Activity 16

Objective

To verify Basic Proportionality Theorem (Thales theorem).

Material Required

Two wooden strips (each of size 1 cm wide and 30 cm long), cutter, adhesive, hammer, nails, bard board, white paper, pulleys, thread, scale and screw etc.

Method of Construction

1. Cut a piece of hardboard of a convenient size and paste a white paper on it.

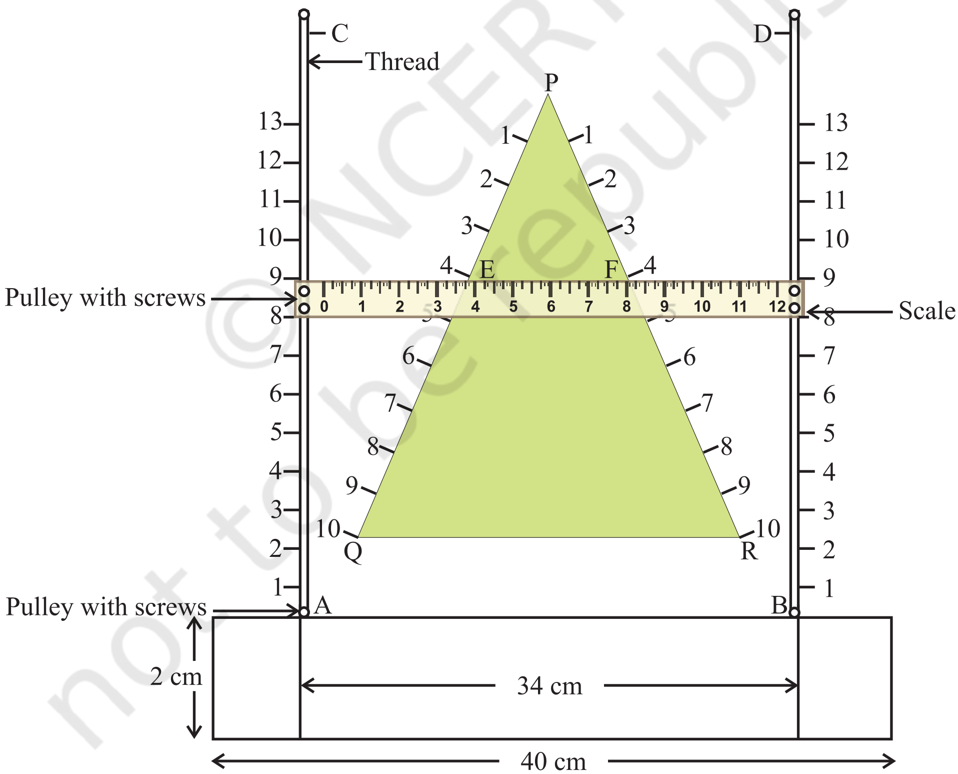

Fig. 1

2. Take two thin wooden strips with markings 1, 2, 3, ... at equal distances and fix them vertically on the two ends of the horizontal strip as shown in Fig. 1 and call them AC and BD.

3. Cut a triangular piece PQR from hardboard (thickness should be negligible) and paste coloured glazed paper on it and place it between the parallel strips AC and BD such that its base QR is parallel to the horizontal strip AB as drawn in Fig. 1.

4. Graduate the other two sides of the triangular piece as shown in the figure.

5. Put the screws along the horizontal strip and two more screws on the top of the board at the points C and D such that A, B, D and C become four vertices of a rectangle.

6. Take a ruler (scale) and make four holes on it as shown in the figure and fix four pulleys at these holes with the help of screws.

7. Fix the scale on the board using the thread tied to nails fixed at points A, B, C and D passing through the pulleys as shown in the figure, so that the scale slides parallel to the horizontal strip AB and can be moved up and down over the triangular piece freely.

Demonstration

1. Set the scale on vertical strips parallel to the base QR of D PQR, say at the points E and F. Measure the distances PE and EQ and also measure the distance PF and FR. It can be easily verified that

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

This verifies Basic Proportionality Theorem (Thales theorem).

2. Repeat the activity as stated above, sliding the scale up and down parallel to the base of the triangle PQR and verify the Thales theorem for different positions of the scales.

OBSERVATION

By actual measurement:

PE = ____________, PF = ____________, EQ = ____________,

FR = ____________

$\frac{PE}{EQ}$ = ________, $\frac{PF}{FR}$ = ____________

Thus, $\frac{PE}{EQ} = \frac{PF}{FR}$. It verifies the Theorem.

APPLICATION

The theorem can be used to establish various criteria of similarity of triangles. It can also be used for constructing a polygon similar to a given polygon with a given scale factor.

Activity 17

OBJECTIVE

To find the relationship between areas and sides of similar triangles.

MATERIAL REQUIRED

Coloured papers, glue, geometry box, scissors/cutters, white paper.

METHOD OF CONSTRUCTION

1. Take a coloured paper of size 15 cm × 15 cm.
2. On a white paper, draw a triangle ABC.
3. Divide the side AB of ΔABC into some equal parts [say 4 parts].
4. Through the points of division, draw line segments parallel to BC and through points of division of AC, draw line segments parallel to AB [see Fig. 1].

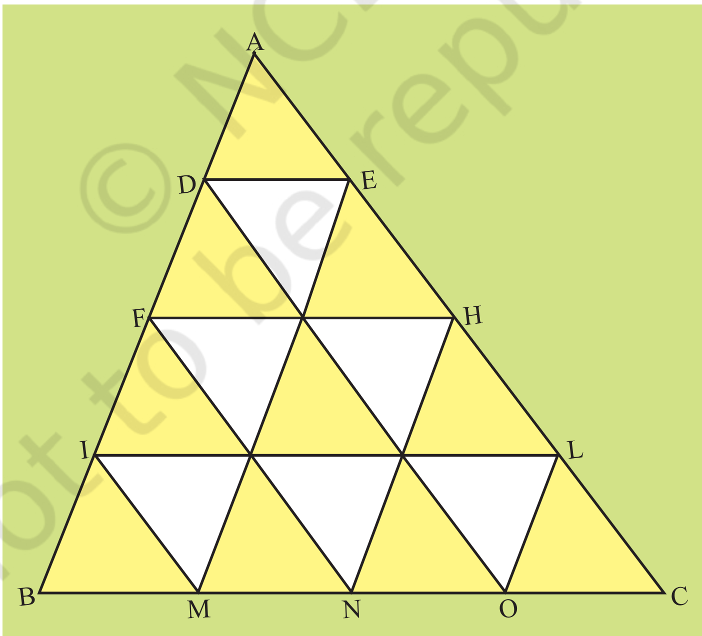

Fig. 1

5. Paste the triangle on the coloured paper.
6. ΔABC is divided into 16 congruent triangles [see Fig. 1].

Demonstration

1. ΔAFH contains 4 congruent triangles, with base FH = 2DE.
2. ΔAIL contains 9 congruent triangles, with base IL = 3 DE = $\frac{3}{2}$ FH.
3. In ΔABC, base BC = 4DE = 2 FH = $\frac{4}{3}$ IL.
4. $\Delta ADE \sim \Delta AFH \sim \Delta AIL \sim \Delta ABC$
5. $$\frac{\text{ar}(\Delta\ \text{AFH})}{\text{ar}(\Delta\ \text{ADE})} = \frac{4}{1} = \left(\frac{\text{FH}}{\text{DE}}\right)^2.$$

$$\frac{\text{ar}(\Delta\ \text{AIL})}{\text{ar}(\Delta\ \text{AFH})} = \frac{9}{4} = \left(\frac{\text{IL}}{\text{FH}}\right)^2.$$

$$\frac{\text{ar}(\Delta\text{ABC})}{\text{ar}(\Delta\text{AFH})} = \frac{16}{4} = \left(\frac{\text{BC}}{\text{FH}}\right)^2$$

Therefore, areas of similar triangles are proportional to ratio of the squares of their corresponding sides.

Observation

By actual measurement:

BC = ______________, IL = ______________, FH = ______________, DE = ______________.

Let area of ΔADE be 1 sq. unit. Then

$$\frac{\text{ar}(\Delta\text{ADE})}{\text{ar}(\Delta\text{AFH})} = \text{\_\_\_\_\_\_\_\_\_\_},\qquad \frac{\text{ar}(\Delta\text{ADE})}{\text{ar}(\Delta\text{AIL})} = \text{\_\_\_\_\_\_\_\_\_\_},$$

$$\frac{\text{ar}(\Delta\text{ADE})}{\text{ar}(\Delta\text{ABC})} = \text{\_\_\_\_\_\_\_\_\_\_},\qquad \left(\frac{\text{DE}}{\text{FH}}\right)^2 = \text{\_\_\_\_\_\_\_\_\_\_},$$

$$\left(\frac{\text{DE}}{\text{IL}}\right)^2 = \text{\_\_\_\_\_\_\_\_\_\_},\qquad \left(\frac{\text{DE}}{\text{BC}}\right)^2 = \text{\_\_\_\_\_\_\_\_\_\_}$$

which shows that ratio of the areas of similar triangles is ______ to the ratio of the squares of their corresponding sides.

APPLICATION

This result is useful in comparing the areas of two similar figures.

Activity 18

Objective

To verify experimentally that the ratio of the areas of two similar triangles is equal to the ratio of the squares of their corresponding sides.

Material Required

Coloured papers, geometry box, sketch pen, white paper, cardboard.

Method of Construction

1. Take a cardboard of a convenient size and paste a white paper on it.
2. Make a triangle (equilateral) on a coloured paper of side x units and cut it out [see Fig. 1]. Call it a unit triangle.
3. Make sufficient number of triangles congruent to the unit triangle using coloured papers.

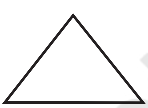

Fig. 1

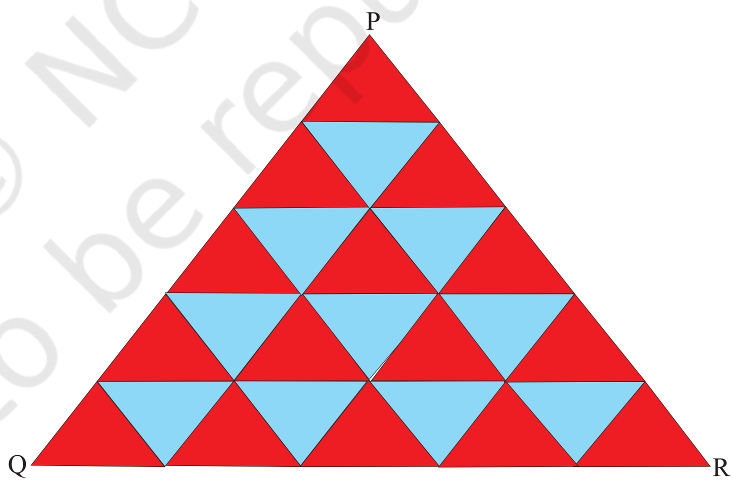

Fig. 2

4. Arrange and paste these triangles on the cardboard as shown in Fig. 2 and Fig. 3.

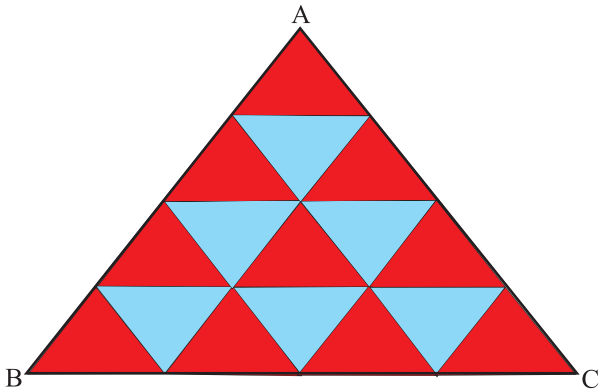

Fig. 3

Demonstration

ΔABC and ΔPQR are similar. Side BC of ΔABC = $(x + x + x + x)$ units = $4x$ units

Side QR of ΔPQR = $5x$ units

Ratio of the corresponding sides of ΔABC and ΔPQR is

$$\frac{BC}{QR} = \frac{4x}{5x} = \frac{4}{5}$$

Area of ΔABC = 16 unit triangles

Area of ΔPQR = 25 unit triangles

Ratio of the areas of ΔABC and ΔPQR = $\frac{16}{25} = \frac{4^2}{5^2}$ = Ratios of the square of corresponding sides of ΔABC and ΔPQR

Observation

By actual measurement:

x = ______. Area of the unit triangle [equilateral triangle in Fig. 1] =_____

Area of ΔABC = ______, Area of ΔPQR = ______

Side BC of ΔABC = ______, Side QR of ΔPQR = ______

BC^2 = ____________, QR^2 = ____________,

AB = ____________, AC = ____________,

PQ = ____________, PR = ____________,

AB^2 = ____________, AC^2 = ____________,

PQ^2 = ____________, PR^2 = ____________,

$$\frac{BC^2}{QR^2} = \text{\_\_\_\_\_\_\_\_\_\_\_\_},\qquad \frac{\text{Area of } \Delta\, ABC}{\text{Area of } \Delta\, PQR} = \text{\_\_\_\_\_\_\_\_\_\_\_\_}$$

$$\frac{\text{Area of } \Delta\, ABC}{\text{Area of } \Delta\, PQR} = \frac{BC^2}{-} = \left(\frac{AB}{-}\right)^2 = \left(\frac{-}{PR}\right)^2$$

Application

This result can be used for similar figures other than triangles also, which in turn helps in preparing maps for plots etc.

Note

This activity can be performed by taking any triangle as a unit triangle.

Activity 19

Objective

To draw a quadrilateral similar to a given quadrilateral as per given scale factor (less than 1).

Material Required

Chart paper (coloured and white), geometry box, cutter, eraser, drawing pins, glue, pins, sketch pens, tape.

Method of Construction

1. Take a cut-out of the given quadrilateral ABCD from a coloured chart paper and paste it on another chart paper [see Fig. 1].
2. Divide the base (here AB) of the quadrilateral ABCD internally in the ratio (given by scale factor) at P [see Fig. 2].
3. With the help of ruler (scale), join the diagonal AC of the quadrilateral ABCD.
4. From P, draw a line-segment PQ||BC, with the help of compasses (set squares or paper folding) meeting AC at R [see Fig. 3].

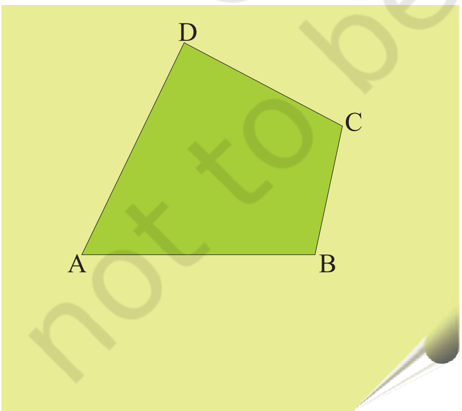

Fig. 1

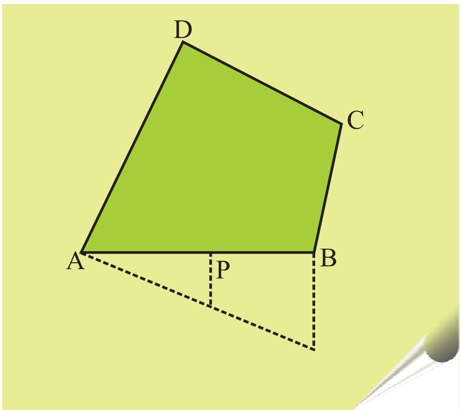

Fig. 2

5. From R, draw a line-segment RS parallel to CD using pair of compasses (set-squares/paper folding) meeting AD in S [see Fig. 4].
6. Colour the quadrilateral APRS using sketch pen.

   APRS is the required quadrilateral, similar to the given quadrilateral ABCD for given scale factor [see Fig. 5].

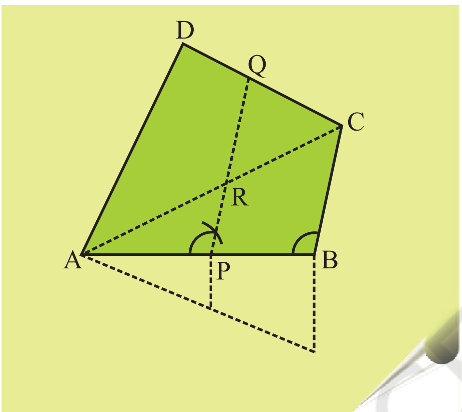

Fig. 3

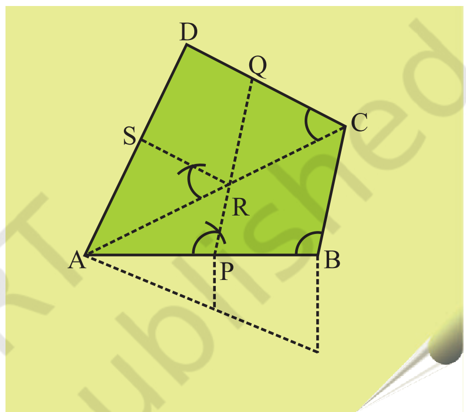

Fig. 4

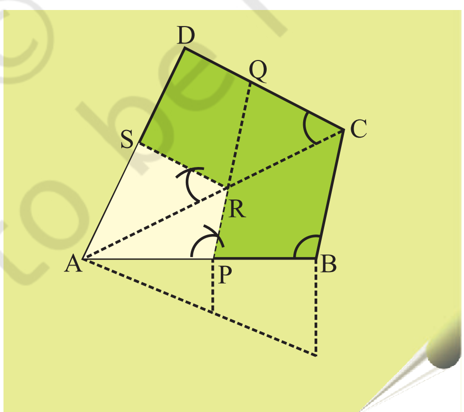

Fig. 5

Demonstration

1. In ΔABC, $PR \parallel BC$. Therefore, $\Delta APR \sim \Delta ABC$
2. In ΔACD, $RS \parallel CD$. Therefore, $\Delta ARS \sim \Delta ACD$
3. From Steps (1) and (2), quadrilateral APRS ~ quadrilateral ABCD

Observation

By actual measurement:

AB = _____, AP = _____, BC = _____,

PR = _____, CD = _____, RS = _____,

AD = _____, AS = _____

$\angle A$ = _____, $\angle B$ = _____, $\angle C$ = _____,

$\angle D$ = _____, $\angle P$ = _____, $\angle R$ = _____,

$\angle S$ = _____.

$\frac{AP}{AB}$ = _______, $\frac{PR}{BC}$ = _______, $\frac{RS}{CD}$ = _______, $\frac{AS}{AD}$ = _______,

$\angle A$ = Angle _____, $\angle P$ = Angle_____, $\angle R$ = Angle_____, $\angle S$ = Angle_____,

Hence, quadrilateral APRS and ABCD are _______.

Application

This activity can be used in day to day life in making pictures (photographs) of same object in different sizes.

Activity 20

Objective

To verify Pythagoras Theorem.

Material Required

Chart paper, glazed papers of different colours, geometry box, scissors, adhesive.

Method of Construction

1. Take a glazed paper and draw a right angled triangle whose base is 'b' units and perpendicular is 'a' units as shown in Fig. 1.
2. Take another glazed paper and draw a right angled triangle whose base is 'a' units and perpendicular is 'b' units as shown in Fig. 2.

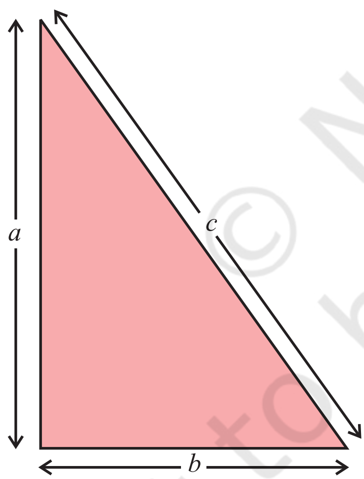

Fig. 1

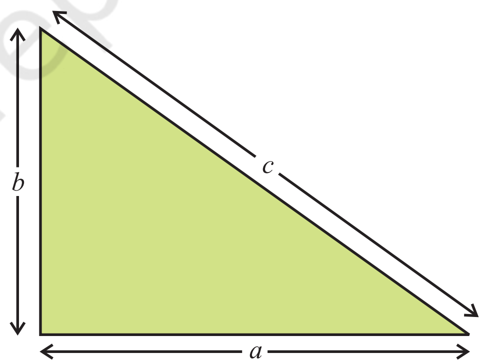

Fig. 2

3. Cut-out the two triangles and paste them on a chart paper in such a way that the bases of the two triangles make a straight line as shown in Fig. 3. Name the triangles as shown in the figure.

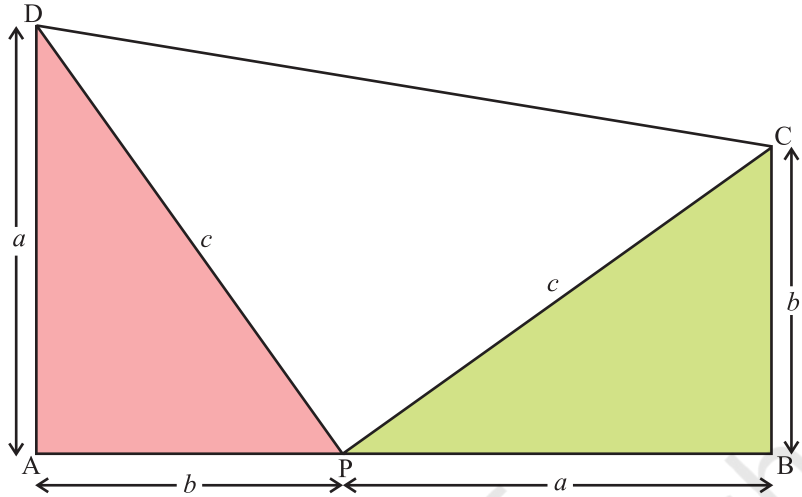

Fig. 3

4. Join CD.
5. ABCD is a trapezium.
6. The trapezium is divided into three triangles: APD, PBC and PCD.

Demonstration

1. ΔDPC is right angled at P.

2. Area of Δ APD $= \frac{1}{2} ba$ sq. units.

   Area of Δ PBC $= \frac{1}{2} ab$ sq. units.

   Area of Δ PCD $= \frac{1}{2} c^2$ sq. units.

3. Area of the trapezium ABCD = ar(ΔAPD) + ar(ΔPBC) + ar(ΔPCD)

So, $\frac{1}{2}(a+b)(a+b) = \left(\frac{1}{2}ab\right) + \left(\frac{1}{2}ab\right) + \frac{1}{2}c^2$

i.e., $(a+b)^2 = (ab + ab + c^2)$

i.e., $a^2 + b^2 + 2ab = (ab + ab + c^2)$

i.e., $a^2 + b^2 = c^2$

Hence, Pythagoras theorem is verified.

Observation

By actual measurement:

AP = _______, AD = _______, DP = _______,

BP = _______, BC = _______, PC = _______,

$AD^2 + AP^2$ = _______, DP^2 = _______,

$BP^2 + BC^2$ = _______, PC^2 = _______,

Thus, $a^2 + b^2$ = _______

Application

Whenever two, out of the three sides, of a right triangle are given, the third side can be found out by using Pythagoras theorem.