \frac{\cos 18^\circ}{\cos 18^\circ} - \frac{\operatorname{cosec} 58^\circ}{\operatorname{cosec} 58^\circ} = 3 - 1 = 2 \end{aligned}$$

(ii) Given  $3 \cos 80^\circ \operatorname{cosec} 10^\circ + 2 \sin 59^\circ \sec 31^\circ$ 

$$\begin{aligned} &= 3 \cos(90^\circ - 10^\circ) \operatorname{cosec} 10^\circ + 2 \sin(90^\circ - 31^\circ) \sec 31^\circ \\ &= 3 \sin 10^\circ \operatorname{cosec} 10^\circ + 2 \cos 31^\circ \sec 31^\circ \\ &= 3(1) + 2(1) \\ &= 3 + 2 \\ &= 5 \end{aligned}$$

(iii) Given

$$\begin{aligned} & \frac{\sin 80^\circ}{\cos 10^\circ} + \sin 59^\circ \sec 31^\circ \\ &= \frac{\sin(90^\circ - 10^\circ)}{\cos 10^\circ} + \sin(90^\circ - 31^\circ) \sec 31^\circ \\ &= \frac{\cos 10^\circ}{\cos 10^\circ} + \frac{\cos 31^\circ}{\cos 31^\circ} \end{aligned}$$

On simplifying we get

$$\begin{aligned} &= 1 + 1 \\ &= 2 \end{aligned}$$

(iv) Given  $\tan(55^\circ - A) - \cot(35^\circ + A)$ 

It can be written as

$$\begin{aligned} \tan(55^\circ - A) - \cot(35^\circ + A) &= \tan[90^\circ - (35^\circ + A)] - \cot(35^\circ + A) \\ &= \cot(35^\circ + A) - \cot(35^\circ + A) \\ &= 0 \end{aligned}$$

(v) Given  $\operatorname{cosec}(65^\circ + A) - \sec(25^\circ - A)$ 

It can be written as

$$\begin{aligned} \operatorname{cosec}(65^\circ + A) - \sec(25^\circ - A) &= \operatorname{cosec}[90^\circ - (25^\circ - A)] - \sec(25^\circ - A) \\ &= \sec(35^\circ + A) - \sec(25^\circ - A) \\ &= 0 \end{aligned}$$

$$\begin{aligned}
 \text{(vi)} \quad & 2 \frac{\tan 57^\circ}{\cot 33^\circ} - \frac{\cot 70^\circ}{\tan 20^\circ} - \sqrt{2} \cos 45^\circ \\
 &= 2 \frac{\tan(90^\circ - 33^\circ)}{\cot 33^\circ} - \frac{\cot(90^\circ - 20^\circ)}{\tan 20^\circ} - \sqrt{2} \left( \frac{1}{\sqrt{2}} \right) \\
 &= 2 \frac{\cot 33^\circ}{\cot 33^\circ} - \frac{\tan 20^\circ}{\tan 20^\circ} - 1
 \end{aligned}$$

On simplifying we get,

$$= 2 - 1 - 1$$

$$= 0$$

(vii) Given

$$\begin{aligned}
 & \frac{\cot^2 41^\circ}{\tan^2 49^\circ} - 2 \frac{\sin^2 75^\circ}{\cos^2 15^\circ} \\
 &= \frac{[\cot(90^\circ - 49^\circ)]^2}{\tan^2 49^\circ} - 2 \frac{[\sin(90^\circ - 15^\circ)]^2}{\cos^2 15^\circ} \\
 &= \frac{\tan^2 49^\circ}{\tan^2 49^\circ} - 2 \frac{\cos^2 15^\circ}{\cos^2 15^\circ}
 \end{aligned}$$

On simplifying we get

$$= 1 - 2$$

$$= -1$$

(viii) Given

$$\begin{aligned}
 & \frac{\cos 70^\circ}{\sin 20^\circ} + \frac{\cos 59^\circ}{\sin 31^\circ} - 8 \sin^2 30^\circ \\
 &= \frac{\cos(90^\circ - 20^\circ)}{\sin 20^\circ} + \frac{\cos(90^\circ - 31^\circ)}{\sin 31^\circ} - 8 \left( \frac{1}{2} \right)^2 \\
 &= \frac{\sin 20^\circ}{\sin 20^\circ} + \frac{\sin 31^\circ}{\sin 31^\circ} - 2
 \end{aligned}$$

On simplifying we get

$$= 1 + 1 - 2$$

$$= 2 - 2$$

$$= 0$$

(ix) Given  $14 \sin 30^\circ + 6 \cos 60^\circ - 5 \tan 45^\circ$

It can be written as

$$14 \sin 30^\circ + 6 \cos 60^\circ - 5 \tan 45^\circ = 14 \left( \frac{1}{2} \right) + 6 \left( \frac{1}{2} \right) - 5 (1)$$

$$= 7 + 3 - 5$$

$$= 10 - 5$$

$$= 5$$

**7. A triangle ABC is right angled at B. Find the value of**

$$\frac{\sec A \sin C - \tan A \tan C}{\sin B}$$

**Solution:**

Since triangle ABC is a right-angled triangle, and right angle at B

Therefore,  $A + C = 90^\circ$

$$\begin{aligned} &\therefore \frac{\sec A \sin C - \tan A \tan C}{\sin B} \\ &= \frac{\sec A (90^\circ - C) \sin C - \tan (90^\circ - C) \tan C}{\sin 90^\circ} \\ &= \frac{\operatorname{cosec} C \sin C - \cot C \tan C}{1} \\ &= \frac{1}{\sin C} \times \sin C - \frac{1}{\tan C} \times \tan C \end{aligned}$$

On simplifying we get

$$= 1 - 1$$

$$= 0$$

8. In each case, given below, find the value of angle A, where  $0^\circ \leq A \leq 90^\circ$ .

(i)  $\sin (90^\circ - 3A) \cdot \operatorname{cosec} 42^\circ = 1$

(ii)  $\cos (90^\circ - 3A) \cdot \sec 77^\circ = 1$

**Solution:**

(i) Given  $\sin (90^\circ - 3A) \cdot \operatorname{cosec} 42^\circ = 1$

On rearranging we get

$$\sin (90^\circ - 3A) = 1/\operatorname{cosec} 42^\circ$$

$$\cos 3A = 1/\operatorname{cosec} (90^\circ - 48^\circ)$$

$$\cos 3A = 1/\sec 48^\circ$$

$$\cos 3A = \cos 48^\circ$$

$$3A = 48^\circ$$

$$\text{Therefore, } A = 16^\circ$$

(ii) Given  $\cos (90^\circ - 3A) \cdot \sec 77^\circ = 1$

$$\cos (90^\circ - 3A) = 1/\sec 77^\circ$$

$$\sin 3A = 1/\sec (90^\circ - 12^\circ)$$

$$\sin 3A = 1/\operatorname{cosec} 12^\circ$$

$$\sin 3A = \sin 12^\circ$$

$$3A = 12^\circ$$

$$A = 3^\circ$$

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